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An Optimization Approach to Continuous Liability Management

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Abstract

Since the 70's both the volatility and level of interest rates have risen. This has lead to an increase in companies' interest rate risks. A stable income source is no longer a guarantee for financial success. To cope with this problem a more active portfolio management has to be employed.

Many tools used in liability management, like interest rate models, use historical data in order to describe the behavior of the market. This implies that a massive amount of financial data needs to be processed to enable sound decision making. Without the help of computers, this problem is difficult for humans to handle. Using optimization algorithms, many different parameters can be analyzed at the same time.

This thesis uses an optimization approach to solve the liability management problem. A method including liquidity risk and interest rate risk is developed based on the concept of linear programming. The usefulness of the method is investigated, using an implementation incorporating the expectations hypothesis for interest rate forecasting and a GARCH model for volatility forecasting.

The method developed in this thesis appears to be efficient in handling the large amount of data. The output from the method can be used as a sound recommendation if satisfactory interest rate forecasts are available. The expectations hypothesis though fails to meet this demand and should be replaced with other, more developed methods.

Sammanfattning

Sedan 70-talet har både räntornas volatilitet och nivå stigit. Detta har lett till en ökning av många företags ränterisker. En stabil inkomstkälla är inte längre en garanti för finansiell framgång. För att bemöta detta problem måste en mer aktiv portföljhantering användas.

Många av de verktyg som används för skuldhantering, som till exempel räntemodeller, använder historisk data för att förutsäga marknadens beteende. Detta antyder att en stor mängd finansiell data behöver behandlas för att möjliggöra rationellt beslutsfattande. Utan hjälp av datorer är detta problem svårt för människor att hantera. Med hjälp av optimeringsalgoritmer, kan många olika parametrar analyseras samtidigt.

Denna uppsats använder en optimeringsansats för att lösa skuldhanteringsproblemet. En metod som inkluderar likviditets- och ränterisk utvecklas, baserad på konceptet linjärprogrammering. Metodens användbarhet undersöks med hjälp av en implementation som innefattar Expectations hypothesis för att förutsäga räntor och en GARCH-modell för volatilitets-förutsägelser.

Metoden som presenteras i denna uppsats verkar vara effektiv i hanteringen av stora datamängder. Utdatan från metoden kan användas som en god rekommendation förutsatt att tillförlitliga ränteförutsägelser finns tillgängliga. Expectations hypothesis visar sig dock inte uppfylla detta krav och borde ersättas med andra, mer välutvecklade metoder.

Acknowledgments

We are specially greatful to Cecilia Bergendahl who is the Chief Financial Officer at Bostads AB Poseidon. She has helped us a lot, not only with the initial idea for this thesis, but also with inspiration and information during the semester. We would also like to thank our supervisor Stefan Sjögren. For his heroic efforts in trying to find time series in the finance lab, we thank Conny Overland.

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Chapter 1

Introduction

1.1 Background

Since the 70's both the volatility and level of interest rates have risen. This has lead to an increase in companies' interest rate risks [Bicksler and Chen, 1986]. A stable income source is no longer a guarantee for financial success. To cope with this problem a more active portfolio management has to be employed. This enables companies to change their strategy in a continuous manner and thereby adjust to changes in the market.

In liability management a number of factors can be considered. The most obvious factor is the current interest rates, which directly determine the cost of loans. However, to make better decisions, future interest rates should also be regarded in order to consider the reinvestment risk of debt. This is the risk that arises from the opportunity cost of issuing debt at lower interest rates in the future. Another factor is the degree of risk aversion for a specific company. This can be used for deciding the risk exposure, affordable to a company.

Many of the developed models for forecasting future interest rates and risk use historical data in order to describe the behavior of the market. To generate sufficiently good estimates, large amounts of data are often needed. This has lead to an increase in the use of computers for financial modeling. The rapid evolution of computational power has lead to that resource demanding optimization methods adopted from mathematical science can be used for financial applications. This enables methods to consider the large number of problems that occur in liability management.

1.2 Problem discussion

The problems involved in liability management require the CFO to make decisions based on many channels of information. To quantify this information is not easy and therefore requires some sort of model to interpret it. One of the most commonly used indicators of the market's expectations is the interest rates. These also affect the cost of debt directly and hence have the largest impact on the liability management strategy. Having this impact, the interest rates give rise to a number of problems in managing the liability portfolio.

The interest rates on different instruments depend on a number of factors. Apart from the current market climate, which affects the market as a whole, different derivatives differ in for example, time to maturity, liquidity and yield. The long-term interest rates are generally higher than the short-term ones. This is often explained by the liquidity premium theory, which states that investors demand an extra premium for binding their assets in long-term investments [Fabozzi, 2002].

When the interest rates are expected to rise, a company wants to have a liability portfolio consisting of instruments with a long time to maturity. The reason for this is that when the interest rate rises, a fixed interest for a long time will keep the interest costs down. On the other hand, if the interest rates are expected to fall, a fixed interest rate for a short period is preferred, since that will give an opportunity to restructure the portfolio at a lower interest rate.

Another perspective on the choice of time to maturity concerns the reinvestment risk of a derivative or a loan. On a loan with a long time to maturity at a fixed interest rate, there is a risk that the interest rates fall. Then the loan will be unnecessarily expensive, since a loan at a lower interest rate could be taken at this time. These relations are displayed in Figure 1.1. This position will also suffer from the risk that a high inflation rate decreases remarkably. In that scenario, the loan will also be expensive, since a large part of the loan was, at the issuing date, expected to have a low real value due to the high inflation rate.

	Interest rates rise	Interest rates fall
Short duration	Debt issued at high interest rate -> high interest costs	Debt issued at low interest rate -> low interest costs
Long duration	Low interest rate retained -> low interest costs	High interest rate retainted -> high interest costs

Figure 1.1: Illustration of the reinvestment risk of debt

A loan with a short time to maturity can also be interpreted as a reinvestment risk. As apposed to a loan with a long time to maturity, one with a short time to maturity will only have its fixed rate for a short time. After that, a new loan will be needed, which will be taken at the future interest rate, which may be higher than the current.

The risk in a forecasted interest rate is often measured using the volatility. Mathematical models like the GARCH model [Fabozzi, 2002] can be used to forecast and analyze volatility. These models require massive amounts of data and calculations to generate reliable forecasts, which makes them infeasible for hand calculations.

The choice of time to maturity does not only depend on the cost of interest. Analogously to the common praxis that projects should be discounted at their opportunity cost of capital [Grinblatt and Titman, 2002], researchers argue that the time to maturity of the loans used for financing a project, often coincides with the expected lifetime of the project according to the preferred habitat theory [Ho and Lee, 2004].

As the previous discussion shows, the choice of instruments in the liability portfolio is non-trivial. The debt can be structured in an almost infinite number of ways. All of these have different costs and react differently to changes in market interest rates, when the debt is renewed. Hence, the structure of the portfolio may have a significant impact on the cost of debt and on the risk of debt. If long debts are issued at a particular time, when interest rates are expected to rise, the option to issue short debt if the interest rates really fall, will be limited. The infinite number of debt structures, together with the limitations just explained and the uncertainty in interest rate forecasts makes it difficult for a human to make appropriate decisions. Using computers, optimization algorithms can be used to help in the analysis of many different parameters at the same time. Advanced models may though require time consuming computations.

1.3 Purpose

The problem formulation points out a number of difficulties in predicting the effects, of different portfolios, on the cost of liabilities. The infinite number of structures for the debt portfolio over time, together with the massive amount of data needed to predict future interest rates and volatility, justify the use of optimization based models.

The decisions on the instruments issued today, will affect the decisions of instruments issued at a later date. Therefore, a continuous management of the liability portfolio that considers future effects of today's decisions is needed to minimize interest costs. A good method for liability management should thus be able to continuously manage and update the portfolio from historical data. To help the CFO[1] in the decision making, it is also essential that the output can serve as an easily interpretable basis for decisions. This also requires that the output can be found in feasible time.

The purpose of this thesis can be stated as:

- To develop an optimization based method for construction and continuous management of liability portfolios using historical data.

