WORKING PAPERS IN ECONOMICS

No 558

Publicly Provided Private Goods and Optimal Taxation when Consumers Have Positional Preferences

Thomas Aronsson and Olof Johansson-Stenman

March 2013

ISSN 1403-2473 (print)
ISSN 1403-2465 (online)
Publicly Provided Private Goods and Optimal Taxation when Consumers Have Positional Preferences

Thomas Aronsson* and Olof Johansson-Stenman+

First version: April 2012
This version: February 2013

ABSTRACT
This paper analyzes optimal differential commodity taxation, together with optimal nonlinear income taxation, in order to deal with positional preferences. It also derives the optimal public provision of private goods both when differential commodity taxation is feasible and when it is not. It is shown that publicly provided non-positional private goods which are (possibly imperfect) substitutes for positional private goods should be used as a corrective instrument even if the tax system is optimal, i.e. even when differential commodity taxation is feasible. An exception is the special case where all consumers contribute equally much to the positional externality, in which the commodity tax constitutes a perfect instrument for internalizing the positional externality.

Keywords: Public provision of private goods, income taxation, commodity taxation, relative consumption, asymmetric information, status, positional goods.

JEL Classification: D62, H21, H23, H41.

** Research grants from the Swedish Research Council (dnr 421-2010-1420) are gratefully acknowledged.
* Address: Department of Economics, Umeå School of Business and Economics, Umeå University, SE – 901 87 Umeå, Sweden. E-mail: Thomas.Aronsson@econ.umu.se
+ Address: Department of Economics, School of Business, Economics and Law, University of Gothenburg, SE – 405 30 Gothenburg, Sweden. E-mail: Olof.Johansson@economics.gu.se
A great portion of the expenses of the higher and middle classes in most countries, and the greatest in this, is not incurred for the sake of the pleasure afforded by the things on which the money is spent, but from regard to opinion, and an idea that certain expenses are expected from them, as an appendage of station; and I cannot but think that expenditure of this sort is a most desirable subject of taxation. If taxation discourages it, some good is done, and if not, no harm; for in so far as taxes are levied on things which are desired and possessed from motives of this description, nobody is the worse for them.

(John Stuart Mill, 1848, Principles of Political Economy, book V, chapter VI)

1. INTRODUCTION

The observation that some goods are consumed largely to signal social status is of course far from new. Moreover, as demonstrated by the initial quotation of Mill, the idea that it may be particularly attractive from a social point of view to tax such goods is not very novel either. Yet, the academic literature on tax differentiation based on the extent by which goods are consumed for status concerns is very limited. Fred Hirsch (1976) coined the term “positional goods” for goods where the utility derived depends primarily on how much one consumes in relation to everyone else.

In the optimal taxation literature, it is most often assumed that consumers derive utility solely from their own absolute consumption of goods, services and leisure. This traditional view has been challenged during the latest decade through a growing empirical literature showing that people seem also to care about their own consumption relative to that of other people.¹ As a consequence, a literature on optimal income taxation in response to relative consumption concerns has gradually evolved since the late 1970s, see Boskin and Sheshinski (1978), Layard (1980), Oswald (1983), Frank (1985), Tuomala (1990), Corneo and Jeanne (1997, 2001), Ljungqvist and Uhlig (2000), Ireland (2001), Brekke and Howarth (2002), Dupor and

¹ This evidence include happiness research (e.g., Easterlin, 2001; Blanchflower and Oswald, 2005; Ferrer-i-Carbonell, 2005; Luttmer, 2005; Clark and Senik, 2010), questionnaire-based studies (e.g., Johansson-Stenman et al., 2002; Solnick and Hemenway, 2005; Carlsson et al., 2007), psychological studies (e.g., Mamot, 2004; Daly and Wilson, 2009) and brain science (e.g., Fliessbach et al., 2007; Dohmen et al., 2011). The study by Stevenson and Wolfers (2008) constitutes a recent exception, claiming that the role of relative income is overstated.
Liu (2003), Abel (2005), Aronsson and Johansson-Stenman (2008, 2010), Wendner and Boulder (2008), Micheletto (2011) and Wendner (2010a, 2010b, 2011). Parts of this literature are based on second-best models where leisure is unobservable and parts are based on nonlinear income taxation. What a large part of this literature has in common is that people are assumed to care about relative consumption but not about relative leisure, such that consumption causes negative externalities on others.\(^2\) Such positional concerns lead to externalities, due to that increased consumption by each individual reduces the relative consumption among other people, and may also imply important challenges for redistribution policy. These arguments for policy intervention are further strengthened by the order of magnitude of the concerns for relative consumption: several studies find that the effect of relative consumption on individual welfare is substantial, i.e. the utility gain to the individual of increased consumption is to a fairly large extent driven by increased relative (instead of absolute) consumption.\(^3\)

However, if different goods are consumed for different reasons, and in particular if they incur positional externalities on others to a varying extent, for which there is clear empirical evidence (e.g., Solnick and Hemenway, 1998; Alpizar et al., 2004; Carlsson et al., 2007), it appears likely that it will be optimal to tax these goods differently. For example, it may be optimal to tax luxury show-off goods such as luxury cars and jewelries higher than, say, insurances, where it is even difficult to observe others’ consumption. As far as we know, the present paper together with Eckerstorfer (2011), which are largely developed independently, constitute the only papers that are dealing with differential commodity taxation in a second-best framework when people care about relative consumption.

Yet, differential commodity taxation is of course not without problems. If the basis for such differentiation is consumption positionality, or the strength of the positional externality, it is often not straightforward to characterize a good as positional or not. For example, private cars are often seen as positional goods: indeed, they are easy to observe and expensive cars are

---

\(^2\) A study by Aronsson and Johansson-Stenman (2013) constitutes a recent exception by analyzing the optimal tax policy in an economy where people both care about their relative consumption and relative use of leisure.

\(^3\) According to the survey-experimental evidence presented by Solnick and Hemenway (1998), Johansson-Stenman et al. (2002), Alpizar et al. (2005), and Carlsson et al. (2007), about 50 per cent (on average) of the utility gain to the individual of an extra dollar spent on consumption (with others’ consumption held constant) is due to increased relative consumption.
typically seen as a status symbol. Still, not all cars are status symbols. This means that one would ideally like to tax some cars at substantially higher rates than others, and such differentiation may be difficult to obtain. Furthermore, one may also foresee substantial manipulations of the care manufacturers, of which some may be welfare reducing. For this reason it makes sense to consider alternative policy measures to deal with such variation of positional externalities. One such possibility, which, as far as we know, has not been analyzed before in the context of relative consumption concerns, is public provision of non-positional private goods, such as insurance and health care. Indeed, it is typically found that a large part of the goods provided by governments are not public good, but rather private goods, which constitutes a puzzle based on conventional public economics.

The present paper presents a framework flexible enough to simultaneously encompass nonlinear income taxes, linear commodity taxes and public provision of private goods, as means of redistribution and correction under relative consumption concerns. Such a framework is motivated because real world tax systems often represent mixtures of a nonlinear income tax and linear commodity taxes, and many types of public expenditure refer to private goods. In fact, a mixed tax system is the most flexible system compatible with the information structure typically characterizing real world economies, where (a) individual ability is private information, (b) income is observable at the individual level, and (c) the commodity trade is anonymous (i.e. only observed at the aggregate level). The main purposes of our paper are to characterize the optimal marginal tax structure as well as analyze the simultaneous use of commodity taxation and public provision of non-positional private goods as instruments for internalizing positional externalities when the income tax is optimal. This will be discussed in greater detail below.

