ESSAYS ON EXCHANGE RATES AND CENTRAL BANK CREDIBILITY

Per-Ola Maneschiöld
Dedication

To my mother and father for their continuous support and love.
Abstract

This thesis contains four separate empirical papers. Paper I tests the behaviour of the volatility of eight Swedish exchange rates over the recent floating period. Various econometric tests are performed in an attempt to identify the presence of ARCH-effects. The sign bias test reveals no significant evidence of asymmetries in the data, which then suggests that it is appropriate to fit the standard ARCH-models to the data. The less restrictive ARCH-models that allow for asymmetric effects do contradict the sign bias test for three of the included exchange rates. This then indicates the importance of combining different tests for examining the same type of effect in the conditional volatility models.

Paper II analyses the behaviour of the eight Swedish exchange rates over the short and long run horizons for significant long memory effects and the possible dynamic effect on the volatility models estimated in paper I. The ARFIMA-GARCH test provides no support for long memory in the Swedish exchange rate. The GPH-estimator provides a significant result in two cases; SEKFIM and SEKGBP. There is then no systematic significant evidence of a long memory property in the Swedish exchange rate. The volatility models, estimated in paper I, were not significantly different upon estimating a long memory structure in the exchange rate data.

Paper III presents empirical evidence on the link between the real exchange rate and the real interest rate differential within a small open economy with an inflation target. Use of the Johansen cointegration technique shows that the real exchange rate and real interest rate are non-stationary but cointegrated. The empirical evidence supports a long-run relationship between real exchange rates and expected real interest differentials in Sweden. The estimated model leads to real exchange rate forecasts that are superior to those generated by a random-walk model. The Diebold and Mariano test also support this superiority in out-of-sample prediction.

Paper IV analyses the issue of central bank credibility for the Swedish Central Bank using the long memory concept. The concept is used to analyse the credibility of the Swedish Central Bank before and after it adopted a strict inflation target for its monetary policy and became relatively independent. If the central bank is very highly
to fully credible, then the memory structure in the inflation rate should be insignificant, i.e. there should be no significant feedback effect from past realised inflation rates on the current inflation rate. Using the GPH-estimator shows that the long memory parameter is significant for the pre-independent period but not so for the post-independent period. Furthermore, the estimates are significantly different from each other using the Chow-test. This evidence strengthens indications that the credibility level for the monetary policy objective of the Swedish Central Bank has increased. The objective by which this increase in credibility has occurred is through the new inflation-target policy regime and an independent central bank.
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Introduction and summary.

The four essays in this thesis deal with topics related to exchange rate theory, monetary policy credibility, and financial econometrics. The empirical conclusions are collected by the use of data from Sweden for two of the essays, from Sweden and Germany for one essay and from Sweden and the US for one essay.

Essay I is concerned with the predictive power of historically-based volatility measures using ARCH-models. These models have firmly established themselves as among the foremost techniques for modelling volatility in financial markets since their introduction by Engle (1982). They have the ability to model the volatility of a series as both conditional and exhibiting periods of relative tranquillity, two of the most important characteristics of financial time series suggested by the current literature.

There are several reasons why one should be interested in trying to predict future volatility. Expectations about future volatility play a crucial role in financial theory. It is, for example, of major importance for market participants to make accurate predictions about future volatility since the volatility is an essential input in asset pricing, hedging and portfolio management. The ability to predict future volatility is also a key element for central banks in their process of conducting monetary policy.

Most of the developed models for forecasting volatility rely generally on the past behaviour of the price of the asset under consideration, i.e. a backward-looking model. The valuation of a derivative is, on the other hand, forward-looking since option prices depend upon expected future volatility. Thus, many economists argue that implied volatility derived from option contracts should have better forecast ability than models based solely on historical data. The implied volatility does not only contain information that is strictly historical, such as publications of macroeconomic indicators of importance for the development of the exchange rate, but it also includes

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1 See Campbell et al (1997) among others.
the market participants beliefs about future events, such as expectations about macroeconomic indicators.

