Similarities and differences between statistical surveillance and certain decision rules in finance.

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The relation between statistical surveillance and certain decision rules in finance

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Abstract

Financial trading rules have the aim of continuously evaluating available information in order to make timely decisions. This is also the aim of methods for statistical surveillance. Many results are available regarding the properties of surveillance methods. We give a review of financial trading rules and use the theory of statistical surveillance to find properties of some commonly used trading rules. In addition, a non-parametric and robust surveillance method is proposed as a trading rule. Evaluation measures used in statistical surveillance are compared with those used in finance. The Hang Seng Index is used for illustration.

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1. Introduction

The purpose of this paper is to investigate the inferential differences and similarities between some methods of statistical surveillance and some prospective decision rules.

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used in finance, and to give a brief review of these financial decision rules from a statistical viewpoint. Furthermore, evaluation measures and utility functions used in statistical surveillance are compared with those used in financial settings.

In the financial market, the aim is to maximize profit. This requires optimal sequential decisions. An indicator is monitored with the aim of detecting the optimal time to trade. The indicator could be the price itself or related to the price. Optimal times to trade are related to regime shifts in the stochastic properties of the indicator. Thus, finding the optimal time to trade is equivalent to the timely detection of a regime shift.

According to the efficient market hypothesis, the financial markets are arbitrage-free and there is no point in trying to determine the optimal transaction time. But when the information about the process is incomplete, as for example when a change point could occur, there may be an arbitrage opportunity, as demonstrated by Shiryaev (2002). Many agents reject the efficient market hypothesis and many studies give support for the profitability of the prospective framework, see for example Sweeney (1986) and Lo (2000).

Since timeliness is crucial in a trading setting, the incoming data should be analyzed online and sequential trading decisions made. In this paper we investigate methods which aim to identify regime shifts, especially turning points. The inference situation is one of surveillance where the aim is to detect a change quickly and safely. It is characterized by repeated decisions and by never accepting the null hypothesis. (For general reviews on statistical surveillance, see Frisén and de Maré (1991), Srivastava and Wu (1993), Lai (1995), and Frisén (2003).) Other names are statistical process control and monitoring.

On-line detection problems are receiving increasing attention in the econometric literature. In Horváth et al. (2004) hypothesis tests are combined with the prospective
aspect of surveillance as a hypothesis is repeatedly tested each time a new observation becomes available.

Neftci (1991), Dewachter (2001), and others have pointed out that many trading rules are ad hoc and that their statistical properties are often unknown. Since the properties of methods of statistical surveillance have been investigated extensively, an integration of surveillance theory and financial decision rules could prove fruitful. Schmid and Tzotchev (2004) state that applications of surveillance methods in finance have been scarce. Lam and Yam (1997) claimed to be the first to link methods of surveillance to technical trading rules. Recent studies using surveillance for financial applications are discussed in Section 3.

“Optimal stopping rules” are based on the assumption that the model is completely known. Then probability theory can be used to find optimal trading times, see e.g. Shiryaev et al. (1994), Shiryaev (1999), Jönsson et al. (2004), and Lai and Lim (2005). In this paper, however, the process includes unknown statistical parameters, and we consider an inferential approach where we want to infer from data whether a regime shift has occurred or not.

In Section 2, regime shifts and model specifications are exemplified. In Section 3 we briefly describe the methodology of statistical surveillance and discuss similarities and differences between methods proposed in the literature on finance and methods of statistical surveillance. In Section 4 some of the methods are applied to the Hang Seng Index. Concluding remarks are given in Section 5.

2. Indicators and regime shifts

The indicator to be monitored can be constructed from one or several processes. The price level of an asset may itself be the indicator. To detect an increased risk level, the estimated variance can be used. The Leverage effect (Black (1976)) motivates
simultaneous monitoring of the mean and variance, as in Schipper and Schmid (2001a).
The arrival time of transactions can be monitored to detect a change in intensity, see for
eexample Zhang et al. (2001) and Sonesson and Bock (2003). Often so-called marks (for
example the volume traded) are available (see for example Dufour and Engle (2000)),
which again motivates multivariate surveillance. For reviews on multivariate surveillance,
see Wessman (1999a) and Frisén (2003). A general type of indicator is the residuals of a
time series model where the change is in the stochastic properties of the residuals. In order
to focus on the decision system we use an anonymous indicator X. In finance, turning
points are of special interest, since it is profitable to sell at highs and buy at lows. The
time of the turn is denoted τ, which is a discrete-valued random variable with probability
function

\[ \pi_i = P(\tau=i), \text{ for } i=1, 2, \ldots. \]

When a distribution of τ is needed, a geometric
distribution on \{1, 2,...\} is used. When necessary a simple standard model conditioned on
\( \tau=i \) is used

\[ X(t) = \mu(t) + \epsilon(t), \quad (1) \]

where \( \mu(t) = E[X(t)|\tau=i] \) is the trend cycle and \( \epsilon(t) \sim iid N[0, \sigma^2] \), \( t=1, 2, \ldots \) conditionally
on \( \tau \). The index \( i \) in \( \mu(t) \) is suppressed when obvious. Model (1) may be too simple for
some financial data. However, the model is often used, and here it is used to emphasize
the inferential issues of on-line turning point detection. The optimal methods are derived
for the simple model, but they are evaluated under realistic conditions in Section 4. The
vector \( \mu \) is determined by \( \tau \). At a peak we have

\[
\begin{aligned}
\mu(1) &\leq \ldots \leq \mu(t), & t \leq \tau \\
\mu(1) &\leq \ldots \leq \mu(\tau) \text{ and } \mu(\tau+1) \geq \ldots \geq \mu(t), & t > \tau
\end{aligned}
\]  

(2)

In the second row of (2), the left hand side set, or the right hand side set, can be empty for
some values of \( \tau \).