As mentioned, earlier studies in this area have so far primarily focused on the implications of positional concerns for optimal redistributive income taxation (e.g., Tuomala, 1990; Aronsson and Johansson-Stenman, 2010; Micheletto, 2011), or public good provision in combination with optimal income taxation (e.g. Aronsson and Johansson-Stenman, 2008). A noteworthy exception is the recent paper by Eckerstorfer (2011), which examines the optimal mix of income and commodity taxation under relative consumption concerns. He finds that such concerns in general affect both the income tax and commodity taxes, and that the

---

4 See also Oswald (1983), who analyzed optimal nonlinear commodity taxation as a means of redistribution and externality correction under relative consumption concerns.
commodity tax system does not fulfill the so called additivity property. The only exception is where all agents contribute equally much to the positional externalities (such that the common reference measure for all individuals is given by the average consumption in the economy as a whole), in which case the principle of targeting applies and the commodity tax targeting the positional good becomes a perfect instrument for correction.\(^5\)

Our study differs from that of Eckerstorfer in at least three ways. First, and foremost, we consider public provision of non-positional private goods as an additional instrument for redistribution and correction. More specifically, we are concerned with non-positional (possibly imperfect) substitutes for positional private goods, and the way in which such instruments may play a supplemental role to the tax system. Just to exemplify, if we assume that certain types of cars or boats represent positional goods,\(^6\) then public transport (appropriately defined) may constitute non-positional substitutes. Similarly, if a degree from certain private schools is a positional good\(^7\), then a well functioning public education system may constitute the non-positional substitute. We show that public provision of such private goods is motivated from the perspective of pure externality-correction as long as different individuals do not generate positional externalities to the same extent (in which case commodity taxation becomes a perfect instrument for such correction). Furthermore, even in the (unlikely) case where all individuals generate positional externalities to the same extent, there is still a second best motive for public provision due to that the positional concerns may vary among different agents. Second, we present the effective marginal tax rate faced by each agent-type at the second best optimum, which is the natural measure of tax distortion when several tax instruments are available, and analyze how positional concerns modify these rates. This will explain how corrective and redistributive aspects of a mixed tax system interact under relative consumption concerns, and has not (to our knowledge) been addressed in earlier comparable literature. Third, we present the results on a format where the policy rules for optimal taxation and public provision directly depend on the extent to which different agent-types are positional, i.e. the strength of the relative consumption concerns, which

\(^{5}\) Veblen (1899) and Duesenberry (1949) argued that people typically compare themselves upwards in the income distribution, in which case the positional externalities are mainly generated by high-income earners. This is also consistent with the empirical findings of Bowles and Park (2005).

\(^{6}\) Empirical evidence suggests that goods which are clearly visible are often more positional than other, less visible, goods.

\(^{7}\) Cf. Solnick and Hemenway (2009).
provides a straightforward generalization of the results on optimal income taxation presented in Aronsson and Johansson-Stenman (2008). This makes it possible to decompose the tax structure into three parts: (i) components that are not due to relative consumption concerns, (ii) a corrective component that reflects the positional externality, and (iii) incentives to relax the incentive constraint by exploiting that agents may differ with regards to positional concerns.

The outline of the study is as follows. In Section 2, we present the decision-problems faced by consumers along with the outcome of private optimization. We assume that consumers differ in ability, and that individual ability is private information. The decision-problem faced by the government is characterized in Section 3, where we utilize the two-type model with asymmetric information between the government and the private sector developed in its original form by Stern (1982) and Stiglitz (1982). This model provides a simple – and at the same time very powerful – framework for capturing redistributive and corrective aspects of taxation and public provision, as well as for capturing the policy incentives caused by interaction between the incentive constraint and the desire to internalize positional externalities. The reason as to why such interaction is important is that policies designed to correct for positional externalities may either contribute to relax or tighten the incentive constraint. In Section 4, we present the results in terms of optimal taxation and public provision of non-positional substitutes to the private positional good. Section 5 summarizes and concludes the paper; proofs are presented in the Appendix.

2. DECISION-PROBLEMS IN THE PRIVATE SECTOR

We start this section by characterizing consumer preferences, and then continue with the commodity demand and labor supply behavior. There are two types of individuals; a low-ability type (type 1) and a high-ability-type (type 2), where the high-ability type is more productive than the low-ability type measured by the before-tax wage rate. The number of individuals of ability-type \( i \) is denoted \( n^i \), and \( N = \sum_i n^i \) denotes the total population.

2.1 Preferences and Relative Consumption Concerns
Each individual derives utility from consuming two private goods; a non-positional good, $c$, and a positional good, $x$, as well as from leisure, $z$. All goods are normal, and the two consumption goods are assumed to be complements in the sense that the marginal utility of consuming the positional good increases with the consumption of the non-positional good, and vice versa. Leisure is defined as a fixed time endowment, $H$, less the hours of work, $l$. We model positional (relative consumption) concerns by assuming that each individual also derives utility from his/her own consumption of the positional good relative to a reference measure. In accordance with the bulk of earlier comparable literature on relative consumption comparisons, we also assume that the relative consumption is given by the difference between the individual’s own consumption and the relevant reference measure.\footnote{See, e.g., Akerlof (1997), Corneo and Jeanne (1997), Ljungqvist and Uhlig (2000), Bowles and Park (2005), Carlsson et al. (2007), and Aronsson and Johansson-Stenman (2008, 2010). Alternative approaches include ratio comparisons (Boskin and Sheshinski 1978; Layard 1980; Wendner and Goulder 2008) and comparisons of ordinal rank (Frank 1985; Hopkins and Kornienko 2004, 2009). Dupor and Liu (2003) consider a flexible functional form that includes the difference and ratio comparisons as special cases. Mujcic and Frijters (2011) compare models with difference comparisons, ratio comparisons and rank-based comparisons without being able to discriminate between them. However, recent evidence by Corazzini, Esposito and Majorano (forthcoming) suggests that not only the rank, but also the magnitudes of the differences play a role.\footnote{The marginal utility associated with the absolute consumption of $x$ (the second argument) must not increase too much if the relative consumption of $x$ (the forth argument) increases.}}

The utility function facing any individual of ability-type $i$ can then be written as (for $i=1,2$)

$$U^i = u'(c^i, x^i + \beta^i g, z^i, x^i - x^R) = u'(c^i, x^i, z^i, g, x^R)$$ \hspace{1cm} (1)$$

where $x^R$ denotes the measure of reference consumption for the positional good, to which we return later, while $g$ represents the publicly provided non-positional substitute to $x$. The parameter $\beta^i \in (0,1]$ measures the degree of substitutability (if $\beta^i < 1$, the publicly provided good is an imperfect substitute to $x$). The function $u'()$ is assumed to be increasing in each argument, which means that $u'()$ is increasing in $c^i$, $x^i$, $z^i$ and $g$, and decreasing in $x^R$ (a property referred to as “jealousy” by Dupor and Liu, 2003). Both $u'()$ and $u''()$ are strictly concave, and $u'()$ is also assumed to be such that $u''_{sg} = \partial^2 u' / \partial x' \partial g < 0$, which places additional restraint on some of the second order cross derivatives of $u'()$.\footnote{The marginal utility associated with the absolute consumption of $x$ (the second argument) must not increase too much if the relative consumption of $x$ (the forth argument) increases.}
that the demand for the positional good decreases in response to public provision; the scenario of main concern here.

The reason as to why we also write out the (quite general) function $u'(\cdot)$ is that many of the result to be derived below do not necessitate any stronger assumption than that the reference measure represents a negative externality; yet, other results necessitate that the relative consumption is measured explicitly, which is where the function $\nu'(\cdot)$ comes in. Notice also that although $x^r$ is an endogenous variable in the model (see below), each individual is assumed to be small relative to the economy as a whole and treat $x^r$ as exogenous, which is conventional in the literature dealing with externalities.

Before we analyze the consumer’s decision problem in greater detail, it is useful to introduce a measure of the extent to which relative consumption is important at the individual level. We will do so by introducing the individual degree of consumption positionality with respect to the positional good. As such, we make use of the assumption that the relative consumption concerns are governed by difference comparisons. Let $\Delta^i = x^i - x^r$ denote the relative consumption of the positional good by ability-type $i$. By using the short notation $e^i = x^i + \beta^i g$, we may then rewrite the first utility formulation in equation (1) as $U^i = \nu'(e^i, e', z', \Delta^i)$. Following Johansson-Stenman et al. (2002), the degree of consumption positionality can now be defined as follows:

$$\alpha^i = \frac{\nu'_e}{\nu'_e + \nu'_\Delta} \in (0,1),$$

where subscripts denote partial derivatives, i.e. $\nu'_e = \partial \nu'/\partial e^i$ and $\nu'_\Delta = \partial \nu'/\partial \Delta^i$. Notice also that combing the functions $\nu'(\cdot)$ and $u'(\cdot)$ in equation (1) gives $u'_e = \nu'_e + \nu'_\lambda$, which means that the denominator of equation (2) represents the marginal utility of consumption of the positional good for ability-type $i$. Therefore, $\alpha^i$ is interpretable as the fraction of the overall utility gain for ability-type $i$ of an additional dollar spent on commodity $x$ that is due to increased relative consumption. In Section 4 below, we show how the optimal tax mix and public provision are directly interpretable in terms of degrees of positionality.
For further use, we also define the average degree of consumption positionality is the economy as a whole as follows:

$$\overline{\alpha} = \frac{\sum_{i} n^i \alpha^i}{N} \in (0,1).$$

(3)

2.1 Consumer Behavior and Equilibrium

The individual budget constraint is given by

$$w^i l^i - T(w^i l^i) = c^i + qx^i$$

where the non-positional private good is treated as the numeraire, whose consumer price is normalized to one. In equation (4), $w^i$ denotes the before-tax wage rate facing ability-type $i$, $T(\cdot)$ is the income tax, and $q$ the consumer price of commodity $x$. The consumer price is given by $q = p + t$, where $p$ is the producer price and $t$ the commodity tax.