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[1] Chief Financial Officer

Chapter 2

Method

For research to be useful it must fulfill certain criteria. If these base criteria are not met, the research will be useless to the community, since it will not add any valid knowledge. Arbnor and Bjerke [Arbnor and Bjerke, 1994] lists seven criteria for scientifically valid research.

- The purpose of the research and the problem should be clearly stated, defined and limited.
- The research procedure used, should be described as thoroughly as possible to enable other researchers to repeat and validate the research.
- The organization of the research should be planned in detail, so that as objective results as possible can be achieved.
- The researcher should report perceived flaws in his procedures and estimate consequences as sincere as possible.
- The analysis made should be adequate enough to evaluate significances and the appropriate techniques should be used.
- Drawn conclusions should be limited to the data used in the analysis and should also be validated by the same data.
- The results will be perceived as more valid if they originate from a researcher with a good scientific reputation, experience and who is known for his integrity.

These criteria were used as a basis for the research conducted and described in this thesis. The problem and purpose of the research are stated in Sections 1.2 and 1.3. The research procedure is further described in Section 2.1 and the simulation procedure is described in Appendix A, to enable other researchers to repeat and validate the research.

2.1 Research procedure

Research can be divided into different phases. By following these different phases, the process will be structured and planned in detail, which will facilitate scientific research. A simple and structured methodology can be divided into the following steps [Ackoff, 1962].

1. Formulation of research area, problem formulation and method
2. Initial information gathering
3. Development of models
4. Information gathering
5. Choice of method
6. Test and verification of results from models
7. Implementation of results

A problem formulation must be exciting and productive [Holme and Solvang, 1997]. Most exciting problems can be found in practical applications. A study concerning a practical topic will often lead to results that can be applied and used in practice. The purpose of this thesis was formulated after an extensive search for interesting practical problems at different companies. The research area was finally chosen in cooperation with the CFO[1] of Bostads AB Poseidon.

During the initial information gathering recent research articles were gathered from databases like JSTOR and S-WOPEC. A number of books and research articles from libraries were also used as a basis for the problem formulation and purpose. To gain knowledge of practical liability management, qualitative information from companies and their operations is important. In order to get that kind of insight, the contact with Bostads AB Poseidon served as a source of information. This company has a large debt portfolio and is highly devoted to liability management[2] and could therefore provide important views on portfolio management in practice. Information was gathered primarily by interviews with Cecilia Bergendahl, Chief Financial Officer at Bostads AB Poseidon.

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[1] Cecilia Bergendahl

Using gathered information, a model was developed with an object-oriented approach. This enabled continuous testing of verification of subfunctions. The tools used in the development process are described in Section 2.2. The model is described in Chapter 3.

Additional data was gathered from the same sources as in the initial information gathering as well as from Reuters and the Federal Reserve. The quantitative financial data used is described in Section 2.3.

The testing of a developed method is crucial for the validation. Chapter 4 describes and discusses chosen methods for validation and performance measures. Appendix A includes the simulation procedures used and the results are stated in Chapter 5.

2.2 Tools

For the implementation of the method presented in Section 3.2, different mathematical software were examined. MATLAB from Mathworks was chosen for its superior functionality. Its handling of matrices and simple syntax enables high speed development. MATLAB includes highly developed functions for analysis of financial time series in the Financial toolbox. It also comprises a GARCH toolbox, which can be used for parameter calibration of GARCH models as well as forecasting of volatility. The Optimization toolbox includes powerful functions for both linear and non-linear optimization.

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[2] Bostads AB Poseidon has received Standard & Poor:s AAA-rating in 2002 [Bergendahl, 2004a]

Since all these functions are included in a single program, the import and export procedures for data are significantly simplified in comparison to using different programs.

2.3 Data

The data used in the simulations are yields of US Treasury securities collected from Federal Reserve [U.S. treasury securities]. The reason for using that data is the availability of long and continuous time series for different instruments. Treasury securities are often used as a reference for market interest rates and the term “interest rates” will be used in this meaning throughout the thesis. The rates used are daily yields from the the time period from January 1, 1982 to November 30, 2004. A problem encountered when using those time series was that Federal Reserve did not issue 20-year bonds between January 1, 1987 and September 30, 1993. Still, it would be favorable to be able to test the model over long time periods and it was decided that the missing values should be extracted from the yield curve in those points by interpolation. That approximation should be good enough for the purpose of this thesis.

Chapter 3

Method for modeling liability management

This chapter covers the information gathered from interviews and the development of a method for modeling liability management. Section 3.1 describes a practical approach to liability management, which is later used as a basis for the development of the method in Sections 3.2-3.4.

In the rest of this thesis, the different terms *method* and *model* will be used frequently. It is important to distinguish between these two terms. *Method* will be used to refere to the method used for liability management. This incorporates all parts of Figure 3.1, which shows a block scheme of the developed method. *Model* referes to the core part of the method, which includes the optimization block in Figure 3.1.

3.1 Practical liability management

To get an insight into how practical liability management can be implemented in practice, Bostads AB Poseidon was studied. They have a well developed liability management and have received Standard & Poor:s AAA-rating [Bergendahl, 2004a], which makes them a suitable company for studying.

Poseidon's finances are continuously supervised by looking at current interest rate exposure and hedging against unexpected changes in the term structure [Bergendahl, 2004b]. The most common financial derivatives used for hedging are interest rate swaps, CAPs and forward rate agreements. Forecasts of the future are made by looking at yield curves and macroeconomic trends. In order to prevent sudden liquidity problems, a maximum of 35% of the liabilities are allowed to expire during a year. Risk is mainly measured by looking at the duration of the portfolio. To avoid high risk taking, there is a company policy stating that the duration should be at least 2 years.

Despite that the length of the company's investments often reach above 50 years, their average duration is slightly above 2 years. This is an indication on their belief that there is money to be saved by using an active portfolio management strategy, instead of matching investment cash flows according to the market segmentation theory.

3.2 Method for liability management

As stated in the problem formulation, large amounts of data need to be processed to make good decisions in liability management. Modeling the system in a computer can help in generating data with reduced complexity as a decision basis. In a good method, the system model should imitate the system as closely as possible without being too complex to implement. The method should also be able to continuously update the output to incorporate both daily and historical data. This enables the model to make decisions based on all available data as well as on previous decisions. The rest of this chapter describes the developed method and specifically the optimization model.

A block scheme of the method is shown in Figure 3.1. The input to the method is an existing portfolio, which may be empty if the method should create an initial portfolio. This enables the optimization model to make decisions on the basis of previous decisions and update the portfolio continuously. The time that constitutes one period can be chosen freely in correspondence with how often a company reviews its portfolio.

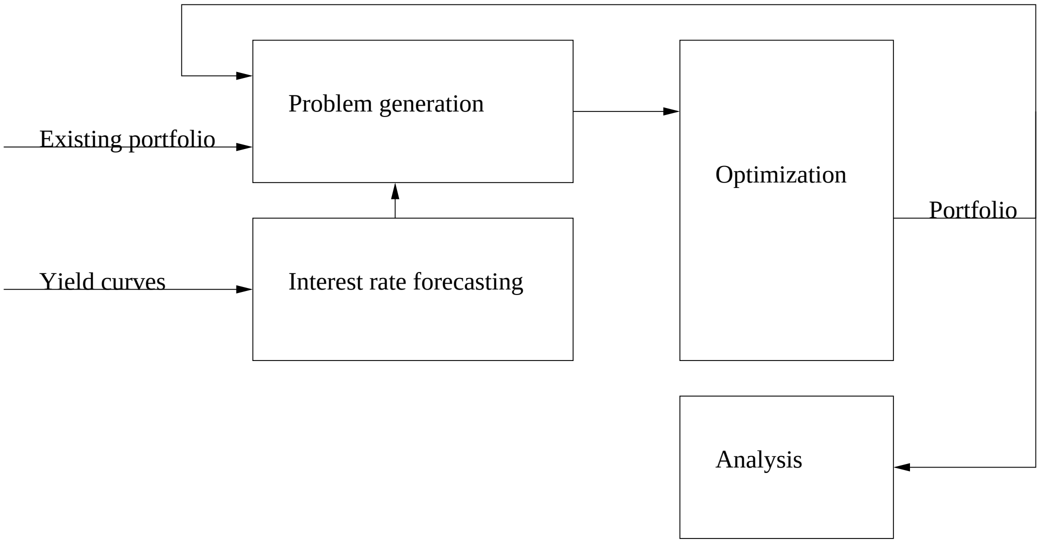

Figure 3.1: Block scheme of the method

The current and historic yield curves are used as input to the forecasting block, which forecasts interest rates from historical data. Using the forecasted interest rates and the initial portfolio, an optimization problem is generated. The details on this are described in the following sections. From the optimization model, a new portfolio is generated, which is used both for analysis and also as input to the method when new data arrives. Before an example of the usage of the method is given, an example of a portfolio over time is shown.