One parametric specification of \( \mu \) is a piecewise linear regression,
\[ \mu(t) = \begin{cases} \beta_0 + \beta_1 \cdot t, & t \leq \tau \\ \beta_0 + \beta_1 \cdot \tau + \beta_2 \cdot (t-\tau), & t > \tau \end{cases} \quad (3) \]

where \( \beta_1 \geq 0 \) and \( \beta_2 \leq 0 \).

For a differentiated process \((Y(t)=X(t)-X(t-1))\), a specification of \(E[Y(t)]=\mu_Y(t)\) is

\[ \mu_Y(t) = \begin{cases} \beta_1, & t \leq \tau \\ \beta_2, & t > \tau \end{cases} \quad (4) \]

The assumption in (4) is used by e.g. Layton (1996), Ivanova et al. (2000), and Layton and Katsuura (2001). For a differentiated process with expected values as in (4), the undifferentiated process can be either independent over time with an expected value as in (3) or a random walk with drift.

At each decision time \(s\), a system is used to decide whether data indicate a turn in \(\mu\) (at an unknown time \(\tau\)) in the time period \(\{1,2,...,s\}\) or not. Thus, the aim is to discriminate between \(C(s)=\{\tau \leq s\}=\bigcup_{i=1}^{\tau} C_i\) where \(C_i=\{\tau=i\}\), and \(D(s)=\{\tau > s\}\).

3. Statistical surveillance and strategies suggested for financial trading decisions

3.1. Measures for evaluations

When evaluating systems for on-line detection of changes it is important to consider the timeliness of the alarms. In finance, the expected return from investments is often used as a measure of performance. The return is often defined as

\[ r(t) = x(t)-x(0), \quad (5) \]

where \(X\) is the (logarithm of the) price, and measures the return of buying at \(t = 0\) and selling at time \(t\).

The expected return \(E[r(t_\lambda)]\) is maximized when \(E[X(t_\lambda)]\) is maximized. This occurs when we call a sell alarm at the peak, in this case at \(t_\lambda=\tau\). Maximizing \(E[r(t_\lambda)]\) is equivalent to minimizing \(|t_\lambda-\tau|\). This is also the aim in statistical surveillance.
The probability of successful detection used for example by Frisén (1992) and Frisén and Wessman (1999) measures the ability to detect a change within \( m \) time units from \( \tau \),

\[
\text{PSD}(m, i) = P(t_{A} - \tau \leq m \mid t_{A} \geq \tau, \tau = i),
\]

(6)
The return in (5) is measured along the log-price scale, whereas PSD(m, i) in (6) is measured along the time scale. Consider the expected difference \( \delta(t_{A}, i) = E[r(t_{A}) - r(\tau) \mid \tau = i] \).

When \( E[X(t)] \) is piecewise linear, then \( P(\delta(t_{A}, \tau) \geq m \cdot \beta_{2} \mid t_{A} \geq \tau, \tau = i) \) is equivalent to PSD(m, i), where \( \beta_{2} \) is the post-turn slope.

When we consider transaction costs, we have a penalty for each alarm, which favors infrequent trading. The influence of transaction costs was discussed by Lam and Wei (2004).

In the specification of utility by Shiryaev (1963),

\[
u(t_{A}, \tau) = \begin{cases} 
    h(t_{A} - \tau), & t_{A} < \tau \\
    a_{1} \cdot (t_{A} - \tau) + a_{2}, & t_{A} \geq \tau 
\end{cases}
\]

(7)
the gain of an alarm is a linear function of the expected delay. The loss of a false alarm is an arbitrary function of the same difference. A specification of \( h \) which is of interest in finance gives

\[
u(t_{A}, \tau) = \begin{cases} 
    b_{1} \cdot (t_{A} - \tau) + b_{2}, & t_{A} < \tau \\
    a_{1} \cdot (t_{A} - \tau) + a_{2}, & t_{A} \geq \tau 
\end{cases}
\]

where \( b_{1} > 0, a_{1} < 0 \) and where \( b_{2} \) and \( a_{2} \) would depend, for example, on the transaction cost. When \( b_{2} = a_{2} = 0 \) and \( E[X(t)] \) is piecewise linear it follows that \( E[u] = E[r(t_{A})] - E[r(\tau)] \), where the expectation is taken with respect to the disturbance. Thus, maximizing the expected utility is the same as maximizing the expected return.