The consumer’s decision-problem it to choose $c^i$, $x^i$ and $l^i$ to maximize the utility function given in equation (1) subject to the budget constraint represented by equation (4), while treating $g$, $x^R$, $w^i$ and $q$ as exogenous. Following earlier literature on optimal mixed taxation (e.g., Christiansen, 1981; Edwards et al., 1994), it is convenient to solve this decision-problem in two stages: in the first, we derive commodity demand and the indirect utility function conditional on the use of leisure (i.e. conditional on the labor supply); in the second, we choose the labor supply to maximize the conditional indirect utility function. The reason for using this particular approach is that the second best problem to be presented in Section 3 below will be written in terms of the conditional demand and indirect utility function. By using the second utility formulation in equation (1), i.e. the function $u^i(\cdot)$, the first stage problem can be written as

$$\max_{c^i,x^i,z^i,g,x^R} u^i(c^i,x^i,z^i,g,x^R) \text{ s.t. } b^i = c^i + qx^i \text{ and } x^i \geq 0,$$

where $b^i$ is a fixed disposable income. The nonnegativity constraint is added to allow for crowding-out due to public provision. Solving the budget constraint for $c^i$ and then substituting into the objective function, the first order condition can be written
\[-u'_i q + u'_i x \leq 0 \quad (= 0, \text{ if } x > 0) \quad (5)\]

where \( u'_i = \partial u_i / \partial c^i \) and \( u'_x = \partial u_i / \partial x^i \). If the nonnegativity constraint does not bind, we can use equation (5) to derive the following condition demand function and then substitute into the direct utility function to obtain the conditional indirect utility function:

\[
x^i = x^i (q, b^i, z^i, g, x^R) \quad (6)
\]

\[
V' = V'(q, b^i, z^i, g, x^R)
= u' (b^i - qx^i (q, b^i, z^i, g, x^R), x^i (q, b^i, z^i, g, x^R), z^i, g, x^R). \quad (7)
\]

In the second stage, we choose \( l^i \) to maximize the conditional indirect utility function given by equation (7), subject to the following relationship between the disposable income and before-tax income: \( b^i = w^i l^i - T(w^i l^i) \). The first order condition for work hours can then be written as

\[
V^i w^i [1 - T'(w^i l^i)] - V^i = 0. \quad (8)
\]

Turning to production, and following the convention in much earlier literature on optimal income taxation and mixed taxation, we assume that output is produced through a linear technology, which is interpretable to mean that the before-tax wage rates and producer price are fixed.

The empirical evidence for how reference groups are formed is scarce. Earlier studies typically assume that each individual compares his/her own consumption with the average consumption in the economy as a whole, although alternative formulations such as upward comparisons (Aronsson and Johansson-Stenman, 2010; Micheletto, 2011) and within-generation comparisons (Aronsson and Johansson-Stenman, 2010) have also been considered. In this paper, we use a general formulation where the reference consumption is defined as a weighted average of the consumption by the two ability-types in the sense that
\[ x^R = \sum_i \zeta^i x^i, \text{ where } \sum_i \zeta^i = 1. \] (9)

Equation (9) encompasses the mean value comparison (\( \zeta^i = n^i / N \) for all \( i \)), in which case all individuals contribute equally to the positional externality, the upward comparison (\( \zeta^2 > n^2 / N \) and \( \zeta^1 < n^1 / N \)), and the downward comparison (\( \zeta^1 > n^1 / N \) and \( \zeta^2 < n^2 / N \)) as special cases. The upward comparison means that the consumption by the high-ability type carries a larger than proportional weight in the reference measure, while the downward comparison analogously means that the consumption by the low-ability type carries a larger than proportional weight. As a consequence, equation (9) also encompasses the extreme cases where each individual only compares his/her own consumption with that of the high-ability type (\( \zeta^2 = 1 \)), and with that of the low-ability type (\( \zeta^1 = 1 \)).

The comparative statics properties of equation (9) can be summarized as follows (for \( i=1,2 \)):

\[
\begin{align*}
\frac{\partial x^R}{\partial b^i} &= \Gamma \zeta^i \frac{\partial x^i}{\partial b^i}, \\
\frac{\partial x^R}{\partial z^i} &= \Gamma \zeta^i \frac{\partial x^i}{\partial z^i}, \\
\frac{\partial x^R}{\partial q} &= \sum_i \zeta^i \frac{\partial x^i}{\partial q}, \\
\frac{\partial x^R}{\partial g} &= \sum_i \zeta^i \frac{\partial x^i}{\partial g}
\end{align*}
\] (10)

where \( \Gamma \) denotes a feedback effect defined as

\[ \Gamma = \frac{1}{1 - \sum_i \zeta^i (\frac{\partial x^i}{\partial x^R})}. \]

We will base our interpretations on the assumption that \( \Gamma > 0. \)\textsuperscript{10} In this case, we have \( \frac{\partial x^R}{\partial b^i} > 0 \) for \( i=1,2 \), \( \frac{\partial x^R}{\partial q} < 0 \), and \( \frac{\partial x^R}{\partial g} < 0 \). The use of leisure by any ability-type, on the other hand, may either contribute to increase or decrease \( x^R \).

3. THE SECOND BEST PROBLEM

The objective of the government is to reach a Pareto efficient allocation. This will be accomplished by maximizing utility of the low-ability type, while holding utility constant for the high-ability type, subject to a self-selection constraint and a resource constraint. The

\textsuperscript{10} This constitutes a stability condition; see Sandmo (1980).
informational assumptions are conventional: the government is able to observe income, although ability is private information. Following the bulk of earlier literature on the self-selection approach to optimal taxation, we assume that the government wants to redistribute from the high-ability type to the low-ability type. This means that the most interesting aspect of self-selection is to prevent the high-ability type mimicking the low-ability type. The self-selection constraint that may bind can then be written as

\[ V^2 = V^2(q, b^2, z^2, g, x^k) \geq V^2(q, b^1, H - \phi l^1, g, x^k) = \tilde{V}^2, \]  

where \( \phi = w^1 / w^2 < 1 \) is the wage ratio. The expression on the right-hand side of the weak inequality is the utility of the mimicker (i.e. a high-ability type who pretends to be a low-ability type). Although the mimicker faces the same combination of disposable and before-tax income as the low-ability type, he/she enjoys more leisure since the mimicker is more productive than the low-ability type. Throughout the paper, mimicker-variables will be denoted by the hat ("\( \hat{\} \)") symbol.

Note that the nonlinear income tax can be used to implement any desired combination of work hours and disposable income for each ability-type (subject to the available information and resources). Therefore, we follow much earlier literature on optimal nonlinear and mixed taxation by formulating a direct public decision-problem in terms of work hours and disposable income instead of in term of parameters of the income tax function. The optimal marginal income tax rates can then be derived by combining the social first order conditions with the private first order conditions for work hours. By using \( T(w^i l^i) = w^i l^i - b^i \) for \( i=1,2, \) the public sector budget constraint becomes

\[ \sum_i n^i \left[ w^i l^i - b^i \right] + t \sum_i n^i x^i(q, b^i, z^i, g, x^k) - Ng = 0. \]  

Equation (12) means that the tax revenue (net of redistribution) is used to finance the publicly provided private good.

The Lagrangean corresponding to the public decision-problem is given by
\[ \mathcal{L} = V^1 + \delta [V^2 - \bar{V}^2] + \lambda [V^2 - \bar{V}^2] + \gamma \left[ \sum_i n' \left( w'i - b' \right) + t \sum_i n' x'i - Ng = 0 \right], \] (13)

where \( x'i \) and \( V'i \) (for \( i = 1, 2 \)) are given by equations (6) and (7), respectively, while \( \bar{V}^2 \) is defined in equation (11). In equation (13), \( \bar{V}^2 \) is the fixed minimum utility for the highability type, while \( \delta \), \( \lambda \) and \( \gamma \) are the Lagrange multipliers associated with the minimum utility restriction, self-selection constraints and budget constraint, respectively. Notice also that \( x^R \) is endogenous and determined by equation (9): as such, the government is assumed to recognize how the reference consumption changes in response to public policy.