In empirical research conducted on currency derivatives, considerable attention has been given to the US foreign exchange market. The general conclusion is that implied volatility on short maturity contracts performs well in forecasting future volatility, even though it is a biased estimator and contains information that is not present in historical volatility. Empirical research using data over longer horizons leads to the general conclusion that neither historical nor implied volatility provides a good forecast of future volatility.²

Most of the work on volatility, both historically-based and implied in currency options, has been conducted on the major traded currencies such as the US dollar. In a paper by Augilar (1999), implied volatility for the Swedish krona, which is a less frequently traded currency, is estimated against volatility measures based purely on historical data, such as the ARCH-models. The implied volatility is outperformed by forecasts based on GARCH-models, i.e. a backward-looking forecast of the volatility. Although the study does specify ARCH-models in the analysis, the focus of the paper is to evaluate volatility implied by option contracts versus volatility based solely on historical data for the Swedish krona against the US dollar and the D-mark. It does not, however, conduct a comprehensive study to model Swedish bilateral exchange rate data using a wider range of the ARCH-family of models. Sweden’s exchange rate and economy have some interesting features³, which make the country of some interest to increase the understanding of exchange rate behaviour and volatility from a relative less frequently traded currency. Essay I undertakes the task to analyse the behaviour of the Swedish bilateral exchange rate data using a wider range of the ARCH-family of models than is the case in Augilar (1999). This is done by examining the eight most important bilateral exchange rates for Sweden.


³ The Swedish economy is a highly open economy with a large proportion of international trade, a highly deregulated financial market, a relatively independent central bank operating an inflation target under a floating exchange rate regime, a relatively high degree of international capital mobility among other things.
Various econometric tests are performed in an attempt to identify the presence of ARCH-effects. Furthermore, the presence of asymmetry in the ARCH-effects is tested using the sign bias test and formal ARCH-models incorporating asymmetric effects. The sign bias test reveals no significant evidence of asymmetries in the data, which in turn suggests that it is appropriate to fit the standard ARCH-models to the data. However, less restrictive ARCH-models, which allow for asymmetric effects, do contradict the sign bias test for three of the included exchange rates. This then indicates the importance of combining different tests for examining the same type of effect in the conditional volatility models. Furthermore, there is evidence in the literature that indicates that the ARCH-effects tend to diminish as the sampling frequency decreases. Using Swedish exchange rates, the results give evidence supporting this hypothesis. However, there were three out of eight exchange rates that did contradict and thereby challenge the proposed hypothesis. The results found in this paper are in general in line with the results found for the major currencies such as the US dollar.

In the last decade, there has been much research and discussion that exchange rates might exhibit a rather special and possibly important property of a long memory process. The research has mainly been conducted on the major traded currencies such as the US dollar, Deutsche mark, and Japanese yen. The results are mixed in the sense that a long memory effect is not in general found in the exchange rates but only for some exchange rates and periods. However, there has been no research on the long memory effect conducted on a less frequently traded currency such as the Swedish krona. The long memory process is a dynamic process, which is not incorporated in standard time series models such as the ARIMA-model. The fractionally integrated ARMA-model, the ARFIMA-model, incorporates this generalisation and makes it a parsimonious and more flexible model for the simultaneous study of both the long memory and short-run dynamics. The second essay analyses the behaviour of the eight Swedish exchange rates over short and long run horizons by the use of the ARFIMA-GARCH model and the Geweke and Porter-Hudak (GPH) estimator. The ARFIMA-GARCH test provides no support for long run memory in the Swedish exchange rate. The GPH-estimator provided a significant result in two cases: the

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4 See Cheung (1993) for a further discussion.
SEKFIM and SEKGBP exchange rates. The significance is at the 5 percent level but at different frequency levels for the GPH-test. There is then no systematic significant evidence of a long memory property in the Swedish exchange rate, which in general is in line with the results found for more frequently traded currencies such as the US dollar. The result was also supported by testing for the memory structure in the randomly rearranged data series evaluated against the memory structure in the original series.

Essay III is concerned with the real exchange rate – real interest rate differential relationship. This relationship is often described as “the most robust relationship in empirical exchange rate models”; see e.g. Meese and Rogoff (1988). This relationship builds on the international parity conditions that link real interest rate differentials and real exchange rates. However, past research on exchange rate determination has been only partly successful in explaining exchange rate movements. Many earlier papers, which model exchange rate movements as a function of real interest differentials and other economic fundamentals, often indicate a statistically significant coefficient for the parameter of the real interest rate differential.5