Instrument maturity [periods]	Weight				
	Period 1	Period 2	Period 3	Period 4	Period 5
1	0.4	0	0.6	0	0
2	0.6	.2	0	0	0.1
3	0	0.1	0	0	0
4	0	0.1	0	0.8	0

Table 3.1: Portfolio over time for three periods and four instruments. The weights constitute the weight of issued value per instrument in that period.

In Table 3.1, a portfolio over time is shown. Assume that one period constitutes one month. The first month, 0.4 is issued in instrument 1 and 0.6 in instrument 2. In the second period, instrument 1 from the first period matures and this weight is distributed over instruments 2, 3 and 4. Since the maturity of instrument 2 is 2, it matures in period three and the same weight is issued in instrument 1. In period four, 0.6 matures from instrument 1 and 0.2 from instrument 2. The whole weight is issued in instrument 4. In period five, 0.1 matures of instrument 3 from period two, which is issued in instrument 2.

Example 3.1 shows a typical usage of the method in continuous liability management.

Example 3.1 *Example on the usage of the method on a monthly basis*

Jan

1. *Historical yield curves, yield curves for January and the current portfolio are used as input to the method.*
2. *The optimization model outputs a portfolio over time that consists of a portfolio for each future time within a specified interval. An example of a portfolio over time is shown in Table 3.1.*
3. *The portfolio over time is used as a basis for decisions on which instruments to issue in January. That is, the decision is based on the post, in the portfolio over time, that refers to the current period.*

Feb

1. *Historical yield curves, yield curves for February and the portfolio from January are used as input to the model.*
2. *The optimization model outputs a portfolio over time that consists of a portfolio for each future time within a specified interval.*
3. *The portfolio over time is used as a basis for decisions on which instruments to issue in February.*

Mar *Continue in the same pattern as in the previous month.*

Using a non-empty initial portfolio, the model can be used for a company with an existing liability portfolio. The choice of initial portfolio, or in this case the existing portfolio, has a huge impact on the performance of the method. In the extreme case, if an initial portfolio is chosen which contains 100 percent liabilities with a long time to maturity, for example 20 years, the method may seem useless. In this case, no changes in the portfolio are allowed in the first 20 years. The problem with long time to maturity on the initial portfolio could be solved using derivatives. This is though outside the scope of this thesis. In a scenario which is not this extreme, the method will be limited to restructuring only a part of the portfolio. The ultimate starting point for the method is when it can freely choose the initial portfolio.

3.3 Assumptions and limitations

The method presented in this chapter is for simplicity restricted to a world of a limited number of financial instruments and no arbitrage, which implies that the problem of finding under- or overvalued instruments is not covered. Since there are an almost infinite number of instruments on the market, modeling all of these would be infeasible. The method is though general in the sense that it easily can be extended to include virtually any instrument or derivative. The instruments used in the implementation were chosen to serve as a reference to the market rates. The size of the debt is assumed to be constant. To reduce the complexity of the model, it is assumed that no repurchases of bonds are possible and that there are no transaction costs. Transaction costs could though be included in the cost function.

According to the Expectations Hypothesis, the interest rates on the market represent the market's expectations on the future interest rates [Ho and Lee, 2004]. Later research by for example [Cox et al., 1981] has though criticized this hypothesis, arguing that other factors[1] are not incorporated in the hypothesis. The basis of the hypothesis, which implies that future interest rates can be (approximately) forecasted from historical data, is still valid [Campbell, 1986]

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[1] These could for example include the liquidity preference

3.4 A linear programming model

The optimization problem of the portfolio decision, is modeled as a linear program. The reason for this is simply that a portfolio is a linear combination of all available instruments and that no parameters with other characteristics are introduced. In other words, the function to be minimized and the constraints are all linear equation systems.

There are two operation modes for the programming model. One is to create an optimal portfolio and the other mode is to iteratively maintain a given portfolio over time, by using all available knowledge of interest rates. At each iteration of the method, the optimization model chooses the portfolio over time which minimizes the cost of the portfolio subject to equality[2] and inequality[3] constraints.

3.4.1 Linear programming

Many real life problems can be modeled as linear functions and optimized using linear programming methods. Linear models are used in many different branches of science. A non-linear problem can often be linearized to yield a linear problem, which approximates the non-linear problem. By such methods, complicated problems can be approximately solved in feasible time. This section describes the mathematical background needed for the following model development.

A linear optimization problem has the following standard form [Nash and Sofer, 1996].

$$\text{minimize} \quad z = c^T x \tag{3.1}$$

$$\text{subject to} \quad Ax = b \tag{3.2}$$

$$x \geq 0 \tag{3.3}$$

where z is called the objective function, A is an m×n constraint matrix, x is a state vector of length n, c is a vector of length n and b is a non-negative vector of length m. All linear programs can be converted to this standard form. Methods for these conversions can be found in [Nash and Sofer, 1996] and will not be further described here. In some implementations of software for solving a linear program, inequalities can also be used in addition to equalities.

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[2] See Section 3.4.5

[3] See Section 3.4.6

The linear program

$$\begin{aligned} \text{minimize} \quad & z = 5x_1 - 5x_2 + 3x_3 && (3.4)\\ \text{subject to} \quad & 3x_1 - 2x_2 + 7x_3 = 7 && (3.5)\\ & 8x_1 + 6x_2 + 6x_3 = 5 && (3.6)\\ & x_1, x_2, x_3 \geq 0 && (3.7) \end{aligned}$$

will in standard form have the following matrices and vectors.

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \quad c = \begin{pmatrix} 5 \\ -5 \\ 3 \end{pmatrix} \quad A = \begin{pmatrix} 3 & -2 & 7 \\ 8 & 6 & 6 \end{pmatrix} \quad b = \begin{pmatrix} 7 \\ 5 \end{pmatrix} \tag{3.8}$$

In Figure 3.2, the bounds of a linear program can be seen. The borders of the figure define the constraints and all points inside this figure are feasible points. The corners of the figure are called extreme points. A basic solution is defined algebraically using the standard form of the constraints according to the following definition [Nash and Sofer, 1996].

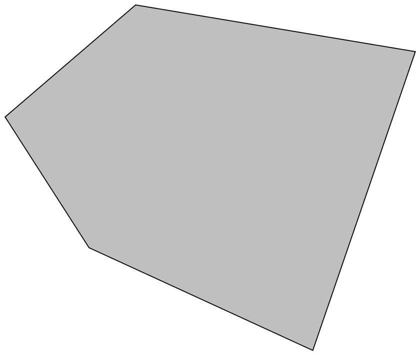

Figure 3.2: Bounds and extreme points of a linear program

Definition 1 *A point x is a* basic solution *if*

1. *x satisfies the equality constraints of the linear program*
2. *the columns of the constraint matrix corresponding to the non-zero components are linearly independent* [4]

Definition 2 *A point x is an* optimal *basic* feasible *solution if*

1. *x is a basic solution*
2. $x \geq 0$
3. *x is optimal for the linear program*

For convex[5] functions, Theorem 1 holds [Nash and Sofer, 1996].