In statistical surveillance the type I error is usually characterized by the average run length until the time of alarm, \( t_{A} \), at no change, \( \text{ARL}^{0} = E[t_{A} \mid \tau = \infty] \). A widely used optimality criteria in the literature on quality control is the minimal \( \text{ARL}^{1} = E[t_{A} \mid \tau = 1] \) for
a fixed ARL\(^0\). Drawbacks with this criterion are discussed by Frisén (2003). ARL\(^1\) only considers immediate changes (τ=1), whereas the conditional expected delay, CED(i) = E(τ_A−τ|τ_A ≥ τ=i), considers different change points. A minimax criterion is the minimum of the maximal CED(i), with respect to τ=i and X_{τ=1}. Brodsky and Darkhovsky (2005) suggest the use of a ratio between a delay measure and a false alarm measure.

3.2. Methods

3.2.1. Benchmark: The full likelihood ratio method

The full likelihood ratio method (LR) is optimal in terms of (7) and fulfills several other optimality criteria, see for example Frisén (2003), and serves here as a benchmark. The alarm criterion of the LR method is

\[
    f_{X|s}(x_s|C(s))/f_{X|s}(x_s|D(s)) = \sum_{i=1}^{S} w(i) \cdot L(s, i) > g_{LR}(s),
\]

where \(L(s, i)=f_{X|s}(x_s|\mu=C_i)/f_{X|s}(x_s|\mu=D)\) is the partial likelihood ratio when τ=i, and \(w(i)=P(τ=i)/P(τ ≤ s)\). The vector \(\mu\) is on the form \(\mu=C_i\) when τ=i (for state \(C_i\)) and on the form \(\mu=D\) when τ>s (for state \(D(s)\)). Thus, the vector \(\mu\) is known given the state, but the state is random.

A likelihood ratio method based on a small change intensity (\(v=P(τ=i|τ ≥ i)→0\)) is the Shiryaev-Roberts (SR) method (Shiryaev (1963) and Roberts (1966)). The SR method can be seen as based on a non-informative generalized prior for the change time since equal weights are used for all components in the likelihood.

3.2.2. Turn detection and the SRnp method

When discussing “sign prediction”, Dewachter (1997) argued that it is the direction of the evolution that is important in finance. Frisén (1994) suggested a non-parametric surveillance approach based only on the monotonicity and unimodality restrictions in (2).
Combined with the weights of the SR method, this is the SRnp method (with \( g_{\text{SRnp}} \) as a constant)

\[
\left(\sum_{i=1}^{n} f_{x_i}(x_i | \mu = \hat{\mu}^D)\right)/f_{x_i}(x_i | \mu = \hat{\mu}^D) > g_{\text{SRnp}}, \tag{9}
\]

The vector \( \hat{\mu}^D \) is the estimator of \( \mu \) under monotonicity restriction \( D \) (no turn), and \( \hat{\mu}^C_i \) is the estimator under restriction \( C_i \) (turn at \( i \)). The estimators give maximum likelihood when the disturbance has a Gaussian distribution (Frisén (1986)). SRnp was described and evaluated in Andersson (2002), Andersson (2004), Andersson et al. (2005a), and Andersson et al. (2005b). So far it has not been used as a financial trading rule, but its possible application for this purpose will be examined in Section 4.

3.2.3. Forecasts and the Shewhart method

Modeling of the financial process as a base for the trading strategy is important. The modeling can be used to forecast the next value, and the difference between the forecast and the last value can then be used in a trading rule (see for example Neely and Weller (2003)).

The Shewhart method signals an alarm as soon as the last partial likelihood ratio \( L(s, s) \) exceeds a constant \( g \), i.e. for the model in (1) we have

\[
x(s)-\mu^D(s) < g_{\text{Shewhart}}. \tag{10}
\]

The method is optimal in terms of (7) when \( C(s) = \{\tau = s\} \).

3.2.4. CUSUM and Filter rules

The CUSUM method of Page (1954) gives a signal as soon as the maximum of the partial likelihood ratios \( L(s, i) \), exceeds a limit. For an independent Gaussian process \( Y \) with constant \( \mu^D_Y \) and \( \mu^C_Y \) as in (4), the alarm criterion for a downward shift is

\[
\sum_{j=1}^{\tau-i} (y(j)-\mu^D_Y) < -(g_{\text{CUSUM}}+k \cdot i), \tag{11}
\]
for some \( i=1, 2, \ldots, s \) where \( g_{\text{CUSUM}} \) is a constant. The optimal value of \( k \) can be expressed as \((\mu^D - \mu^C) / 2\). The CUSUM method satisfies the minimax criterion in section 3.1.

Lam and Yam (1997) propose a generalized Filter rule (GFR). At decision time \( s \), a peak signal for the process \( X \) is given when

\[
\left( x(s) - x(s-i) \right) / x(s-i) < (g_{\text{GFR}} + k_{\text{GFR}}) \cdot i \tag{12}
\]

for some \( i=1, 2, \ldots, s \), where \( g_{\text{GFR}} \) and \( k_{\text{GFR}} \) are chosen constants. We will now show that GFR is approximately equivalent to the CUSUM method. If we let \( Y(t) = \ln X(t) - \ln X(t-1) \), the alarm criterion of the CUSUM method can be expressed as

\[
\left( x(s) - x(s-i) \right) / x(s-i) < \exp\{-g_{\text{CUSUM}}\} \cdot \frac{D_{\text{Y}} \exp\{(k - \mu) \cdot i\}}{2} \tag{13}
\]