The optimal tax structure is governed by the first order conditions for \( l^1 \), \( b^1 \), \( l^2 \), \( b^2 \) and \( t \), which are written as follows:

\[ l^1 : \quad -V^1 z + \lambda \bar{V}^2 z + \gamma n \left[ w^1 i - t \frac{\partial x^1}{\partial z} \right] - \Omega \frac{\partial x^R}{\partial z} = 0, \] (14a)

\[ b^1 : \quad V^1 b - \lambda \bar{V}^2 b + \gamma n \left[ -1 + t \frac{\partial x^1}{\partial b^1} \right] + \Omega \frac{\partial x^R}{\partial b^1} = 0, \] (14b)

\[ l^2 : \quad -(\delta + \lambda) V^2 z + \gamma n \left[ w^2 i - t \frac{\partial x^2}{\partial z} \right] - \Omega \frac{\partial x^R}{\partial z} = 0, \] (14c)

\[ b^2 : \quad (\delta + \lambda) V^2 b + \gamma n \left[ -1 + t \frac{\partial x^2}{\partial b^2} \right] + \Omega \frac{\partial x^R}{\partial b^2} = 0, \] (14d)

\[ t : \quad V^1 q + (\delta + \lambda) V^2 q - \lambda \bar{V}^2 q + \gamma \sum_i n' \left[ x'^i + t \frac{\partial x'^i}{\partial q} \right] + \Omega \frac{\partial x^R}{\partial t} = 0. \] (14e)

where subscripts attached to the conditional indirect utility function denotes partial derivative. Since the analysis to be carried out below does not necessarily require that the publicly provided private good is optimally chosen, we refrain from presenting a first order condition for \( g \) here. Instead, we return to the publicly provided private good in Section 4.

The final term on the right hand side of equations (14a)-(14e) appears because the reference consumption is endogenous and determined by equation (9). The variable \( \Omega = \frac{\partial \mathcal{L}}{\partial x^R} \) measures the partial welfare effect on an increase in the reference consumption, ceteris
paribus, and we will refer to $\Omega$ as the “shadow price of reference consumption” in what follows.

4. OPTIMAL TAXATION AND PUBLIC PROVISION

Subsection 4.1 characterizes the shadow price of the reference consumption, and explains how this shadow price depends on the degrees of positionality defined above. We will then address optimal taxation, where commodity and income taxation are analyzed in subsection 4.2, while subsection 4.3 concerns the effective marginal tax rates (the overall marginal rate following an increase in the before-tax income). Finally, in subsection 4.4, we analyze public provision of the non-positional private good and, in particular, the usefulness of such public public provision when both the income tax and commodity tax are optimally chosen.

4.1 Shadow Price of Reference Consumption

The shadow price of reference consumption in equations (14a)-(14b) plays a key role in terms of optimal taxation and public provision. It defined as follows:

$$\Omega = \frac{\partial L}{\partial x^R} = V^1_x + (\delta + \lambda)V^2_x - \lambda \dot{V}^2_x + \gamma t \sum_i n_i \frac{\partial \bar{x}'_i}{\partial x^R}.$$  (15)

Although the first two terms on the right hand side of equation (15) are negative, due to that increased reference consumption leads to lower utility for both ability-types ($x^R$ only affects individual utility through the argument $x' - x^R$), the third term is positive while the sign of the fourth term is ambiguous. As a consequence, the right hand side of equation (14) can be either positive or negative.

For the analysis to be carried out below, it is useful to express $\Omega$ in terms of the degrees of consumption positionality defined in subsection 2.1. As such, we extend the corresponding measure of shadow price presented by Aronsson and Johansson-Stenman (2008) into a model with more than one consumption good and a mixed tax system. Denote ability-type $i$'s compensated (Hicksian) demand for the positional good by $\bar{x}'_i$, and let $\alpha^2$ denote the degree of consumption positionality by the mimicker (which is defined in the same general way as
the corresponding measure for the true ability-type $i$ in equation (2)). We can then characterize $\Omega$ as follows:

**Proposition 1.** The shadow price of reference consumption over the shadow price of the public budget constraint can be written as

$$\frac{\Omega}{\gamma} = \frac{1}{\Psi} \left[ -Nq\bar{\alpha} + \lambda^* q\left( \hat{\alpha}^2 - \alpha^i \right) + \sum \sum \frac{\partial^2 x}{\partial \tilde{\alpha} R^i} \right],$$

where $\lambda^* = \lambda^2 / \gamma > 0$ and $\Psi = 1 - \sum \alpha^i (\hat{\alpha}^R / \hat{b}^i)$.

Proof: see the Appendix.

In Proposition 1, we have expressed the shadow price of reference consumption in terms of public funds, simply because it will appear in this particular form in the formulas for optimal taxation and public expenditure to be presented below. The expression within square brackets on the right hand side of equation (16) decomposes the (normalized) shadow price of reference consumption into three distinct parts: (i) an effect through the average degree of consumption positionality, (ii) an effect through the difference in the degree of positionality between the mimicker and the low-ability type, and (iii) a tax base effect due to the commodity tax. In Aronsson and Johansson-Stenman (2008), only the first two effects were present; let be in a different form than here, since their study made no distinction between different private consumption goods and was, instead, based on the assumption that the private consumption as a whole constitutes a positional good. The interpretations given below are based on the assumption that the scale factor $1/\Psi$ is positive, in which case the sign of $\Omega/\gamma$ is determined by the sign of the expression within square brackets. We return to the intuition behind this scale factor below and also explain why it is reasonable to assume that it is positive.

The average degree of consumption positionality, $\bar{\alpha}$, has a negative effect on $\Omega/\gamma$. This component reflects the consumption externality and appears in equation (16) because an increase in $x^R$ contributes to reduce the utility faced by each ability-type by lowering the
relative consumption (in equation (1), this effect goes through \( x^i - x^R \) in the function \( \nu^i(\cdot) \)). Notice that this negative effect becomes stronger if \( \alpha \) increases; the intuition is that the higher the average degree of consumption positionality, ceteris paribus, the greater will be the welfare loss faced by the consumers due to the positional externality. The second term within square brackets reflects an incentive for the government to relax the self-selection constraint by exploiting that the mimicker and the low-ability type may differ with respect to positional concerns. If the mimicker is more positional than the low-ability type, if follows that an increase in \( x^R \) leads to a larger utility loss for the mimicker than for the low-ability type. As such, there is an incentive for the government to relax the self-selection constraint by implementing policies that lead to increased reference consumption, which shows up as a larger shadow price in equation (16). If the mimicker is less positional than the low-ability type, the argument for reducing the reference consumption is analogous, and the second term within square brackets is negative in that case.

Notice also that the first and second terms within the square bracket are proportional to the consumer price of the positional good, \( q \). To see the intuition behind this result, use equations (1), (2) and (8) to derive \( V^i_{s} = -\alpha^i u^i_{s} \) and \( u^i_{b} = V^i_{b} \), which in combination with equation (3) gives

\[
q \alpha^i = -MRS^i_{s,b} = -\frac{V^i_{s}}{V^i_{b}}.
\]

Therefore, the individual’s marginal willingness-to-pay to avoid the externality depends on the product of the consumer price and the individual degree of consumption positionality. As such, the consumer price of the positional good will influence the strengths by which the average degree of consumption positionality and the measure of the positionality difference between the mimicker and the low-ability type affect the shadow price of reference consumption.

The tax base effect depends on how the conditional compensated demand for the positional good responds to an increase in the reference consumption, i.e.

\[
\frac{\partial x^i}{\partial x^R} = \frac{\partial x^i}{\partial x^R} - MRS^i_{s,b} \frac{\partial x^j}{\partial b^j} \text{ for } i=1,2
\]

The reason as to why the tax base effect is measured in terms of the compensated (instead of uncompensated) demand is that equation (16) presupposes that the income tax is optimal.
This is seen in the Appendix, where we derive equation (16) by combining equations (14b), (14d) and (15). Note that the tax base effect can be either positive or negative, depending on whether $x^R$ is complementary with the positional good (in the sense that $\partial x^i / \partial x^R > 0$) or substitutable for the positional good (in which case $\partial x^i / \partial x^R < 0$).

The scale factor $1/\Psi$ in equation (16) is due to that $\Omega$ directly affect the first order conditions for $b^1$ and $b^2$, which is seen from equations (14b) and (14d). Since the positional good is normal by assumption, i.e. $\partial x^i / \partial b^i > 0$, it follows that the government can reduce the consumption of the positional good and, therefore, the reference consumption solely by collecting income tax revenue which, in turn, exacerbates the use of public expenditure to correct for positional externalities. As such, this effect is expected to scale up $\Omega/\gamma$ in the sense of magnifying the benefit (cost) of decreasing (increasing) $x^R$. In a simplified model with only one private consumption good and where each individual contributes equally much to the positional externality (so $\zeta^i = n^i / N$) – the case considered in Aronsson and Johansson-Stenman (2008) – this intuition leads the whole way, since such a model implies that $1/\Psi = 1/(1 - \bar{\alpha}) > 1$. In the present model, on the other hand, we cannot rule out that $\Psi = 1 - q\sum \alpha' (\partial x^R / \partial b^i) < 0$; let be that this outcome is very unintuitive (as it would imply that the individual utility loss of increased reference consumption is associated with a social welfare gain). If we disregard this possibility, in which case $\Psi \in (0,1)$ by the assumptions made earlier, it follows that $1/\Psi > 1$. As such, the intuition presented above applies. We concentrate on this (highly plausible) case in the discussion of tax and expenditure policy below.