Theorem 1 *If a convex function has a local minmum x^*, this is also a global minimum for the function*

3.4.1.1 Simplex algorithm

The simplex algorithm was developed in the 1940's. It is a method, commonly used to solve linear programs in many different areas. The method was early applied to economic problems which explains the use of terms like shadow price and cost function. The simplex algorithm in an iterative algorithm, which jumps between the extreme points along the constraints until an optimal basic feasible solution is found. Details of the simplex algorithm can be found in [Nash and Sofer, 1996] and will not be further described here. The complexity of the simplex algorithm is high (possibly as high as $\mathrm{O}\begin{pmatrix} n \\ m \end{pmatrix}$)[6], which decreases its usability for large-scale problems.

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[4] See a book in linear algebra for a definition of linearly independent

[5] See [Nash and Sofer, 1996] for a definition of convex functions

[6] O denotes big ordo, which is a measure of complexity of an algorithm. In an algorithm of $\mathrm{O}(n^2)$, the number of operations required to solve a problem increases by the square of an increase in the number of inputs.

3.4.1.2 Interior point algorithms

Interior point methods are a class of algorithms which generate points in the interior of the feasible region, hence the name. These methods emerge from the barrier methods, which use a barrier to stay inside the feasible region. The idea is that a penalty term is added to the objective function, which increases dramatically when close to a constraint. Current algorithms have complexity of $O(\sqrt{n}L)$ (where n is the number of variables and L is the length of the input - the number of bits used to represent the problem data), which makes them much more suitable for large-scale problems than the simplex algorithm. The downside of the interior point methods is that they do not find basic feasible solutions. The solutions are though, with well conditioned problems, close to the basic feasible solutions.

3.4.2 State variables

A linear program is formulated by specifying the structure and contents of the matrices and vectors described in Section 3.4.1. The rest of this chapter is dedicated to the formulation of the optimization model.

The first step in building the linear program is specifying the state vector x. It contains the weights of all instruments at all times t in the time period specified and is structured as in Equation 3.9.

$$x = \begin{pmatrix} x_{1,1} & x_{2,1} & \dots & x_{M,1} & x_{1,2} & x_{2,2} & \dots & x_{M,2} & \dots & x_{M,N} \end{pmatrix}^T \tag{3.9}$$

where M is the number of instruments and N is the length of the time period.

In the case where there are existing liabilities, new instruments are defined with the corresponding time to maturity to match those liabilities. These are placed at the beginning of the vector and are handled separately. The reason for this is that if new instruments would be added to extend the original problem, this would lead to an unnecessary increase in complexity. The size of the matrices in the optimization problem would increase by roughly $(NK)^2$, where K equals the number of added instruments.

3.4.3 Cost function

The cost function is a linear function of the weights in the portfolio. These are specified using the state vector x as formulated in Section 3.4.2. This section describes the cost function, that is minimized in the optimization.

Definition 3 *Let P, I and w(i,p) be defined as*

$P = \{\textit{all periods}\}$

$I = \{\textit{all instruments}\}$

$w(i,p) = \{\textit{weight of instrument } i \in I \textit{ in period } p \in P\}$

Assume that a one year zero coupon bond with a face value of $f = \$110$ is sold in year 0 for $p = \$100$. The yield y of the bond can be calculated from Equation 3.11.

$$p = \frac{f}{1+y} \tag{3.10}$$

$$100 = \frac{110}{1+y} \tag{3.11}$$

The cost can then be calculated as

$$c = \frac{f-p}{1+r} = \frac{p(1+y)-p}{1+d} = \frac{py}{1+d} = \frac{100y}{1+d} \tag{3.12}$$

where d is the appropriate discount rate. Since the company is assumed to keep a constant debt, the appropriate discount rate is not dependent on the company's activities. Instead the risk free rate is used. If the bond is a treasury bond, the yield of this bond could be used as an approximation the risk free rate. Hence, $d = y$ and substituting this into Equation 3.12, the cost is

$$c = \frac{p((1+y)-1)}{1+y} \tag{3.13}$$

A similar discussion for a two year bond issued year 0, gives a cost of

$$c = \frac{p((1+y)^2-1)}{(1+y)^2} \tag{3.14}$$

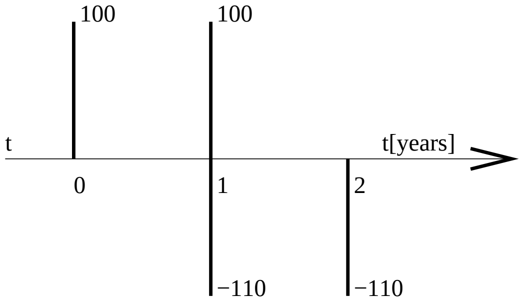

Figure 3.3: One year bonds issued at time zero and one

Definition 4 *Let* $y_{x,t}$ *denote the yield of an x year bond issued at time t*

Using the notation from Definition 4, the cost of issuing two consecutive one year bonds as in Figure 3.3 can be calculated as in Equation 3.15

$$c = \frac{p((1+y_{1,0})-1)}{(1+y_{1,0})} + \frac{p((1+y_{1,1})-1)}{(1+y_{1,1})(1+y_{1,0})} \tag{3.15}$$

Generalizing this, the total cost of a portfolio over time can be calculated as

$$c = \sum_{i \in I, p \in P} \frac{w(i,p)((1+y_{(t_m-t_i),t_i})^{t_m-t_i}-1)}{(1+y_{(t_m-t_i),t_i})^{t_m-t_i}(1+y_{t_i,1})^{t_i}} \tag{3.16}$$

where t_m is the maturity year and t_i is the issuing year. To enable decision making every month instead of every year, the issuing year and maturity year can be expressed in months instead.

Definition 5 *Let T denote the end of the simulated period*

The coefficients in the cost function have to constitute the total cost of an instrument as opposed to the yearly cost. This generates problems close to the end of the period. If an instrument's maturity date exceeds T, it is not fair to directly compare its cost with another instrument, with a shorter time to maturity, issued at the same time. The long instrument will carry a higher total cost, without the inherit gain from the long time to maturity. This issue is solved as follows.

If the expiration time exceeds T for a single element in the sum in Equation 3.16:

1. Use the yield of a $T - t_i$ month bond at time t_i instead of $y_{(t_m - t_i),t_i}$ in Equation 3.16.
2. Use $T - t_i$ instead of $t_m - t_i$ in Equation 3.16.

Note that all times are given in months, and the monthly yields are interpolated. The method used for interpolation, which is specific for the implementation, is specified in Section 4.3.1.1. The discounting was done using the discrete time method. Continuous discounting could also be used with negligible differences in the result.

The discounting of cash flows to net present value, introduces the effect that distant cash flows have less significance in the optimization. This is an important effect, which will make the model favor a decrease in near future costs over a decrease in distant costs. This effect is favorable, since distant, future interest rates are harder to predict accurately.

3.4.4 Cost of volatility

To this point, no consideration of the risk involved when managing a debt portfolio has been included in the model. Therefore, a parameter corresponding to the cost of the uncertainty in interest rate forecasting is incorporated. Just adding the risk parameter to the cost function is not correct, because companies have different risk policies. To be used properly, a degree of risk aversion of a company, must be considered. This can be achieved by multiplying the risk with a constant stating this risk aversion (or risk taking). The cost function including risk, can then be expressed as

$$c_{t,total} = \gamma_t \sigma_t^2 + c_t \tag{3.17}$$

where γ_t is the constant defining the degree of risk aversion, σ_t^2 is the estimated risk and c_t is the cost described in Section 3.4.3. This will render a first order approximation to the level of impact that risk has on the cost for a certain amount of risk taking.

3.4.5 Equality Constraints

The sum of the weights of the individual instruments issued in one period can obviously at no time exceed the total portfolio weight, 1. Hence, the sum of the weights in one period is less than or equal to one. Since the size of debt is assumed to be constant, new debt has to be immediately issued to cover maturing debt. Therefore the sum of issued debt and old debt in a period has to be larger than or equal to one. These two inequalities can be written using the following equality

$$\sum_{i \in I} w(i,p) = 1, \quad \forall p \in P \tag{3.18}$$

which is the the first equality constraint.

In accordance with the previous discussion, new debt has to be issued to cover maturing debt. Therefore, the sum of maturing debt, which can be seen as a negative issuing, and issued debt in a period have to equal zero.