Let \( \exp\{-g_{\text{CUSUM}}\} = (1 + g_{\text{GFR}})^{-i} \) and \( \exp\{(k - \mu) \cdot i\} = (1 + k_{\text{GFR}})^i \). We approximate \((1 + k_{\text{GFR}})^i\) by \((1 - k_{\text{GFR}} \cdot i)\) and \((1 + g_{\text{GFR}})^{-i}\) by \((1 - g_{\text{GFR}})\). If we make these substitutions in (13), then

\[
\left( x(s) - x(s-i) \right) / x(s-i) < (1 - g_{\text{GFR}}) \cdot (1 - k_{\text{GFR}} \cdot i)^{-1}.
\]

If \( g_{\text{GFR}} \cdot k_{\text{GFR}} \cdot i \approx 0 \), then (13) approximately equals (12). Thus, it follows that GFR is approximately minimax optimal under certain conditions.

A special case of GFR is obtained when \( k_{\text{GFR}} = 0 \). This is the widely used Filter rule, FR (Alexander (1961), see also Taylor (1986) and Sweeney (1986)). According to Lam and Yam (1997), the FR calls a peak alarm when

\[
\left( \max_{t \leq s} \{x(t)\} - x(s) \right) / \max_{t \leq s} \{x(t)\} > g_{\text{FR}}
\]

where \( g_{\text{FR}} \) is a constant. Lam and Yam showed that FR, used on \( X(t) \), is equivalent to a special case of CUSUM used on \( Y(t) = \ln X(t) - \ln X(t-1) \). Hence, when \( Y(t) \) is independent with a Gaussian distribution and \( \mu^D_{\text{Y}} = (\mu^D_{\text{Y}} - \mu^C_{\text{Y}}) / 2 \), FR has the same properties as the CUSUM method. The \( \mu^D_{\text{Y}} = (\mu^D_{\text{Y}} - \mu^C_{\text{Y}}) / 2 \) implies that \( \mu^C_{\text{Y}} = -\mu^D_{\text{Y}} \), i.e. symmetry. The performance of the CUSUM depends on which shift size (measured by \( (\mu^D_{\text{Y}} - \mu^C_{\text{Y}}) \)) the
method is designed to detect, and thus the FR method is minimax optimal for a symmetric turn. The FR is the same as the Trading range break.

The performances of GFR and FR were evaluated by Lam and Yam (1997) for different combinations of \( g_{CUSUM} \) and \( k \), using 24 years of daily data on the Hang Seng Index. For some combinations, GFR had a better return than FR. In the case of GFR, however, no discussion was made regarding the relation between \( k \) and the size of the shift.

3.2.5. Moving averages in surveillance and finance

The moving average method of surveillance (see for example Wetherhill and Brown (1991) and Frisén (2003)) gives an alarm when the partial likelihood ratio \( L(s, s-m) \) exceeds a constant. When \( C(s) = \{ \tau = s-m+1 \} \), the moving average method with window width \( m \) is optimal (Frisén (2003)). For an independent process with a Gaussian distribution, the alarm criterion is

\[
\sum_{i=s-m+1}^{s} \left( x(i) - \mu^D(s) \right) < g_{MAR}, \tag{14}
\]

where \( m \) is the window width and \( g_{MAR} \) is a constant.

Moving average rules (several variants have been suggested) may be the most commonly discussed trading rule. The rule used by e.g. Neftci (1991), Brock et al. (1992), and Neely (1997), calls a peak alarm (sell signal) as soon as the difference between two overlapping moving averages is below a limit.

\[
\frac{1}{m} \sum_{i=s-m+1}^{s} x(i) - \frac{1}{n} \sum_{i=s-n+1}^{s} x(i) < g_{MAR}, \tag{15}
\]

where the narrow window has width \( m \) and the wide window has width \( n \). The limit is usually set to zero. Dewachter (1997) and Dewachter (2001) referred to this rule as the “oscillator rule”. By expressing (15) as \( \sum_{i=s-m+1}^{s} (x(i) - \hat{\mu}^D(s)) < g'(s)_{MAR} \), where
\[ \hat{\mu}^D(s) = \left( \sum_{i=s-n+1}^{s-m} x(i) \right) / (n-m) \] and \( g'(s)_{\text{MAR}} = m \cdot n \cdot g(s)_{\text{MAR}} / (n-m) \), we see that (15) is the surveillance method in (14) with \( \mu^D(s) \) replaced by the moving average of \( (n-m) \) past observations. A special case of (15), which is often considered, is when \( m=1 \),

\[ x(s) - \frac{1}{n} \sum_{i=s-n+1}^{s} x(i) < g(s)_{\text{MAR}}. \] (16)

By expressing (16) as \( x(s) - \hat{\mu}^D(s) < g'(s)_{\text{MAR}} \), where \( \hat{\mu}^D(s) = \left( \sum_{i=s-n+1}^{s-1} x(i) \right) / (n-1) \) and \( g'(s)_{\text{MAR}} = (n/(n-1)) \cdot g(s)_{\text{MAR}} \), we see that (16) is the Shewhart situation in (10).

The optimality of (15) and (16) is not so clear-cut, as \( \mu^D(s) \) is estimated. Andersson and Bock (2001) demonstrated that in the case of cyclical processes, a moving average does not always preserve the true time of the turning point. This causes a delay of the signal.