### 4.2 Commodity Taxation

It is useful to start by a general characterization of commodity taxation, and then go into greater detail about how the underlying policy incentives are affected by positional concerns. Effective marginal tax rates (capturing overall tax distortions faced by each ability-type) will be addressed in the next subsection.

Let $\bar{x}^R = \sum_i \zeta^i \bar{x}^i$ denote the reference consumption based on compensated (Hicksian) demand. By the properties of the compensated demand, we have
If the income and commodity taxes are optimal, we can use equations (14b), (14d) and (14e) to characterize the tax on the positional good follows:

\[
\frac{\partial x^R}{\partial q} = \frac{\partial x^R}{\partial q} + \sum_i \frac{\partial x^R}{\partial b'} x' = \sum_i \varepsilon_i \left[ \frac{\partial x^i}{\partial q} + \frac{\partial x^i}{\partial b'} x' \right] < 0.
\]  

(17)

Equation (18) decomposes the commodity tax formula into two parts. The first term within square brackets reflects an incentive for the government to relax the self-selection constraint by exploiting that the mimicker and the (mimicked) low-ability type may prefer different consumption bundles. If the mimicker consumes more of commodity \( x \) than the low-ability type, so \( \hat{x}^2 > x^1 \), then this effect works to increase the commodity tax; the opposite applies if \( x^1 > \hat{x}^2 \). Note that this effect has nothing to do with positional concerns, i.e. it would be present also in the special case where the consumers do not derive utility from their relative consumption, i.e. the special case where \( \Omega = 0 \). As such, this policy incentive is well understood from earlier research (e.g., Edwards et al. 1994) and will not be further discussed here.

The second term is proportional to the shadow price of reference consumption, \( \Omega / \gamma \), and arises because positional concerns give rise to externalities. We explained in the context of Proposition 1 that \( \Omega / \gamma \) can be either positive or negative, as it reflects a mixture of pure externality correction and incentives to relax the self-selection constraint by exploiting that the mimicker and the low-ability type may differ with respect to the degree of consumption positionality. To see these different incentives more clearly, we combine equations (16) and (18) to derive the following result:
Proposition 2. If the income tax is optimal, then the optimal commodity tax can be written on the following additive form:

\[
\begin{align*}
t & = \frac{1}{D} \left[ \lambda^* (x^i - \hat{\lambda}^i) + \frac{qN}{\Psi} \frac{\partial \lambda^R}{\partial q} \left\{ \alpha - \frac{\lambda^* (\hat{\alpha}^i - \alpha^i)}{N} \right\} \right] \\
\end{align*}
\]  

(19)

where \( D = \sum \left[ n^i \frac{\partial \lambda^W}{\partial q} + n^i \frac{\partial \lambda^R}{\partial q} \frac{1}{\Psi} \right] \).

Proof: see the Appendix.

In equation (19), the variable \( D \) reflects the total effect of price on compensated demand, which also contains the indirect effect through the externality, \( x^R \). If the reference consumption is weakly complementary with the positional good in the sense that \( \frac{\partial \alpha^i}{\partial x^R} \geq 0 \), then \( D \) is unambiguously negative; otherwise, \( D \) can be either positive or negative. In what follows, we will assume that \( D < 0 \), which is the case that makes most intuitive sense.

Equation (19) means that the optimal commodity tax reflects a mixture of three distinct policy components; (i) an incentive to relax the self-selection constraint through channels that do not reflect positional concerns, (ii) an incentive to internalize the positional externality, and (iii) an incentive to relax the self-selection constraint by exploiting that the mimicker may either be more or less positional than the low-ability type. We have already briefly commented on incentive effect (i) above. Incentive effect (ii) depends on the average degree of consumption positionality, \( \bar{\alpha} \), without any reference to differences in the degree of consumption positionality between individuals. As such, this term reflects pure externality correction and works to increase the tax on the positional good. Finally, incentive effect (iii) depends on the difference in the degree of consumption positionality between the mimicker and the low-ability type. If the low-ability type is more positional than the mimicker, i.e. if \( \alpha^1 > \hat{\alpha}^2 \), increased reference consumption will hurt the low-ability type more than it hurts the mimicker. As such, the government may then relax the self-selection constraint through a higher commodity tax, meaning that incentive effects (ii) and (iii) jointly contribute to increase the tax on the positional good. On the other hand, if \( \hat{\alpha}^2 > \alpha^1 \), meaning that the
mimicker is more positional than the low-ability type, the government may relax the self-selection constraint through a policy that leads to increased reference consumption. In this case, we can no longer rule out that positional concerns leads to a lower tax on the positional good.

The following result is an immediate consequence of Proposition 2:

**Corollary 1.** If (i) the self-selection constraint does not bind, meaning that $\lambda = 0$, or (ii) the utility function takes the form $v^i(c', x', \beta, z', x^R) = b'(a(c', x' + \beta g, x' - x^R), z')$, the optimal commodity tax simplifies to read

$$t = P \frac{\partial x^R}{\partial q} \tilde{\alpha}$$

(20)

where $P = qN / D\Psi$.

Part (i) of the corollary is obvious; part (ii) follows because if the sub-utility function $a(\cdot)$ is common to all consumers, and if $\beta' = \beta$ (for $i=1,2$) in which case the degree of substitutability between $x$ and $g$ is the same for all consumers, then $x^l = \tilde{x}^2$ and $\alpha^l = \tilde{\alpha}^2$. As a consequence, incentive effects (i) and (iii) referred to above vanish. Note also that this holds despite that the function $b'(\cdot)$ may differ between the ability-types. Since $P < 0$ by the assumptions made earlier, we can interpret equation (19a) such that the optimal commodity tax is positive, and that an increase in the average degree of consumption positionality leads to an increase in the optimal commodity tax, ceteris paribus. Differences in the degree of consumption positionality between individuals no longer directly affect the commodity tax.

**4.3 Effective Marginal Tax Rates**

In a mixed tax system, it is useful to derive an overall measure of optimal tax distortion. Such a measure is given by the effective marginal tax rate, which measures how the sum of the income and commodity tax payments by the individual responds to a small increase in the before-tax income. The total tax paid by ability-type $i$ is given by
\( \tau^i = T(w'l') + t\alpha'(q, b^i, z^i, x^R) \). \hspace{1cm} (21)

By differentiating equation (21) with respect to \( l' \), using \( b^i = w'l' - T(w'l') \) and \( z^i = H - l^i \), and dividing by \( w^i \), we obtain

\[
\tau^i s = \frac{1}{w^i} \frac{\partial \tau^i}{\partial l'} = T'(w'l') + t \left[ \frac{\partial \alpha^i}{\partial b^i} (1 - T'(w'l')) - \frac{1}{w^i} \frac{\partial \alpha^i}{\partial z^i} \right].
\] \hspace{1cm} (22)

We can then use the first order conditions given by equations (14a)-(14e) to analyze how positional concerns modify the effective marginal tax rates by comparison with earlier literature on optimal mixed taxation.

Let \( MRS_{z,b}^i = V_z^i / V_b^i \) denote the marginal rate of substitution between leisure and disposable income for ability-type \( i \) (for \( i=1,2 \)), while \( \hat{MRS}_{z,b}^2 = \hat{V}_z^2 / \hat{V}_b^2 \) denotes the corresponding marginal rate of substitution for the mimicker. For notational convenience, define \( \tau^i s \) for \( i=1,2 \) to be the effective marginal tax rate faced by ability-type \( i \) in the absence of any positional concerns, i.e. in the conventional two-type model of optimal mixed taxation where \( \Omega = 0 \), i.e.

\[
\tau^i s = \frac{\lambda^i}{w'n} \left[ MRS_{z,b}^1 - \hat{MRS}_{z,b}^2 \theta \right]
\] \hspace{1cm} (23a)

\[
\tau^i s = 0.
\] \hspace{1cm} (23b)

The right hand side of equation (23a) is familiar from earlier research: the government may relax the self-selection constraint by exploiting that the mimicker typically has flatter indifference curves in \( (wl-b) \) space than the mimicker, in which case a positive effective marginal tax rate imposed on the low-ability type leads to a relaxation of the self-selection constraint.\(^{11}\) There is no corresponding mechanism by which marginal taxation of the high-ability type relaxes the self-selection constraint, which explains why the right hand side of equation (23b) is zero.