Definition 6 *Let m(i,p) and n(i,p) be defined as*

m(i,p) $= \{$*weights of instrument* $i \in I$ *that mature in period* $p \in P\}$

n(i,p) $= \{$*weights issued (new) debt of instrument* $i \in I$ *in period* $p \in P\}$

The second equality constraint can be written as:

$$\sum_{i \in P} m(i,p) + n(i,p) = 0, \quad \forall p \in P \tag{3.19}$$

The initial portfolio, from the input, is also defined using equality constraints.

3.4.6 Inequality Constraint

To bring more reality into the model, a constraint ξ, defining the maximum percentage of the portfolio, allowed to expire during the nearest year, as described in Section 3.1, is introduced.

Definition 7 *Let P(t,y) be defined as*

P(t,y) = {the y nearest following periods p ∈ P starting at time t}

Including the constraint in the model the inequality can be written as

$$\sum_{p \in P(t,y)} m(i,p) \leq \xi, \quad \forall P(t,y) \in P \tag{3.20}$$

This inequality will have significant impact on the outcome of the model. Over some time periods where it may be optimal to roll over the shortest possible instrument, the solution of the optimization problem will suggest that longer instruments should be used. Hence, the model considers the risk involved when ”putting all eggs in the same basket” and avoids it by spreading the portfolio over more bonds and of different maturities.

Chapter 4

Method validation and implementation

4.1 Method validation

It is important to keep in mind that the validity of the model does not incorporate the performance of implementation-specific details, like the risk measure and interest forecasting method. These are just choices of methods for preprocessing historical data, *before* it is sent to the optimization model. Instead, it is the method incorporating all parts, not necessarily choosing these specific details that is the main focus of this thesis. The central part of the method, which is not replaceable, is the optimization *model* which is thus most important in the validation.

The first part of the model to validate is the single optimization step. Since the stated optimization model is linear, it is also convex [Nash and Sofer, 1996]. A local minimum for a convex problem is also a global minimum according to Theorem 1. Hence, it can be assumed that the found solution minimizes the cost in each optimization.

However, because each optimization step only considers a limited portion of the future, the total cost over a longer period will not necessarily be minimal. When a second optimization is performed it will consider new data at the end and lose data in the beginning since time has moved forward. Hence, the data is not exactly the same and the optimization can find a way of restructuring the portfolio just a time period later. Then it may not be possible to do so because no bonds are expiring. Because of the difficulties involved in predicting interest rates in the distant future, it is unrealistic to consider long time horizons in each optimization step. A time horizon much longer than the length of the instruments is also questionable, since all instruments will have matured during that time. Hence, if it is assumed that the model minimizes the cost in each optimization step, then it can be argued that the total cost over time is minimized in a reasonable sense.

4.2 Performance measures

When analyzing the results from simulations, the comparison criteria are of high importance for drawing accurate conclusions. Badly suited criteria can lead to inaccurate or even completely wrong conclusions. The single most important measure is the cost of the portfolio, since this is what the method is minimizing. All costs are measured at their net present value, but as we will see in Section 4.2.2.1, even this standardized measure can be a bit ambiguous.

A measure often used in practice to classify a portfolio is the duration of the portfolio. The pros of using duration for classification is that it is easy to understand, and that it explains a lot of the behavior of a portfolio. A long duration will for example be desired when interest rates are expected to rise. The cons of using duration as a classification measure is that it does not capture the behavior of a portfolio completely. For example, a portfolio where half consists of four year bonds and half consists of two year bonds, will have the same duration as a portfolio completely consisting of three year bonds. These two portfolios will react differently to shifts in the yield curve and changes in the convexity[1] of the yield curve. To be consistent with practically used methods, duration was used as the basic classification measure. Since the interesting approach in this thesis is the change of a portfolio over time, the duration was also measured over time.

The comparison of durations over time is non trivial. Delays in the results cause problems in finding a fair measure which captures the connection between duration and cost. In the following sections, different performance measures are discussed.

---

[1] The second derivative of the yield curve, which defines its curvature

4.2.1 Optimal portfolio

The aim of the method described in Section 3.2 is to minimize the cost of a portfolio over time. This implies that the optimization will be able to find a portfolio with minimum cost over a time interval, given that all interest rates are deterministic. This portfolio is necessary to perform comparisons of simulation results. It can be used to measure the deviations of the simulation results in terms of, for example, cost and duration. The definition of an optimal portfolio is

Definition 8 *Optimal portfolio*
An optimal portfolio is a portfolio which minimizes the cost of debt over some predetermined interval.

4.2.2 Portfolio comparison

The comparison of portfolios is an important topic to cover since it acts as the foundation to build the evaluation of the model upon. In this section, the procedures and most important parameters for comparison are discussed in order to facilitate the performance evaluation in Chapter 5.

4.2.2.1 Comparing cost

The performances of two portfolios are compared mainly using the cost over a specified time interval. To find a procedure to use for this action is not as trivial as it may seem. The time value of money must be considered in some way, but discounting all cash flows to the same date will make distant interest rate expenditures insignificant. Hence, this may not be a fair measure when a minimization of the interest costs over an interval is sought. This leads to the conclusion that another discounting procedure could be considered as well. Instead, the cash flows can be discounted to the same date as the corresponding bond was issued and these values are then summed. This method of comparing costs may be a more fair way of measuring the performance of the model, since the cost function[2] minimizes these values in each optimization step. In order to consider multiple ways of comparing costs, both of these methods were used in the analysis.

4.2.2.2 Comparing duration

Deviations between different measurements are often analyzed using the *mean square error(MSE)* which is calculated using Equation 4.1

$$\frac{1}{N}\sum_{n=1}^{N}||x(n)-\hat{x}(n)||^2 \tag{4.1}$$

where $\hat{x}(n)$ is the measured value of $x(n)$ and N is the number of observations. The benefit of using this method instead of just taking the mean of the errors, is that positive and negative errors will not cancel out. Using the MSE for comparison between results from the model proposed in this thesis would though generate unfair results. This is because the MSE method does not consider delays between the data compared.

Example 4.1 *Assume that two series A and B are to be compared using MSE. The series A and B are defined as in equation 4.3.*

$$\begin{aligned} A &= (0,0,10,10,10,0\ ,0,0) && (4.2)\\ B &= (0,0,0\ ,10,10,10,0,0) && (4.3) \end{aligned}$$

Comparing the two series A and B we get an MSE of:

$$MSE=\frac{1}{8}(10^2+10^2)=25 \tag{4.4}$$

The two series are in fact the same, with the only difference that the second series is delayed by one index. Still, the error is quite large.

---

[2] See 3.4.3

These types of delays will occur often in the analysis of the model proposed in this thesis. Assume that the model is used for continuous portfolio management and that this is compared to the optimal portfolio. Further assume that an instrument is taken with a time to maturity of 1 month in continuous case and that the optimal portfolio at the same time chooses an instrument with a time to maturity of 2 months. A sudden expected increase in interest rates will lead to that an instrument with a long time to maturity is preferred. Both models will choose this as soon as possible. The second model will though not be able to change instrument until one month after the first one, leading to the effects discussed previously.

To solve this problem, we propose a method which we choose to call the *floating mean square error (FMSE).* In this method, the errors are calculated from a floating mean before they are squared which smooths abrupt transitions in the model. This will decrease the impact of short delays on the analysis. The floating mean square error is defined as.

Definition 9 *Floating mean square error (FMSE)*

$$FMSE = \frac{1}{N}\sum_{n=1}^{N}\left|\left|\frac{1}{2M+1}\sum_{m=-M}^{M}x(n+m) - \frac{1}{2M+1}\sum_{m=-M}^{M}\hat{x}(n+m)\right|\right|^2 \tag{4.5}$$

Where $M \in \mathbb{N}$ is the smoothing windows size, and N is the number of observations. We assume that $x(n)$ and $\hat{x}(n)$ are zero for all $n < 0$ and $N < n$.

What remains is to choose an appropriate value for M. If M is chosen too low, the delays will not be smoothed out. If M on the other hand is chosen too large, the resolution of the error calculations will reduce significantly, leading to an inaccurate performance analysis. If the model is assumed to work well, the majority of the delays will occur due to choices of instruments where the time to maturity does not differ significantly. If M is chosen so that adjacent instruments are smoothed, it could be expected that the majority of the delays will be smoothed out, retaining a sufficiently high resolution.