More recent work has, however, in general been unable to establish this long-run relationship between the real exchange rate and the real interest differential using more sophisticated empirical techniques.6 In the Campbell and Clarida (1987) paper there was evidence that the expected real interest differentials could only partly explain the real exchange rate movement in the dollar. This inability to fully explain this relationship is due to the fact that the real interest rate differential has not been persistent enough and that its variance innovation has not been large enough to account for much of the fluctuation in the real exchange rate. In the Meese and Rogoff (1988) paper, the authors test for cointegration between real exchange rates and long-term interest rate differentials and find that they cannot reject the null hypothesis of non-cointegration between the variables. They interpret this result as an indication of the possible omission of one or more variables from the exchange rate – interest rate

5 See Frankel (1979), Hooper and Merton (1982) and Shafer and Loopesko (1983) among others.

6 Two of the most well known recent papers that have been unable to establish this long-run relationship are Campbell and Clarida (1987) and Meese and Rogoff (1988).
differential relationship that have a large variance. They suggest that this possible omitted variable might be the expected value of some future real exchange rate. This suggestion of an important missing variable is also in line with the Campbell – Clarida result.

Hooper and Merton (1982) showed that the equilibrium real exchange rate could be posited to be a linear function of a constant and the cumulated current account. Blundell-Wignall and Browne (1991) used this fact in a recent paper in which they indicated that there is evidence that there may be a cointegrating relationship between real exchange rates and real interest rates. This cointegrating relationship is shown to depend on the inclusion of a proxy for the expected future real exchange rate. The proxy that was used was a linear combination of a constant and the difference in the share of the cumulated current account relative to GDP.

In an environment with an increasing degree of openness and flexibility of international capital as well as a greater independence of the central bank with an inflation target, it is increasingly relevant to re-examine the link between the real exchange rate and the real interest rate differential. As Meese and Rogoff (1988) cited, in such an environment one might expect that this robust relationship would be even more robust. Essay III applies the Johansen cointegration technique and the model developed in Edison and Pauls (1993) and Wu (1999) to re-examine this relationship between the real exchange rate and the real interest rate differential in Sweden with the U.S. as the base country with the inclusion of the suggested proxy for the expected future real exchange rate. The empirical results show that the real exchange rate and real interest rate are non-stationary but cointegrated. Thus, the results support a long-run relationship between the real exchange rates and the real interest differentials in Sweden. This result is in line with Wu (1999) but not with Edison and Pauls (1993). In addition, empirical evidence is provided to show that our error-correction framework leads to real exchange rate forecasts that are superior to those generated by a random-walk model. The Diebold and Mariano test also support this superiority in out-of-sample prediction.

The fourth essay focuses on the issue of central bank credibility for the Central Bank of Sweden (Riksbanken) using the concept of long memory. During the last
decade, the concept of credibility has become a central concern not only in the scholarly literature on monetary policy but also in practical central-banking circles. The credibility issue of monetary policy focus on the resolution of the so-called inconsistency problem\(^7\) in the conduct of monetary policy, as identified by Kydland and Prescott (1977) and Barro and Gordon (1983a, 1983b). Suggestions for removal of the inflation bias implied by the inconsistency problem include among other things (1) the building up of an anti-inflationary reputation by the public sector\(^8\), and (2) an institutional reform aimed at establishing an independent and anti-inflationary central bank(er)\(^9\). These suggestions were aimed at strengthening the credibility of the policy objective of central banks and their independence.

Although the concept of credibility has become of central concern, there appears to be no generally agreed-upon definition of the term in either central bank circles or the academic literature. In central bank circles, one often hears definitions of a purely pragmatic nature; such as for example that a central bank is credible if people believe it will do what it says\(^10\). In the academic literature, however, credibility is often identified with one of three things: strong aversion to inflation, incentive compatibility, or precommitment\(^11\). In the literature too, a distinction is made between the credibility of the policy objective of the central bank, i.e. the credibility of its “conservativeness”, and the credibility of its independence from the political sphere. It has, furthermore, been recognised that there may be a difference between the de facto status (political) of the central bank and its de jure status (legal), that might affect belief in the independence of the central bank. Several attempts to formalise, measure and estimate the degree of informal/political as opposed to formal/legal

\(^7\) The “traditional” inconsistency problem arises when a policymaker is tempted to raise output and employment above its “natural rate” level by creating unanticipated monetary shocks. See Välimä (1996) for a further discussion.

\(^8\) See Barro et al (1983b) for a further discussion.

\(^9\) Rogoff (1985) and Lohmann (1992) have shown that separating the objective function of the central banker from the objective function of the policymaker is possible, costless and credible in equilibrium.