Another method based on moving averages is the EWMA (exponentially weighted moving averages) method. The optimality of this method is analyzed by Frisén and Sonesson (2006).

### 3.2.6. Rules based on Hidden Markov Models

In a Hidden Markov Model, HMM, the process has different properties for different states and a first-order time-homogenous Markov process governs the switching between the states. Examples of the use of HMM in finance are given by Marsh (2000), Dewachter (1997), and Dewachter (2001). In financial applications the hidden Markov chain often have two states, \( J(t) = \{1, 2\} \), the expansion and recession phases. The process depends on the states, so that for (4) we have \( E[Y(t)|J(t)=i] = \beta_i \). Sometimes a HMM is referred to as a Markov-switching or regime switching model.

A natural alarm statistic is based on the one-step-ahead predicted expected value of the differentiated process \( Y \) conditional on past values, with alarm rule

\[ E[Y(s+1)|y_s] = P(J(s+1)=1|y_s) \cdot \beta_1 + P(J(s+1)=2|y_s) \cdot \beta_2 < c, \] (17)
where \( P(J(s+1)=1|y_s)=p_{11}\cdot P(J(s)=1|y_s)+p_{21}\cdot P(J(s)=2|y_s) \). This is hereafter denoted the HMR method (Hidden Markov Rule). The alarm limit in (17) is usually set to zero. A related statistic is the posterior probability, \( P(J(s)=2|x_s) \) (see Hamilton (1989)). The posterior probability was also used by Rukhin (2002) in a retrospective setting for estimating the change point time.

Frisén and de Maré (1991) showed that when the two states are complements to each other, rules based on the posterior probability are equivalent to the LR method in (8). Thus, the HMR method is ED optimal. However, when HMR and LR have different aims (classification and change detection respectively), the methods imply different properties (see Andersson et al. (2005b)).

3.2.7. The Zarnowitz Moore method and multivariate surveillance

The methods by Zarnowitz and Moore (1982) are explicitly stated as sequential signal systems for business cycles but have also been used as trading rules for financial series by Boehm and Moore (1991) and Moore et al. (1994). Their methods are multivariate (with \( X(s) \) as the process of interest and \( L(s) \) as a one-dimensional leading index) and only utilize information from before the decision time \( s \) by a rule of “natural ordering” of statements. This method can be regarded as a multivariate Shewhart method, since only the last observation is used for the alarm. For reviews on multivariate surveillance, see e.g. Wessman (1999b), Ryan (2000), Andersson (2005), and Sonesson and Frisén (2005).

3.2.8. Methods for statistical surveillance used as trading rules in finance

Lam and Yam (1997) claimed to be the first to discuss using methods for statistical surveillance as financial trading rules. After that, Yashchin et al. (1997) and Philips (2003) advocated the use of the CUSUM method for this purpose. Beibel and Lerche (1997) and Shiryaev (2002) used the theory of optimal surveillance to derive trading rules
for continuous time processes, and Schmid and Tzotchev (2004) used different types of
EWMA methods to detect changes in a discrete time interest rate model.

Severin and Schmid (1998), Severin and Schmid (1999), Schipper and Schmid
(2001a), and Schipper and Schmid (2001b) compared the performance of different
versions of the CUSUM, EWMA, and Shewhart methods with respect to detecting
changes in GARCH processes (generalized autoregressive conditional heteroscedasticity),
which are used to describe volatility in financial markets. Whereas Schipper and Schmid
(2001b) aimed at detecting a change in the variance of a GARCH process, the aim in
Schipper and Schmid (2001a) was to simultaneously detect an additive outlier and a
changed variance. In Severin and Schmid (1998) and Severin and Schmid (1999), the
CUSUM, EWMA, and Shewhart methods were compared with respect to a change in the
mean of an ARCH(1) process. Schipper and Schmid (2001b) used the CUSUM and
EWMA methods on the following indicators: the squared observations, the logarithm of
the squared observations, the conditional variance, and the residuals of a GARCH model.
Schipper and Schmid (2001a) used the EWMA method on each of these indicators to
monitor the level of the process. Here the ARL$^1$ for a fixed ARL$^0$ was used as evaluation
measure.

Sliwa and Schmid (2005) suggested methods for monitoring of the cross-covariances
of a multivariate ARMA process to monitor data on the Eastern European stock markets
by different EWMA methods.

Steland (2002) monitors both a GARCH and an independent process with a Gaussian
distribution for a change in the drift. The indicator under surveillance is a non-parametric
kernel estimator of $\mu$ (estimated by an exponentially weighted moving average), and only
the latest estimated value is used in the surveillance (a Shewhart-type method). The
performance is measured by $\text{ARL}^1$ for a fixed $\text{ARL}^0$. A similar approach was used in Steland (2005) to monitor a smooth but nonlinear change.

Blondell et al. (2002) suggested a CUSUM method with re-estimation of parameters for the detection of the turns in a cyclical mean level with volatility regime shifts of financial time series.

