Abstract

This thesis consists of four papers in which we discuss various kinds of Lie bialgebra structures, their connection with solutions of the classical Yang-Baxter equation and explicit quantization.

In the first paper, we present the theory of rational solutions of the classical Yang-Baxter equation for a simple compact real Lie algebra $\mathbf{g}$. We prove that, up to gauge equivalence, any rational solution has the form $X(u,v) = \frac{\Omega}{u-v} + t_1 \wedge t_2 + ... + t_{2n-1} \wedge t_{2n}$, where Ω denotes the quadratic Casimir element of $\mathbf{g}$ and $\{t_i\}$ are linearly independent elements in a maximal torus $\mathbf{t}$ of $\mathbf{g}$. The quantization of these solutions is also emphasized.

In the second paper we investigate the rational solutions of the CYBE for $o(n)$ from the point of view of orders in the corresponding loop algebra. In the case of so-called singular vertices, we use the list of connected irreducible subgroups of $SO(n)$ locally transitive on the Grassmann manifolds IG_k^n of isotropic k-dimensional subspaces in $\mathbb{C}^n$, obtained by E. Vinberg and B. N. Kimel'fel'd. New arguments based on the analysis of the structure of the stationary subalgebra of a generic point allow us to find several rational solutions in $o(7)$, $o(8)$ and $o(12)$.

The third article is focused on some Lie bialgebra structures on parabolic subalgebras. Given a complex simple finite-dimensional Lie algebra $\mathbf{g}$ with fixed root system, there exists a so-called classical Drinfeld-Jimbo r-matrix, r. Consider any parabolic subalgebra $P_S \subseteq \mathbf{g}$ defined by a subset S of the set of simple roots. We prove that the Lie bialgebra structure on $\mathbf{g}$ defined by r can be restricted to P_S. Moreover, it turns out that the corresponding classical double $D(P_S)$ is isomorphic to $\mathbf{g} \oplus \mathbf{Red}(P_S)$, where $\mathbf{Red}(P_S)$ denotes the reductive part of P_S.

Finally, in the fourth article, we study classical twists of Lie bialgebra structures on the polynomial current algebra. We focus on the structures induced by so-called quasi-trigonometric solutions of the classical Yang-Baxter equation. We give complete classification for sl_2 and sl_3. For the sl_2 case we also emphasize quantization. We obtain a two-parameter twist of the quantum affine algebra and of the Yangian. Consequently, we determine the deformed quantum R matrices which correspond to quasi-trigonometric and rational solutions in sl_2.

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