Changing the scenario described above, where two instruments with different time to maturity are issued at the same time, the instruments are now issued at different times. To give the delay effect earlier discussed, the instruments have to overlap in time. Assume that the instruments are issued at a discrete time t_i in months. If the issuing date of instrument one is fixed and $L1 > L2$, the issuing time of instrument two is uniformly distributed in the interval $0 \leq t_2 \leq L_1 - L_2$ where L_1 and L_2 are the lengths of the instruments respectively. If M is chosen as $\lfloor \frac{(L_1 - L_2)}{2} \rfloor$, where $\lfloor\ \rfloor$ denotes rounding down to closest integer, the window will overlap the two transitions with a probability of one. Using the two shortest instruments as instrument one and two should thus give a good trade off between smoothing transitions and resolution.

4.3 Implementation

In this section, the implementation used for testing and evaluating the liability management method is presented. The implementation was developed in MATLAB. The choice of forecasting method and risk measure are described. The implementation of the algorithms are however not described, since they are dependent on the chosen software.

4.3.1 Forecasting interest rates

The optimization model requires input of the cost of the available bonds, over the time horizon used in the optimization. The expectations theory can be used to approximate future interest rates by iteratively calculating the forward rates. It states that "the movement of the yield curve should be dependent on market expcetations, which is the relationship between the forward rate and the spot rate" [Ho and Lee, 2004]. The most common form of this statement is the *unbiased expectations hypothesis*, which states that the expected future spot rate is equal to the forward rate. The hypothesis can be stated as

$$E[R(t + nP, 1)] = F(t + kP, 1, n - k), \quad k = 0, 1, ..., n - 1 \tag{4.6}$$

where E[.] is the expectation operator.

This prediction is quite rough but can still be used for implementation in this thesis, because the model itself is not dependent on the specific details of how interst rates are forecasted. Implementation of more advanced term structure models, like the CIR-model [Mele and Fornari, 2000] is beyond the scope of this thesis.

The expectations theory approach requires interest rates for all discrete times to maturity, that is, one month, two months, three months rates and so forth. Since those are unavailable, they can be interpolated from the yield curve using splines[3], to render a reasonable approximation.

4.3.1.1 Using splines for interpolation

Polynomials of different orders can be used to interpolate between discrete values. The problem with this method is that the polynomials tend to generate large deviations from the expected interpolation. This problem can be solved using splines.

Splines can be used as a powerful tool for interpolating between discrete values. A spline is a linear combination of piecewise continuous polynomials. A commonly used method which ensures that the curve passes through all points is cubic splines [Fabozzi, 2002]. Other methods have been proposed, for example exponential splines [Vasicek and Fong, 1981]. We choose to use cubic splines as recommended by Fabozzi [Fabozzi, 2002]. An implementation of cubic splines interpolation can be found in MATLAB and will not be discussed here. Interested readers can refer to [Fabozzi, 2002], which includes a chapter covering this method in depth.

4.3.2 Risk measurement

The optimization model requires input of the interest rate risk for the cost function. This risk is approximated by the volatility of the interest rates and a GARCH model[4] is used for estimating it. A GARCH(1,1) model is used, but other risk measures could easily be incorporated instead. To get reliable estimates of volatility from GARCH models, sample sizes of 200 or more are generally required [Bollerslev et al., 1994]. This places a lower bound on the required amount of historical data needed.

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[3] See section 4.3.1.1

4.3.2.1 GARCH model

The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model is extensively used for estimating volatility in financial markets [Fabozzi, 2002]. It captures the well known characteristic of fat tails in the statistical distribution of many financial time series, better than the normal distribution and is therefore more favorable. The GARCH model is popular because it considers random yield shocks as well as serial dependence in yield volatility. One opposition to GARCH, however, is that the level of yield is not incorporated.

The standard GARCH(1,1) model can be written as

$$y_t - y_{t-1} = \varepsilon_t \tag{4.7}$$

$$E[\varepsilon_t^2] = \sigma_t^2 = a_0 + a_1\varepsilon_{t-1}^2 + a_2\sigma_{t-1}^2 \tag{4.8}$$

where ε_t is the daily yield change, $E[\cdot]$ is the expectation operator and a_0, a_1 and a_2 are parameters to be estimated. Hence, volatility in this period depends on both yield change and yield volatility in the last period.

---

[4] See section 4.3.2.1

Chapter 5

Results

5.1 The optimal portfolio

The optimal portfolio was found by following the description in the simulation procedure in Appendix A. Characteristics of the optimal portfolio over time are depicted in Figure 5.1, where duration over time is shown. The behavior is clearly as expected. When interest rates are falling, the portfolio is rolling over 3-month Treasury bills and when a rise is imminent, all debt is placed in an instrument with maturity long enough to avoid most effects of the interest rate rise. Section 4.1 states that the portfolio is optimal for the cost function with given constraints. This statement coincides with the results shown here. The cost of this portfolio should obviously be the lowest, when compared to all other simulated values, and this is confirmed by calculations. See Figures 5.2 and 5.3 for a visualization of this, where cost is calculated in the two different ways earlier described in Section 4.2.2.1. The flat, lower surface illustrates the cost of the optimal portfolio. Values of $\gamma > 20$ are not shown, since the cost is just monotonically increasing for this interval.

5.2 Performance of the model

In order to evaluate the performance of the implementation, the parameters must be chosen to represent a certain portfolio strategy. First, ξ is selected

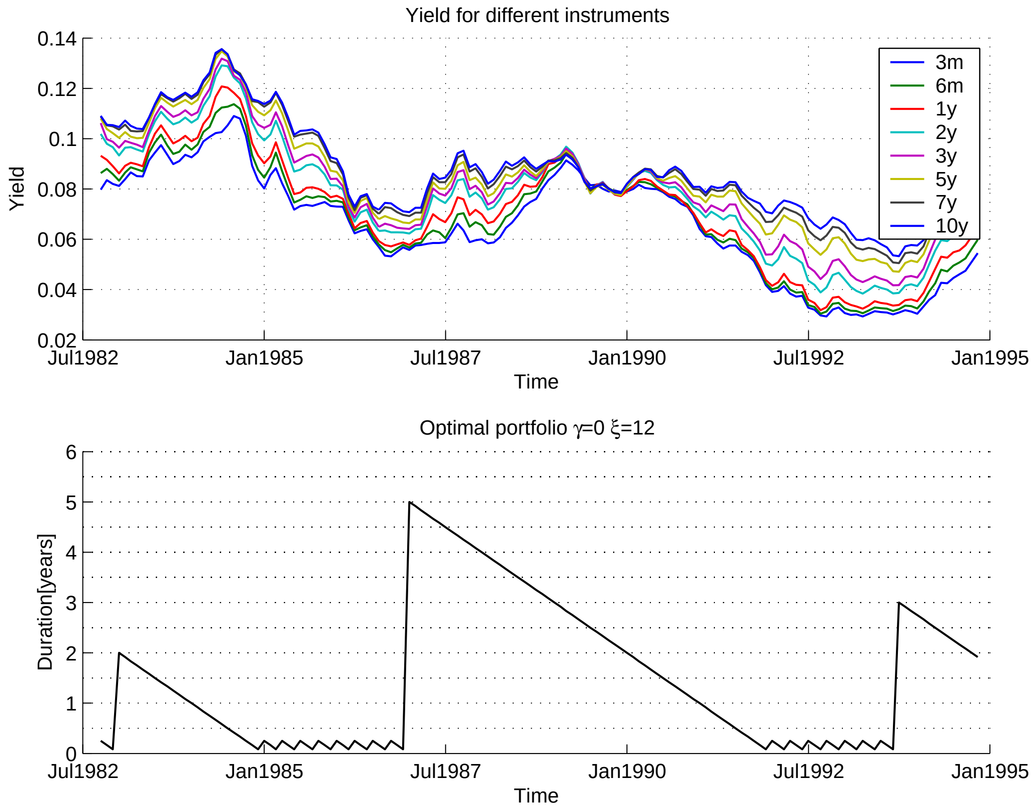


Figure 5.1: Yield and the optimal portfolio over time

as 0.35, to serve as an example. As is shown in Figure 5.2 and 5.3, the other limits behave similarly. From Figure 5.2 and 5.3, a suitable value of γ can be chosen. The objective is to select the one that has minimized the cost for $\xi = 0.35$ over the data series used. It can be seen that there are valleys in the plots for $4 \leq \gamma \leq 7$ and $2 \leq \gamma \leq 4.5$ respectively. Therefore, $\gamma = 4$ is a proper value in order to capture the lowest cost in both aspects.

