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Geometrical and percolative properties of spatially correlated models

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Abstract

This thesis consists of four papers dealing with phase transitions in various models of continuum percolation. These models exhibit complicated dependencies and are generated by different Poisson processes. For each such process there is a parameter, known as the intensity, governing its behavior. By varying the value of this parameter, the geometrical and topological properties of these models may undergo dramatic and rapid changes. This phenomenon is called a phase transition and the value at which the change occur is called a critical value.

In Paper I, we study the topic of visibility in the vacant set of the Brownian interlacements in Euclidean space and the Brownian excursions process in the unit disc. For the vacant set of the Brownian interlacements we obtain upper and lower bounds of the probability of having visibility in *some* direction to a distance r in terms of the probability of having visibility in a *fixed* direction of distance r. For the vacant set of the Brownian excursions we prove a phase transition in terms of visibility to infinity (with respect to the hyperbolic metric). We also determine the critical value and show that at the critical value there is no visibility to infinity.

In Paper II we compute the critical value for percolation in the vacant set of the Brownian excursions process. We also show that the Brownian excursions process is a hyperbolic analogue of the Brownian interlacements.

In Paper III, we study the vacant set of a semi scale invariant version of the Poisson cylinder model. In this model it turns out that the vacant set is a fractal. We determine the critical value for the so-called existence phase transition and what happens at the critical value. We also compute the Hausdorff dimension of the fractal whenever it exists. Furthermore, we prove that the fractal exhibits a nontrivial connectivity phase transition for $d \geq 4$ and that the fractal is totally disconnected for $d = 2$. In the case $d = 3$ we prove a partial result showing that the fractal restricted to a plane is totally disconnected with probability one.

In Paper IV we study a continuum percolation process, the random ellipsoid model, generated by taking the intersection of a Poisson cylinder model in $\mathbb{R}^d$ and a subspace of dimension k. For $k \in \{2, 3, ..., d-2\}$, we show that there is a non-trivial phase transition concerning the expected number of ellipsoids in the cluster of the origin. When $k = d-1$ this critical value is zero. We compare these results with results obtained for the classical Poisson Boolean model.

Keywords: continuum percolation, Brownian interlacements, Brownian excursions, Poisson cylinder model, fractal percolation.