\(^{11}\) Strictly speaking, this interpretation presupposes that the preferences do not differ between ability-types.
Returning to the general model where the consumers are concerned with their relative consumption, i.e. where \( \Omega \neq 0 \), and with equations (24a)-(24b) at our disposal, we have derived the following result:

**Proposition 3.** The effective marginal tax rates can be characterized as (for \( i = 1, 2 \))

\[
\tau_i' = \tau_i + \frac{1}{\Psi w' n'} \left[ \lambda^* (x^1 - \hat{x}^2) \sum_i n_i' \frac{\partial \bar{x}^i}{\partial x^R} + (1 - \eta) N \left( -\bar{x}^* + \frac{\lambda^* (\hat{\alpha}^2 - \alpha')}{N} \right) \right] \frac{\partial x^R}{\partial z^i} \tag{24}
\]

where \( \eta = \frac{q N R^k}{\Psi D} \frac{\partial x^R}{\partial q} \sum_i n_i' \frac{\partial \bar{x}^i}{\partial x^R} \).

Proof: see the Appendix.

In equation (24), all terms within the square bracket are multiplied by

\[
\frac{\partial \bar{x}^R}{\partial z^i} = \frac{\partial x^R}{\partial z^i} - MRS^i_{z,b} \frac{\partial x^R}{\partial b^i} = \frac{\xi^i}{\Gamma} \left[ \frac{\partial x^i}{\partial z^i} - MRS^i_{z,b} \frac{\partial x^i}{\partial b^i} \right],
\]

showing how the reference consumption measured in terms of compensated demand responds to increased use of leisure by ability-type \( i \). Consider first the case where leisure and the reference consumption are substitutes in the sense that \( \frac{\partial x^R}{\partial z^i} < 0 \), which also encompasses the special case where utility is separable in \( z^i \) (so \( \frac{\partial x^R}{\partial z^i} = -MRS^i_{z,b} \frac{\partial x^R}{\partial b^i} < 0 \)). The first term within the square bracket appears due to that a tax base effect of \( x^R \) affects the shadow price of reference consumption according to equation (16). As such, this component serves to reduce the social loss of positional concerns by exploiting that the reference consumption can be used to affect the tax revenue raised through the commodity tax. If increased use of leisure reduces the reference consumption, and if the commodity tax is positive (in which case \( \hat{x}^2 > x^1 \)), this effect contributes to increase the effective marginal tax rate if the reference consumption is substitutable for \( x^i \) in the sense that \( \frac{\partial x^i}{\partial x^R} < 0 \), and decrease the effective marginal tax rate if \( \frac{\partial x^i}{\partial x^R} > 0 \) meaning that the reference consumption is complementary with \( x^i \).
The second component in square brackets depends on the average degree of consumption positionality, \( \bar{\alpha} \), and the difference in the degree of positionality between the mimicker and the low-ability type, \( \hat{\alpha}^2 - \alpha^i \). Both these variables are, in turn, multiplied by \( 1 - \eta \), which reflects the feedback effect of \( x^R \) on the compensated demand for the positional good. By the assumptions made earlier, \( \eta > 1 \) \((<1)\) if \( \partial x^R / \partial \bar{\alpha} \) \((-0)\) meaning that the feedback effect reinforces (weakens) any tax and expenditure policy designed to reduce \( x^R \). For the case considered here, where \( \partial x^R / \partial z^i < 0 \) due to substitutability between \( x^i \) and \( z^i \), we can see that the average degree of consumption positionality unambiguously contributes to increase the effective marginal tax rate. Furthermore, the government may, in this case, relax the self-selection constraint through a higher effective marginal tax rate is the low-ability type is more positional than the mimicker (\( \alpha^i > \hat{\alpha}^2 \)), and a lower effective marginal tax rate if the mimicker is more positional than the low-ability type (\( \hat{\alpha}^2 > \alpha^i \)). As before, the former situation provides an incentive for the government to exploit that increased reference consumption hurts the low-ability type more than it hurts the mimicker, while the latter instead means that the mimicker is hurt more than the low-ability type by increased reference consumption.

Instead, if leisure and the reference consumption are complements in the sense that \( \partial x^R / \partial z^i > 0 \), the qualitative effect of the average degree of consumption positionality and the difference in the degree of positionality between the mimicker and the low-ability type, respectively, will be the opposite to those discussed above. In this case, a lower (instead of higher) effective marginal tax rate leads decreased reference consumption through decreased use of leisure.

Finally, consider once again the special case with common sub-utility functions and leisure separability, where the following result is an immediate consequence of Proposition 3:

**Corollary 2.** If \( v'(c^i, x^i + \beta^i g, z^i, x^R - x^i) = b'(a(c^i, x^i + \beta g, x^R - x^i), z^i) \), then the effective marginal tax rate simplifies to read (for \( i=1,2 \))

---

12 Again, this presupposes that the feedback effect is not strong enough to dominate, so \( 1 - \eta > 0 \).
\[
\tau' = \tau_{s} + \frac{(1 - \eta)N \bar{\alpha}}{\Psi_{W'n}^{i} MRS_{s,b}^{i} \frac{\partial x^{R}}{\partial b}}.
\]  

(25)

This means that positional concerns will contribute to an increase in the effective marginal tax rate.

4.4 Public Provision

Define by \( MRS_{g,b}^{i} = V_{g}^{i} / V_{b}^{i} \) for \( i=1,2 \) and \( M\hat{R}S_{g,b}^{2} = \hat{V}_{g}^{2} / \hat{V}_{b}^{2} \) to be the marginal rate of substitution between the publicly provided good and disposable income for ability-type \( i \) and the mimicker, respectively. Our first result is summarized as

**Proposition 4.** If the income tax is optimal, the policy rule for public provision can be characterized as

\[
\frac{\partial \mathcal{L}}{\partial g} \frac{1}{\gamma} = \sum_{i} n_{i} \left[ MRS_{g,b}^{i} - 1 \right] + \lambda^{i} \left[ MRS_{g,b}^{i} - M\hat{R}S_{g,b}^{2} \right] + t \sum_{i} n_{i} \frac{\partial x^{i}}{\partial g} + \frac{\Omega}{\gamma} \frac{\partial x^{R}}{\partial g}. 
\]  

(26)

Proof: see the Appendix.

To begin with, note that the cost benefit rule in Proposition 4 does not require that the commodity tax is optimal; instead, it applies for any given commodity tax, \( t \). This provides a suitable starting point by characterizing the incentive effects underlying policy without comparing their relative size of individual terms. The first term on the right hand side measures the sum of the consumers’ marginal-willingness-to-pay for the publicly provided good less the direct marginal cost in terms of lost disposable income. The second term reflects the difference in the marginal-willingness-to-pay for the publicly provided good between the low-ability type and the mimicker. As such, this effect serves to relax the self-selection constraint: if the low-ability type is willing to pay more (less) than the mimicker, there is an incentive for the government to provide more (less) than it would otherwise have done.
Now, recall that the publicly provided good is defined as a non-positional substitute for the positional good, \( x \). As such, the consumer’s marginal-willingness-to-pay for the publicly provided good depends on the marginal-willingness-to-pay for the “non-positional services” provided by commodity \( x \), as well as on the degree of substitutability between \( g \) and \( x \), i.e. \( \beta^i \). This is easily seen by using equation (1) to derive

\[
MRS_{g,b}^i = \frac{V_g}{V_b} = \frac{\beta^i}{\beta^i} = \frac{u_x^i}{u_x^i} = \beta^i \frac{u_x^i}{u_x^i} = \beta^i q(1-\alpha^i)
\]

in which \( MRS_{g,c}^i = u_x^i / u_c^i \), and where we have used \( MRS_{g,c}^i = q \) from the consumer’s first order condition given by equation (5). We can interpret \( q(1-\alpha^i) \) as the non-positional benefit of the positional good for ability-type \( i \), which if multiplied by \( \beta^i \) gives the marginal-willingness-to-pay for the publicly provided good by ability-type \( i \). Therefore, \( MRS_{g,b}^i \) may either exceed, or fall short of, unity, which means that the first term on the right hand side of equation (26) can be either positive or negative. The same applies to the second term on the right hand side, the sign of which both depends on whether the mimicker is more or less positional than the low-ability type, and whether the publicly provided good is a stronger or weaker substitute for the (high-ability) mimicker than for the low-ability type.

To be able to interpret the first and second terms on the right hand side in terms of positional concerns, it is instructive to consider the case where \( \beta^i = \beta^2 \), meaning that the degree of substitutability between the publicly provided good and the positional private good is the same for both ability-types. We can then rewrite equation (26) to read

\[
\frac{\partial \mathcal{L}}{\partial g} \bigg|_1 = \sum_i n^i \left[ \beta q(1-\alpha^i) -1 \right] + \lambda^i q \beta \left( \hat{\alpha}^2 - \alpha^1 \right) + t \sum_i n^i \frac{\partial \tilde{\alpha}^i}{\partial g} + \Omega \frac{\partial \tilde{\alpha}^g}{\partial g}. \tag{27}
\]

The second best motive to exploit the difference in the degree of positionality between the mimicker and the low-ability type is now clearly visible from the second term on the right hand side. If the mimicker is more positional than the low-ability type, so \( \hat{\alpha}^2 > \alpha^1 \), the mimicker will benefit less than the low-ability type from the publicly provided good, simply because the publicly provided good is a non-positional substitute to commodity \( x \). As such, the government may relax the self-selection constraint through increased public provision. It,
on the other hand, the low-ability type is more positional than the mimicker, we have the opposite incentive to reduce public provision.