The impact of the limit, ξ, is illustrated in Figure 5.4a and b. It can be seen that the peaks of the duration occur at approximately the same times, but the values are a bit different. The large difference in the first peak may seem quite strange, but recalling the discussion in Section 3.4.6, it is rather obvious. Instead of issuing the whole portfolio in two year bonds, as in the optimal portfolio (a), the portfolio is distributed over more and longer maturities. Remember that with $\xi = 0.35$, the portfolio is only allowed to consist to 35% of bonds with $0 \leq maturity \leq 1$ years, an equal amount

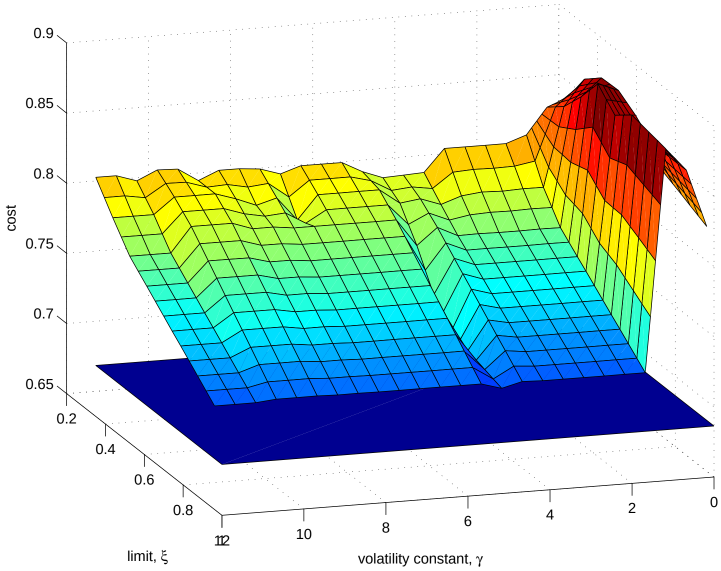


Figure 5.2: Cost of simulated portfolios discounted to issuing date, with $0.35 \leq \xi \leq 1$ and $0 \leq \gamma \leq 12$

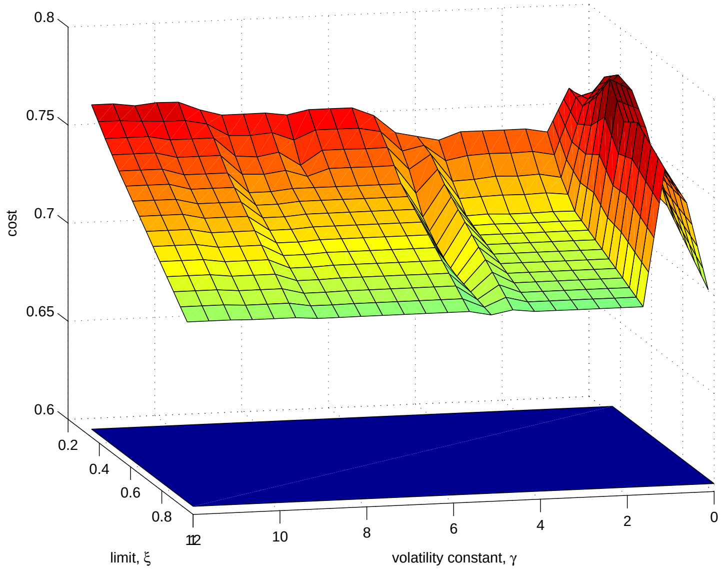
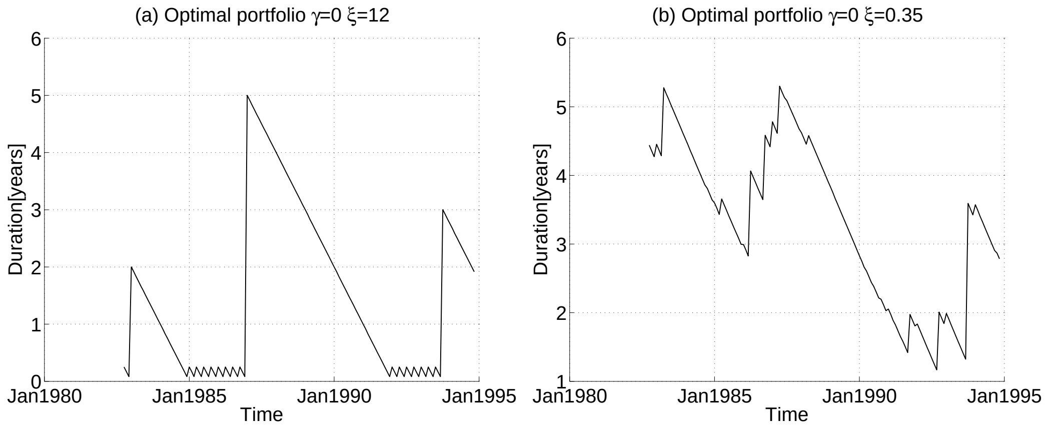


Figure 5.3: Cost of simulated portfolios discounted to time 0, with $0.35 \leq \xi \leq 1$ and $0 \leq \gamma \leq 12$

within $1 \leq maturity \leq 2$ and the rest above 2. This implies that all debt cannot be issued in two year bonds and that the average duration should therefore exceed two years.



	$\xi = 12$	$\xi = 0.35$
optimal	0.6863	0.7534
forecast,$\gamma = 4$		0.8173

Table 5.1: Total cost of portfolios

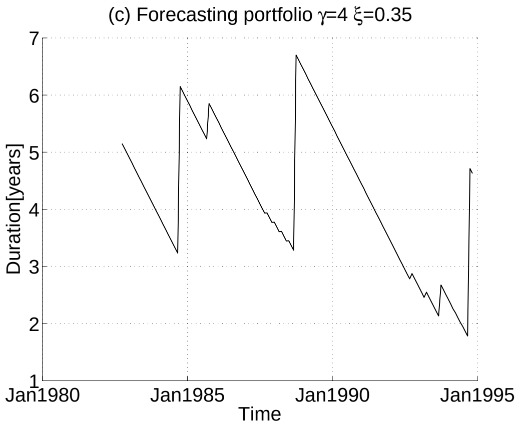


Figure 5.4: Duration of different portfolios

The overall performance of the implementation of the model can be seen in Figure 5.4, where (c) is the duration over time for the portfolio with the chosen parameters ($\xi = 0.35$ and $\gamma = 4$) and the cost is shown in Table 5.1. Comparing this plot with the yield curve in Figure 5.1, the response in terms of changed duration when interest rates move is seen.

Introducing forecasted interest rates into the model results in some deviations in the portfolio behavior. In Figure 5.4(c), peaks in the duration appear at different times compared to the optimal portfolio with $\xi = 0.35$ in Figure 5.4(b). These differences result in an increased cost compared to the optimal portfolio.

A measure of the deviations between the duration curves using forecasted interest rates and the optimal portfolio is illustrated by using the concept of floating mean square error (FMSE), earlier described in Section 4.2.2.2. The resulting plot is shown in Figure 5.5. It can be seen that the lowest error occurs for $\gamma = 0$, suggesting that volatility does not need to be considered in the model. This is not the case since the cost, which clearly is the best performance measure, is higher for that volatility constant as seen in Figures 5.2 and 5.3. Further, a valley in the interval $2 \leq \gamma \leq 4$ shows that the duration over time is relatively similar to the optimal portfolio in that region. Also, this valley coincides with the one in Figure 5.3, indicating that discounting to time 0 may be the best way of measuring cost.