4. Illustration by data on the Hang Seng Index

A common way to evaluate decision rules in financial literature is using one or several case studies. Here the case studies will not be used to decide which method is the best, but instead to illustrate several difficulties with evaluation by case studies. Some of the methods described above are applied to data on the Hang Seng index (HSI), which was also used by Lam and Yam (1997). HSI is a market-value weighted index of the stock prices of the 33 largest companies on the Hong Kong stock market. The weight each stock is assigned in the index is related to the price of the stock. HSI can thus be seen as the price of a stock portfolio. Usually, as here, the values reported are the logarithms of the prices. The values of HSI for the period from February 10, 1999 to June 26, 2002 are shown in Figure 1. The series is divided into two periods so that several aspects of methods and evaluations can be illustrated. All days are not trading days and the trading days are numbered consecutively. Period I goes from February 10, 1999 to May 28, 1999 (day 0–71). Period II goes from May 31, 1999 to June 26, 2002 (day 72 –828).
Figure 1. Daily observations of HSI from February 10, 1999 to June 26, 2002. The limit between the periods (I and II) is marked with a dashed vertical line.

4.1. Specification of the trading rules

The statistical surveillance methods that will be evaluated are SRnp (alarm rule (9)) and CUSUM (alarm rule (11)). The CUSUM method is very similar to the financial trading rule GFR in (12), as was discussed in Section 3.2.4. The HMM-based method to be evaluated is HMR (alarm rule (17)).

4.1.1. Parameter values

The different methods need different parameters in the alarm statistic, such as the trend cycle ($\mu(t)$), the variance ($\sigma$), and the transition probabilities ($p_{11}$ and $p_{22}$). The data from period I (the first peak) will be used to estimate the parameters in question.

For SRnp, the estimation of $\mu$ is non-parametric and non-informative weight are used for the intensity. Thus, it suffices to estimate the variance of the process.

For CUSUM(opt), the optimal CUSUM in (11), we obtain the value of $k$ from the estimated parameters of $\mu^c_Y$ and $\mu^d_Y$. For CUSUM, we also use another set of parameters.
In Lam and Yam (1997) the value of $k$ was determined from a long period to yield maximum profit. CUSUM(L&Y best) is the CUSUM method with their best values of the alarm limit and $k$.

In HMR, we need estimates of the transition probabilities $p_{12}$ and $p_{21}$, in addition to the parameter estimates mentioned earlier. Here we use the maximum likelihood estimates under assumption of constant transition probabilities.

4.1.2. Controlling false alarms

In surveillance, the alarm limit is often determined so as to get control over false alarms, but also other approaches are possible.

In Lam and Yam (1997) no discussion was made regarding the false alarm rate. Instead, the alarm limit is determined to yield maximum profit in a long period. This alarm limit is used in the CUSUM(L&Y best) method. For the SRnp and CUSUM(opt) methods, the limits are determined to give a maximal total return for period II.

The limit zero in the HMR alarm rule (17) is often described as a "natural" limit and used in the evaluations below. However, other limits have been used with reference to transaction costs.

In a practical online monitoring situation, where the process is continually observed, the parameters must be estimated by observations that are available at the current time point. We illustrate this aspect by not re-estimating the parameters when evaluating the methods with respect to period II. Thus, the parameters estimated by observations from period I are also used in period II.

4.2. Results

After having bought the stock at the start, we sell it at the first sell-signal, i.e. at the alarm for a peak. After a peak alarm is given, the aim is to detect a forthcoming trough in
order to buy again, and so on. We assume that it is possible to buy or sell at the price at the alarm time $t_A$. For all methods except HMR, only observations past the previous alarm are used. For HMR all past observations within the evaluation period are used for the updating of the posterior probability. There are several aspects to consider when evaluating a monitoring system, among them timeliness. Direct measures such as those described in Section 3.1 require a precise definition of what is a turning point. For this series of observations we have not used that approach. Instead, timeliness can be measured indirectly, as the amount gained by detecting the change at the “right” time. Such a measure is the return (5) which is used here.

Transaction costs and interest rate, earned by having money in the bank, are not reported here. Their exact values depend on several circumstances, and for this series they would have negligible impact compared with the stock return. Transaction costs are proportional to the number of transactions and, as seen in Table 2, the HMR method would have the largest costs. Also the earned interest rate would be disadvantageous for the HMR method, as the periods when the asset is not held are shorter than for the other methods. However, both effects are too small to have any substantial influence on the comparison of returns.

4.2.1. Period I

Period I (day 0–71) is used for estimation, and therefore the illustration of the methods as regards this period can be seen as an in-sample performance.

The alarm systems (CUSUM, HMR, SRnp) are applied to the observations in period I. All methods start the monitoring at February 10, 1999 ($t = 0$). The times and returns of sell-signals are reported in Figure 2.

In Table 1 the return of buying one unit of the index at $t = 0$ and selling it at the sell-signal is reported. SRnp is the only method that reacts to a slight dip around day 30. A sell
signal alarm is given at day 27, and a buy signal is then given at t=34. Since this dip was a single occasion during a long expansion phase, SRnp gets a small total return for period I. The CUSUM(opt) has a slightly better return than the CUSUM(L&Y best) due to parameter values which are exactly adopted to that period. The HMR method yields a smaller return because the alarm is given late.

