

Sampling and Fourier-Laplace Transforms in Several Complex Variables

Niklas Lindholm

Department of Mathematics, Chalmers University of Technology
and Göteborg University, SE-412 96 Göteborg, Sweden

Abstract

In the first paper we study the Laplace transform on Hardy spaces on a class of convex domains in \mathbb{C}^n . We obtain a Paley-Wiener theorem which states that the Laplace transform is an isomorphism from the Hardy space to a normed space of entire functions in \mathbb{C}^n . This characterizes the entire functions of exponential type which occur as Laplace transforms, and we have equivalence in norm between the two spaces. In the analysis we use the Fantappiè transform and the Borel transform to rewrite the Laplace transform and reduce the problem to theorems in one complex variable.

The second and third paper are devoted to sampling and interpolation in the generalized Fock space in \mathbb{C}^n , which is a weighted L^p -space of entire functions. In one complex variable there is a complete characterization of all sampling sequences in the spaces in question. In several variables, a criterion of the same type cannot be sufficient for sampling. In the case of a 2-homogeneous, plurisubharmonic weight function in \mathbb{C}^n , we will demonstrate that a natural generalization of the density condition from one variable is necessary for sampling. To do this, we consider the Bergman kernel in the Fock space and obtain several results about its asymptotic behaviour. These results give asymptotic estimates for the eigenvalues of a certain Toeplitz concentration operator, which allow us to draw conclusions about density for a sampling sequence.

We also consider the problem of general sampling measures. Unlike the case in one variable, in several variables there are many different scales of sampling measures. In the third paper we will discuss some necessary conditions which are geometric obstructions for sampling measures. For instance, a sequence of points which is sampling cannot locally be contained in an algebraic hypersurface of low degree. We will also prove a complete description of sampling measures in \mathbb{C} .

Keywords: Fock space, sampling, interpolation, Bargmann space, concentration operator, Bergman kernel, Toeplitz operator, Laplace transform, Paley-Wiener theorems, Fantappiè transform, Borel transform, exponential type

2000 Mathematics Subject Classification: 46E20, 32A15, 32A30, 30E05, 32A25