So far, we have analyzed public provision conditional on the commodity tax, i.e. we have not yet assumed that the commodity tax is optimal. An interesting question then arises: what is the supplemental role of publicly provided private goods when both the income tax and the commodity tax are optimal? To be more specific, is the commodity tax a perfect instrument for targeting the welfare consequences of positional concerns, or is there still a corrective or redistributive role for public provision due to that the consumers have positional preferences? We have derived the following result:

**Proposition 5.** If the income and commodity taxes are optimal, the policy rule for the publicly provided good becomes

\[
\frac{\partial L}{\partial g} = \sum_i n^i \left[ \beta q (1-\alpha') - 1 \right] + 
\sum_i n^i (\partial \tilde{x}^i / \partial q) \sum_i n^i (\partial \tilde{x}^i / \partial q) + 
\sum_i n^i (\partial \tilde{x}^i / \partial g) \sum_i n^i (\partial \tilde{x}^i / \partial g)
\]

Two implications are immediate from equation (28). First, if all individuals contribute equally to the positional externality, so \( \zeta' = n' / N \), then the shadow price of reference consumption, i.e. \( \Omega / \gamma \), will vanish from equation (28). In this case, the commodity tax will be a perfect instrument for internalizing the positional externality, which means that public provision will not be used for externality correction. Second, even if we were to assume that all individuals contribute equally much to the externality, positional concerns will, nevertheless, directly affect the cost benefit rule for the publicly provided good through the third term on the right hand side of equation (28). As such, there is still a second best incentive to adjust the public provision in response to differences in the degree of positionality between the mimicker and the low-ability type.

Note that \( \zeta' = n' / N \) means that the reference consumption is given by the mean value of the positional good measured for the economy as a whole, i.e. \( \bar{x} = \sum_i n' x' / N \). Although this
assumption is very common in earlier comparable studies on optimal taxation under relative consumption concerns, it runs counter to arguments put forward by Veblen (1899) and Duesenberry (1949) as well as the empirical evidence presented in Bowles and Park (2005), which imply that people tend to compare themselves upwards, i.e. with those who can afford to spent more on the positional good. This suggests that the consumption of the positional good by the high-ability type might carry a larger than proportional weight in the welfare measure. Below, we discuss this case more thoroughly in terms of public provision.

If the consumption of the positional good by the high-ability carries a larger than proportional weight in the reference measure, we have \( \zeta^2 > n^2 / N \). Let us formalize this idea by assuming that \( \zeta^2 = n^2 / N + \kappa \), where \( \kappa \in (0, 1 - n^2 / N] \) the degree to which the weight attached to the high-ability type exceeds the share of high-ability individuals. By using \( \sum_i \zeta^i = 1 \), we can then rewrite the fourth term on the right hand side of equation (28) as

\[
\frac{\Omega}{\gamma} \frac{\kappa N}{\sum_i n'(\hat{\lambda}^i / \partial q)} \left[ \frac{\partial x^i}{\partial g} \frac{\partial x^2}{\partial q} - \frac{\partial x^1}{\partial g} \frac{\partial x^2}{\partial q} \right]. \tag{29}
\]

Consider first the case where \( \Omega / \gamma < 0 \), which means that increased reference consumption leads to lower social welfare, ceteris paribus, which appears to be a plausible assumption. Therefore, since \( \hat{\lambda}^i / \partial q < 0 \), we have

\[
\frac{\Omega}{\gamma} \frac{\kappa N}{\sum_i n'(\hat{\lambda}^i / \partial q)} > 0,
\]

meaning that the whole expression (29) is positive if the expression within square brackets is positive, in which case the fourth term on the right hand side of equation (28) contributes to increase the optimal provision of the non-positional good, \( g \). Since \( \hat{\lambda}^i / \partial g < 0 \) for \( i = 1, 2 \) by the assumptions made earlier, the role of \( g \) as a supplemental instrument for correction now becomes clear. More specifically, two mechanisms contribute to increased public provision: (i) the more price-sensitive the compensated demand for the positional good by the low-ability-type, and the less price-sensitive the compensated demand for the positional good by the high-ability type, and (ii) the more substitutable \( x^2 \) for \( g \), and the less substitutable \( x^1 \) for \( g \). The intuition is that, with upward comparisons, the commodity tax is too low for internalizing the externality generated by the high-ability type; it is also higher than necessary for correcting the externality generated by the low-ability type. As such, two corrective aspects of public provision can be established, which are related to the mechanisms discussed.
above: (i) to compensate the low-ability type for excessive commodity taxation, and (ii) to reduce the consumption of the positional good by the high-ability type. Note also that these incentives for public provision are particularly strong if \( \kappa \) reaches the upper limit, i.e. if \( \kappa = 1 - n^2 / N \) so \( \zeta^2 = 1 \), in which case all positional externalities are generated by the high-ability type.

Under the less likely scenario with downward comparisons (where the consumption of the positional good by the low-ability type carries a larger than proportional weight in the reference measure), or if the shadow price of reference consumption is positive (i.e. \( \Omega / \gamma > 0 \)), analogous policy incentives will follow, with the modification that mechanisms (i) and (ii) discussed above work to reduce (instead of increase) the optimal public provision of \( g \).

Notice also that if the income and commodity taxes are optimal, the corrective motive for public provision will vanish if either ability-type is crowded out by the tax system, i.e. if either \( x^1 = 0 \) or \( x^2 = 0 \) when \( g = 0 \). This is seen from equation (28), where the final term on the right hand side is zero if either \( x^1 = 0 \) or \( x^2 = 0 \). The intuition behind this result is straightforward: if the consumption of the positional good by one of the ability-types is zero at a conditional second best optimum where \( g = 0 \), it follows that the commodity tax will be used to fully internalize the positional externality generated by the other ability-type. As such, there is no need to use public provision as a supplemental instrument for correction.

5. CONCLUSION

In this paper, we examine the simultaneous use of optimal mixed taxation and publicly provided private goods under relative consumption concerns, where one of the private consumption good is, in part, a positional good. More specifically, we consider public provision of a non-positional (possibly imperfect) substitute for the positional private good under optimal taxation. The analysis is based on an extension of the two-type model for optimal mixed taxation; here extended by positional preferences as well as a publicly provided private good.
The appearance of positional concerns gives rise to a corrective motive for public policy, due to that such concerns lead to negative externalities, as well as an incentive for the government to relax the self-selection constraint by exploiting differences in the degree of positionality between consumer-types. Our results imply that the optimal commodity tax on the externality generating good as well as the effective marginal tax rates become higher the more positional people are on average, ceteris paribus. These optimal taxes are generally higher (lower) if the mimicker is less (more) positional than the low-ability type, since an increase in the reference consumption in that case leads to a tightening (relaxation) of the self-selection constraint.

Furthermore, we show that publicly provided private goods of the type discussed above should in general be used as a corrective instrument even if the tax system is optimal; an exception is the special case where all consumers contribute equally much to the positional externality, in which the commodity tax constitutes a perfect instrument for internalizing the positional externality. However, such a scenario would imply that the appropriate measure of reference consumption is given by the average consumption of the positional good in the economy as a whole, which runs counter both to arguments put forward by Veblen (1899) and Duesenberry (1949) as well as to more recent empirical evidence in favor of upward comparisons. We show that if people tend to compare themselves upwards in the ability-distribution in the sense that the consumption of the positional good by the high-ability type carries a larger than proportional weight in the reference measure, pure externality correction motivates increased public provision (a) the more price-sensitive the demand for the positional good by the low-ability-type relative to the corresponding demand by the high-ability type, and (b) if the publicly provided private good reduces the high-ability type’s demand for the positional good more than it reduces the low-ability type’s demand. Finally, even if we were to consider the special case where all consumers contribute equally to the positional externality, we are still able to derive a novel second best motive for public provision of a non-positional substitute to the positional private good: if the mimicker is more (less) positional than the mimicked low-ability type, increased (decreased) public provision contributes to a relaxation of the self-selection constraint.

Future research may take several directions, and we will briefly discuss two of them here. First, our analysis is based on a static model, despite that many (presumably) positional goods such as cars, boats and houses have durable goods properties. As such, publically provided non-positional substitutes to such goods ought to have durable good character as well.
Second, a dynamic model also allows for much more flexible measures of the reference consumption at the individual level (cf. Aronsson and Johansson-Stenman 2010), and it would be interesting to analyze how optimal taxation and public provision of non-positional private goods interact in such a flexible environment. We hope to address these issues in future research.