Table 1. Summary of the returns of period I, February 10, 1999 to May 28, 1999

<table>
<thead>
<tr>
<th>Method</th>
<th>Time of sell-signal</th>
<th>Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>SRnp</td>
<td>27, 56</td>
<td>0.28</td>
</tr>
<tr>
<td>CUSUM (opt)</td>
<td>56</td>
<td>0.36</td>
</tr>
<tr>
<td>CUSUM (L&amp;Y best)</td>
<td>58</td>
<td>0.35</td>
</tr>
<tr>
<td>HMR</td>
<td>62</td>
<td>0.33</td>
</tr>
</tbody>
</table>

4.2.2. Period II

In the comparison of the methods for Period II (day 72–828), the parameter estimates obtained in Period I are used again. It is assumed that one unit of the index is bought at the first buy signal in period II and then sold and bought repeatedly during the whole period.

In Period II, when the CUSUM methods (especially CUSUM(opt)) use parameter estimates which do not agree so well with the data in this period, the return is smaller than for SRnp, which does not use estimates of trends. The HMR method is very different from the others since it does not require strong evidence for an alarm but classifies into either state. On a plateau (where the return is almost independent of any trading strategy) the expected value in (17) fluctuates around zero, i.e. the HMR alarm statistic fluctuates around its alarm limit. On long and monotone upward (or downward) stretches, however
(where one would get the best return by not doing any transactions), the expected value is far from zero and no alarm is given for small fluctuations. This adaptability seems to work very well, and the HMR method has the best return in Period II.

Figure 2 presents the return at each sell signal, and the total return is reported in Table 2.

\[ \text{Figure 2. Values of the returns at each sell-signal during the whole period. The limit between the periods (I and II) is marked with a dashed vertical line. SRnp (--- *), CUSUM(opt) (----- B), CUSUM(L&Y best) (--- △), HMR (--- #).} \]

\[ \text{Table 2. Summary of the return of period II, May 31, 1999 to June 26, 2002.} \]

<table>
<thead>
<tr>
<th>Method</th>
<th># of sell signals</th>
<th>Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>SRnp</td>
<td>37</td>
<td>0.19</td>
</tr>
<tr>
<td>CUSUM (opt)</td>
<td>27</td>
<td>0.15</td>
</tr>
<tr>
<td>CUSUM (L&amp;Y best)</td>
<td>24</td>
<td>0.20</td>
</tr>
<tr>
<td>HMR</td>
<td>42</td>
<td>0.30</td>
</tr>
</tbody>
</table>

4.2.3. Case studies versus Monte Carlo methods

The use of a single case study for evaluation can be questioned. How well does the actual data set represent the process of interest? How extreme is the outcome of the
process? If it is very extreme, then the resulting alarm time is very rare. However, it is impossible to know whether an outcome is extreme or not, unless several examples are available or the process is replicated. If we perform a study where we replicate the process by Monte Carlo methods, we can make statements about the properties of a method under the assumptions made in that study.

4.3. Robustness of alarm systems used in finance

The interpretation of results of case studies should be made with care, and we will now point out some sensitive issues.

4.3.1. Transition probabilities

The HMR method depends on the value of the probabilities of staying in the current state (p11 for expansion and p22 for recession, respectively). In the HMR method, as well as in most HMM-based methods (for example Dewachter (1997), Hamilton (1989), and Marsh (2000)), constant transition probabilities are used. Another approach is to allow duration dependence, which means that the intensity changes with time spent in the regime (for example Maheu and McCurdy (2004)). When only uncertain information about the intensity is available, non-informative weights for the change point time prevents the risk of serious misspecification. This approach is used by the SRnp method.

4.3.2. Alarm limits

For the HMR method, the alarm limit determines the possible lack of symmetry between the two states (expansion and recession). For the other methods it determines the amount of evidence required for any action. In this latter situation, the alarm limit determines the sensitivity of the alarm statistic. A low sensitivity can be motivated by a wish to decrease the frequency of false alarms and thus the transaction costs. In the study above, transaction costs were small compared with the return.
4.3.3. Slopes within the phases

Marsh (2000) addressed the problem of instability of the estimates of $\beta_1$ and $\beta_2$ over different time periods. In Figure 1 it is seen that the cycles vary a lot. It was shown in Andersson et al. (2004) that even a small misspecification of $\mu$ results in long delays for early turns.

When the parameters are determined from the same data that are being monitored, as in period I, the methods which use these estimates give better returns than the nonparametric SRnp method (Table I). However, when these estimates are used in period II, the situation is quite different. The non-parametric method SRnp performs well, even though very few assumptions are made. The other methods, however, which all assume known parameter values, perform badly in some periods where the parameters differ much from the true ones, and the total return is not so impressive for period II (see Table 2) as for period I (see Table 1). When no assumption is made regarding the parametric form of the curve, as in the SRnp method, the surveillance system is robust against changes in the characteristics of the curve over time.

The optimal value for $k$ in CUSUM is $\frac{(\mu^D_\gamma - \mu^C_\gamma)}{2}$. The other parameter of the alarm limit, $g_{\text{CUSUM}}$, can be determined to regulate the false alarm property. Lam and Yam (1997) do not discuss using the optimal criterion for $k$. Instead, the maximum return is considered for combinations of both parameters ($k$ and $g$). For period II we conclude that the value of $k$, estimated from period I, does not fit whereas Lam and Yams value of $k$ fits better. This underlines once more the risks of parametric assumptions.