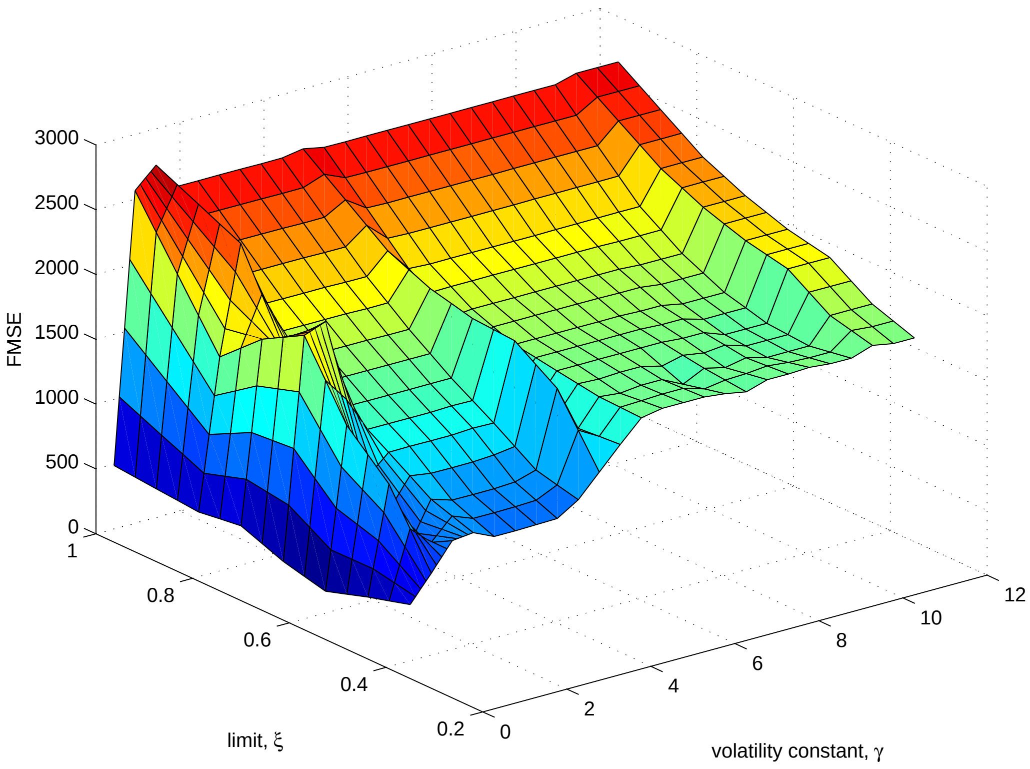


Figure 5.5: Floating Mean Square Error compared to the optimal portfolio for $0.35 \leq \xi \leq 1$ and $0 \leq \gamma \leq 12$

One of the criteria for a computer based methods is that the solution should be reached within feasible time. The specific implementation used in this thesis found a solution in 1-2 minutes.

Chapter 6

Conclusions

The massive amount of financial data available today, justify the use of computer based models to support decision making in the financial area. The decisions on the instruments issued today will affect the decisions of instruments issued at a later date. Therefore, a continuous management of the liability portfolio that considers future effects of todays decisions, is needed to minimize interest costs.

As can be seen in Section 5.1, the method, with observed interest rates, generates a reasonable solution. The structure of the portfolio reacts well to changes in the interest rates, using long instruments over peaks and short instruments in slopes. The results show that the portfolio is optimal for the cost function with given constraints. Hence, with observed future interest rates, the method will return an optimal portfolio over time, which can be used for portfolio management. Given satisfactory forecasts of future interest rates, the method can be used as a sound recommendation on how to construct or change the structure of the liability portfolio. The choice of instruments made by the method with a limit can be quite easily interpreted, but would be impossible for any human to make. Additionally, when using computers as an aid in decision making, personal judgment must of course be used. The usage of numerical methods can sometimes yield inaccurate solutions due to for example badly conditioned data.

In Section 5.2, it is noted that using forecasted interest rates results in deviations in portfolio structure. The reason for these deviations is the poor performance of the expectations theory's ability to predict future interest rates. This is because, as shown in the results, the model minimizes the cost when given a series of interest rates. Hence, the expectations theory is inadequate for practical use with this method.

As shown in the result, the volatility constant γ affects the output from the model. The simulations do however not show if the cost of risk introduced in the model really reduces the risk in liability management. It could be the case that the volatility just adjusts the bad forecasts from the expectations hypothesis, without really reducing the risk. To validate this assumption, studies of the method using other forecasting methods and other data would be required. The method in itself is though solid against this kind of flaws, since a proper calibration of the model would reduce these effects using a low γ. An alternative way for the risk introduction into the model, could also be considered in order to change the impact of the volatility constant.

To be usable in practice, the method must find a solution within feasible time. None of the simulation times exceeded two minutes, which clearly is a reasonable time consumption.

The specific implementation described in this thesis, where a GARCH model and the expectations theory has been used for measuring volatility and forecasting interest rates respectively, can be further improved by using a more advanced forecasting method. The method of measuring volatility may also be questioned, even though it is very commonly used today. Further, a more thorough validation of the model presented in this thesis, by testing on more financial time series is necessary. Comparisons against other models would result in a measure of the performance of the model compared to these. It should also be mentioned that the basis for the model, a cost function, two equalities and one inequality, could possibly be extended to a more advanced model, though more resource demanding.

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Appendix A

Simulation procedure

Simulations are needed in order to test and validate the implementation of the model. The linear optimization problem formulated in Section 3.4 was implemented using MATLAB 6.5.1 and the large scale algorithm in matlab's linprog was used. The resulting program was then tested and its output verified. The testing procedure is summarized in Figure A.1.

Since at least 200 observations are needed for the GARCH model [Bollerslev et al., 1994], the historical data for the GARCH model was limited to 200 observations. Since the longest instrument used in the simulations is 10 years, this horizon was also used for each optimization. The time interval for simulations was limited by the available data on different instruments. The results of the simulations cover the time period between October 1982 and November 1994. The reason for this is that the optimal portfolio needs 10 years of future data in each optimization step. Hence, in the last optimization it uses the data from November 1994 to November 2004 and that is the end of the available time series. Step 1 is used for simulating a portfolio that is iteratively updated each month and uses known data. The time horizon used for optimizing each month is 10 years. In this step, multiple optimal portfolios are generated, first by varying the limit $0.35 \leq \xi \leq 1$, and then setting it to $\xi = 12$. The reason for simulating with $\xi = 12$ is that it implies that it is possible to let the whole portfolio expire during the same month. In reality, this means that the liquidity problem is set aside, but this will provide the structure of the optimal portfolio over time.

1. Find optimal portfolio iteratively, optimizing over a time t each iteration, *known data* used
2. Simulate different portfolios iteratively, optimizing over a time t each iteration and include volatility in the cost, *forecasted data* used
3. Compare the results from step 2 and 1

Figure A.1: Simulation Procedure

The reason for varying the limit $0.35 \leq \xi \leq 1$ is that Poseidon uses 0.35 as their annual limitation and, hence, reflects a situation in real life. However, it should be noted that Bostads AB Poseidon is a municipality owned company and their degree of risk aversion may therefore not be representative for the whole market. It should also be noted that no limits below 0.35 is used. This is because the number of financial instruments considered in this thesis is very limited, which will result in difficulties finding a feasible solution for low expiration limits. With more instruments, the limit could be lowered.

The procedure for constructing a portfolio using forecasted data in step 2 is made by same principles as the portfolio above, except for the difference in data and adding volatility to the cost. The data used was forecasted from the yield curve using the expectations theory in each point in time. The volatility is added as described in Section 3.4.4 and the coefficient, γ, was varied between 0 and 100. The reason for varying both the limit and the volatility coefficient is to see where the cost is lowest and to be able to calibrate the model.

Simulations of 67 values of γ and 7 values of ξ over 146 periods each were conducted, which sums up to 68474 simulations. With a simulation time of 1-2 minutes for each combination of variables, this would take about 1711 hours on a single desktop computer. Therefore, the simulations were distributed over about 80 computers.