**APPENDIX**

*Proof of Proposition 1*

Let \( \Delta_i = x^i - x^R \) and use equations (1) and (7) to derive

\[
V_{x^i} = u'_{x^i} = -u'_{\Delta_i}
\]

(A1)

where subindices denote partial derivatives. From equations (A1), (3) and (5), and by using \( V_{b^i} = u'_{c} \), we can then establish that

\[
V_{x^i} = -\alpha' u_{x^i} = -\alpha' \frac{u'_{c}}{u'_{c}} u_{c} = -\alpha' q V_{b^i}.
\]

(A2)

Substituting into equation (15) gives

\[
\Omega = -\alpha' q V_{b^i} - (\delta + \lambda) \alpha^2 q V_{b^i} + \lambda \hat{\alpha}^2 q \hat{V}_{b^i}^2 + \gamma t \sum n' \frac{\tilde{\alpha}^i}{\tilde{\alpha}^R}.
\]

(A3)

By solving equation (14b) for \( V_{b^i} \) and equation (14d) for \( (\delta + \lambda) V_{b^i} \) and then substituting into equation (A3), we obtain

\[
\Omega = -\alpha' q \left[ \lambda \phi \hat{V}_{b^i}^2 + \gamma n' \left( 1 - t \frac{\tilde{\alpha}^i}{\tilde{\alpha}^R} \right) - \Omega \frac{\tilde{\alpha}^R}{\tilde{\alpha}^R} \right] - \alpha^2 q \gamma n' \left( 1 - t \frac{\tilde{\alpha}^2}{\tilde{\alpha}^2} \right) - \Omega \frac{\tilde{\alpha}^R}{\tilde{\alpha}^R} + \lambda q \hat{\alpha} \hat{V}_{b^i}^2 + \gamma t \sum n' \frac{\tilde{\alpha}^i}{\tilde{\alpha}^R}.
\]

(A4)

Collecting \( \Omega \)-terms on the left hand side and rearrangements give

\[
\Omega \left( 1 - q \sum \alpha' \frac{\tilde{\alpha}^R}{\tilde{\alpha}^b} \right) = -\gamma q \sum \alpha' n' + \lambda q \hat{V}_{b^i}^2 [\hat{\alpha}^2 - \alpha^2] + \gamma t \sum n' \frac{\tilde{\alpha}^i}{\tilde{\alpha}^R}.
\]

(A5)

Use equation (A2) to define \( MRS_{x^i,b} = V_{x^i} / V_{b^i} = -q \alpha' \) and calculate
\[
\frac{\partial x^i}{\partial x^k} = \frac{\partial x^j}{\partial x^k} - MRS_{x^j,x^b} \frac{\partial x^i}{\partial b_i} \quad \text{for } i, j = 1, 2.
\]

Substituting into equation (A5) and dividing by \( \gamma \) gives equation (16). \( \blacksquare \)

**Proof of Proposition 2**

Rewrite equation (18) to read

\[
t \sum_i n^i (\partial \tilde{x}^i / \partial q) = \lambda^* (x^1 - \tilde{x}^2) - \frac{\Omega}{\gamma} \frac{\partial \tilde{x}^R}{\partial q} .
\]

Use equation (16) to substitute for \( \Omega / \gamma \) in equation (A7)

\[
t \sum_i n^i (\partial \tilde{x}^i / \partial q) = \lambda^* (x^1 - \tilde{x}^2) - \left[ \frac{1}{\Psi} \left( -Nq\alpha + \lambda^* q (\dot{\alpha}^1 - \alpha^1) + t \sum_i n^i \frac{\partial \tilde{x}^i}{\partial x^R} \right) \right] \frac{\partial \tilde{x}^R}{\partial q} .
\]

Collecting \( t \)-terms and rearranging gives equation (19). \( \blacksquare \)

**Proof of Proposition 3**

Consider first the effective marginal tax rate of the low-ability type. Use equations (8), (14a) and (14b) to define the marginal income tax rate

\[
T'(w^{11}) = \frac{1}{n^1 w^1} \left[ \lambda^* (MRS^1_{z,b} - \tilde{MRS}_{z,b} \phi) + m^1 \frac{\partial \tilde{x}^1}{\partial z^1} + \frac{\Omega}{\gamma} \frac{\xi^1}{\Gamma} \frac{\partial \tilde{x}^1}{\partial z^1} \right]
\]

where

\[
\frac{\partial \tilde{x}^1}{\partial z^1} = \frac{\partial x^1}{\partial z^1} - MRS^1_{z,b} \frac{\partial x^1}{\partial b^1} .
\]

Substituting equation (A9) into equation (22) gives

\[
w^{11} \tau^{11} = \frac{1}{n^1} \left[ \lambda^* (MRS^1_{z,b} - \tilde{MRS}_{z,b} \phi) + \frac{\Omega}{\gamma} \frac{\xi^1}{\Gamma} \frac{\partial \tilde{x}^1}{\partial z^1} \right] .
\]

Finally, by substituting equations (16) and (19) into equation (A10) gives equation (24). \( \blacksquare \)

**Proof of Proposition 4**

Differentiating the Lagrangean in equation (13) with respect to \( g \) gives
\[
\frac{\partial L}{\partial g} = V^i + (\delta + \lambda) V^2 - \lambda \hat{V}^2 + \gamma \left[ t \sum n_i \frac{\partial x^i}{\partial g} - N \right] + \Omega \frac{\partial x^R}{\partial g} .
\]  
(A11)

By using \( MRS_{g,b}^i = V^i / V_b^i \) for \( i = 1,2 \), and \( \hat{MRS}^2_{g,b} = \hat{V}^2 / \hat{V}_b^2 \) and substituting into equation (A11), we have

\[
\frac{\partial L}{\partial g} = MRS_{g,b}^1 V^i + (\delta + \lambda) MRS_{g,b}^2 V^2 - \lambda \hat{MRS}^2_{g,b} \hat{V}^2 + \gamma \left[ t \sum n_i \frac{\partial x^i}{\partial g} - N \right] + \Omega \frac{\partial x^R}{\partial g} .
\]  
(A12)

Solve equation (14b) for \( V^i_b \) and equation (14d) for \( (\delta + \lambda) V^2_b \), and substitute into equation (A12)

\[
\frac{\partial L}{\partial g} = MRS_{g,b}^1 \left[ \lambda \phi \hat{V}^2 + \gamma n_1 - \gamma n_1 t \frac{\partial x^i}{\partial b^i} - \Omega \frac{\partial x^R}{\partial b^i} \right] + MRS_{g,b}^2 \left[ \gamma n_2 - \gamma n_2 t \frac{\partial x^2}{\partial b^2} - \Omega \frac{\partial x^R}{\partial b^2} \right] - \lambda \hat{MRS}^2_{g,b} \hat{V}^2_b .
\]  
(A13)

Define

\[
\frac{\partial \hat{x}^i}{\partial g} = \frac{\partial \hat{x}^i}{\partial g} - MRS_{g,b}^i \frac{\partial \hat{x}^i}{\partial b^i} \quad \text{for} \ i = 1,2
\]  
(A14)

\[
\frac{\partial x^R}{\partial g} = \frac{1}{\Gamma} \sum \zeta^i \frac{\partial \hat{x}^i}{\partial g} .
\]  
(A15)

Substituting equations (A14) and (A15) into equation (A13), dividing by \( \gamma \) and rearranging gives equation (26).  

**Proof of Proposition 5**

By substituting equation (18) into equation (27), while using

\[
\frac{\partial x^R}{\partial g} = \frac{1}{\Gamma} \sum \zeta^i \frac{\partial \hat{x}^i}{\partial g} \quad \text{and} \quad \frac{\partial x^R}{\partial q} = \frac{1}{\Gamma} \sum \zeta^i \frac{\partial \hat{x}^i}{\partial q} ,
\]

we obtain
\[
\frac{\partial \mathcal{L}}{\partial g} = \sum_i n_i \left[ \beta q (1 - \alpha_i) - 1 \right] + \lambda^i q \beta \left( \ddot{\alpha}^i - \alpha_i \right) \\
+ \left[ \frac{\lambda^i (\dot{\alpha}^i - \ddot{\alpha}^i)}{\sum_i n_i (\dot{\alpha}^i / \delta q)} - \frac{\Omega}{\gamma} \sum_i n_i (\dot{\alpha}^i / \delta q) \right] \sum_i n_i \dddot{x}^i \\
+ \frac{\Omega}{\gamma} \sum_i \dot{x}^i \frac{\partial \dddot{x}^i}{\partial g}.
\]

(A16)

Rearrangement gives equation (28). \(\blacksquare\)

REFERENCES