4.3.4. Autocorrelation

In statistical surveillance there are several ways of dealing with dependent data (see Pettersson (1998) and Frisén (2003) for reviews).
The methods discussed in this paper do not take account of any dependency structure in the alarm statistics. A dependency structure of the disturbance term appears to be present, however, and a first order autoregressive process describes the disturbance term well. Lam and Yam (1997) calculated the $\text{ARL}^0$ analytically, but the possibility of a dependency structure was not discussed. Many methods are constructed under the assumption of independent observations. As a result, they are useful if the dependency is slight but might be misleading if the dependency is strong. The effects of such misspecification are studied for example by Kramer and Schmid (1997) and Schöne et al. (1999).

One approach is to use a method designed for independence but to adjust the alarm limit so that the false alarm property is correct for the dependent process. Such methods are often referred to as “modified control charts” (Kramer and Schmid (1997)). This may work well but is not optimal.

Another approach is to calculate the residuals of an estimated model of the dependency structure and then monitor these. Such methods are often referred to as “residual charts” (Kramer and Schmid (1997)). It was demonstrated by Pettersson (1998) that this results in an approximation of the full likelihood ratio method.

A general approach in statistical surveillance is to eliminate an unknown parameter by a pivot statistic. In finance it is common to differentiate the observations in order to eliminate the dependency. This is done under assumption of a random walk. If the undifferentiated process is independent, however, the first difference $Y$ will be a MA(1) process.

As is done in Section 3.2.1 for an independent process, it is possible to derive the likelihood ratio statistic for a process with a dependency structure. The LR-statistic for a change in the mean of a stationary AR(1) process with normally distributed disturbances
was derived in Pettersson (1998). The partial likelihood ratio $L(s, i)$ for a change in the mean of a MA process where the disturbance term has a Gaussian distribution is derived in Petzold et al. (2004).

Further improvements can be expected by using methods of surveillance that takes the dependency structure into account.

5. **Concluding remarks**

A desirable property of any prospective monitoring method is that a change is detected quickly (timeliness) and without too many unmotivated signals of change (safety). In the general theory of statistical surveillance, the aim is to optimize the method with respect to these properties. Thus, the aims of trading rules in finance agree with those of statistical surveillance.

The common strategies suggested for financial trading decisions, which have been investigated in this paper are all special cases of well-known methods of statistical surveillance.

The Filter rule (or trend breaking rule) FR has been shown to be a special case of the CUSUM method. GFR is a close approximation of CUSUM that does take the aspect of the relation between the slopes of the upward and downward trends into account. It is well known that the CUSUM method has minimax optimality properties. Thus, the FR and GFR methods will have approximate minimax properties under certain conditions.

Methods based on moving averages are common and relatively simple to construct. One method, the oscillator method, which is widely used for trading, consists of comparing two overlapping moving averages of different lengths. In this paper it has been demonstrated that this approach is equivalent to a moving average method described in the literature of surveillance, where the in-control mean is estimated by a moving average. A special case often considered is when the current observation is compared with a simple
moving average of observations. It has been shown that this method is similar to the Shewhart method of surveillance where, again, the in-control mean is estimated by a moving average. The optimality properties of these two surveillance methods are known. They have good properties for detecting large recent changes. This does not, however, necessarily imply that the moving average trading rule is optimal, as the result depends on the properties of the moving average estimator. In any case, the method cannot be considered suited to detect small changes which occur gradually, since only the last observations are used.

Several rules for financial trading use a hidden Markov model approach. The inferential structure of the HMR method is equivalent to that of the optimal LR method when the specifications of the states that are to be discriminated between are the same. This requires, however, that we use knowledge of the type of the next turn. Thus, the method is also optimal in the sense that the expected utility is maximized. However, the results from the monitoring also depend on the knowledge of (or the method for estimating) parameters and the distribution of $\tau$.

Since the aim of a financial decision rule generally is to maximize the expected return (adjusted for the risk exposure and transaction costs), return measures are natural to use. Most of the results in the theory of statistical surveillance have been developed solely on the use of the timeliness scale. However, timeliness and return are, as we have demonstrated, closely related.

Improper assumptions regarding the process under surveillance may have great impact on performance. Thus, single case studies are very sensitive to how representative the chosen data are. The application to the Hang Seng Index demonstrated for the HMR method a lack of robustness against errors in the estimation of the transition probabilities.
When the distribution of the time of the change $\tau$ is unknown, non-informative weights can be used in order to avoid the risk of serious misspecification. This is what the SRlin and SRnp methods use. One way of avoiding the risk of seriously misspecifying the regression is to use the non-parametric approach of the SRnp method. The case studies illustrate the advantage of the SRnp method when the current turn has a different shape than the previous one.

Since there are many problems to deal with in the implementation of surveillance in finance, further research is needed. Much effort has been made on the modeling aspect of processes related to finance, such as volatility, the arrival time of transactions, and smooth transitions between regimes. However, much remains to be done as regards the implementation of these models in a decision system, although the most recent research is very promising. The use of knowledge on statistical surveillance for the construction of financial trading rules will certainly be of value.

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<th>Title</th>
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<td>2007:1</td>
<td>Andersson, E.</td>
<td>Effect of dependency in systems for multivariate surveillance.</td>
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<td>2007:3</td>
<td>Bock, D.</td>
<td>Consequences of using the probability of a false alarm as the false alarm measure.</td>
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