\(^10\) The central bankers, given that many countries recently adopted an inflation target as their monetary policy regime, often take the degree of dedication to price stability as synonymous with credibility. See Blinder (2000).

\(^11\) See Blinder (2000) for a more formal discussion about the definition of credibility in the case of a central bank.
independence of the central bank have been considered\textsuperscript{12}. The influence of the political sphere on the independence of the central bank can also be attributed to the objectives or instruments of the central bank policy, i.e. a central bank might have “instrument independence” but not “objective independence”\textsuperscript{13}. Yet another way for the political sphere to obtrude upon the independence of a central bank is through the appointment of its board members\textsuperscript{14}.

In a paper by Blinder (2000)\textsuperscript{15}, two main issues about central bank credibility were addressed: first, why is credibility so important to central bankers? and; second, how can a central bank create or enhance credibility? Blinder (2000) proposed seven reasons why credibility is important\textsuperscript{16} and the central bankers favour four of them, namely: Greater credibility (1) makes disinflation less costly; (2) helps hold down inflation once it is low; (3) makes it easier to defend the currency when necessary; and (4) helps garner public support for central-bank independence. Most economists agree on the two first reasons for why credibility is important, i.e. reducing the costs of disinflation, and keeping inflation low\textsuperscript{17}.

\textsuperscript{12} See Cukierman (1992) and Blinder (2000) for a further discussion.

\textsuperscript{13} The Reserve Bank of New Zealand Act from 1989 implies “instrument independence” but not “objective independence” because the Reserve Bank of New Zealand is free to choose by what means the established objective is to be achieved. See Välilä (1996) and Walsh (1995). In a similar fashion, the government sets the goal for the Bank of England but the Monetary Policy Committee sets the instrument (see King, 1998). A similar separation of goal and instrument is to be found in Sweden, where the Swedish Central Bank has formalised the government-established goal to an inflation rate of 2\% +/- 1\% over a two-year period. The Swedish Central Bank sets the instrument. Similar arrangements are to be found in other countries that have the same framework for the monetary policy.

\textsuperscript{14} See Lohmann (1992) and Välilä (1996).

\textsuperscript{15} The paper builds on a survey (a questionnaire) mailed to the heads of 127 central banks (84 responded implying a response rate of 66 percent) and to a similar sized sample of academic economists who specialise in monetary economics or macroeconomics.

\textsuperscript{16} The seven reasons set forth by Blinder (2000) for importance of credibility are (1) Reducing the costs of disinflation (credibility hypothesis); (2) Helping to keep low inflation (a version of the credibility hypothesis); (3) Flexibility to change tactics; (4) Serving as a lender of last resort; (5) Defending the exchange rate; (6) A duty to be open and truthful; and (7) Public support for central bank independence.

\textsuperscript{17} Beyond that the economists have markedly different rankings than the central bankers. See Blinder (2000) for a more detailed discussion.
For the second issue in the paper by Blinder (2000), i.e. methods of building or creating credibility\(^\text{18}\), the views of the central bankers and the economists are in general closely aligned. Establishing a history of living up to its word is ranked as the most important method, followed by central bank independence; two of the methods most strongly emphasised in the scholarly literature\(^\text{19}\) (1) precommitment and (2) incentive-compatible contracts were rated as least important by both groups. It would appear then that the respondents of the questionnaire feel that central bankers earn credibility from the market participants more by building a track record for honesty and inflation aversion than by limiting their discretion via commitment technologies or by entering into incentive-compatible contracts.

In the last decade, many countries have adopted an inflation target\(^\text{20}\) as their monetary policy regime. Since the central bankers in those countries often equate the degree of dedication to price stability as synonymous with credibility from the market participants\(^\text{21}\), a perfectly credible central bank will then have no feedback effect from previously realised outcomes in the monetary policy making process, i.e. in its targeted inflation process. In other words, there will be no significant serial autocorrelation or memory in the inflation process, i.e. past realised monetary policy and inflation rates will not significantly affect expectations about and the outcome of the current monetary target and associated policy.

The long memory parameter for the Swedish inflation series is found to be significant for the pre-independent period but not so for the post-independent period using the GPH-estimator. Furthermore, the estimates are significantly different from each other using the Chow-test. The estimates of the randomly rearranged series are insignificant for both periods with the long memory estimate of the pre-independent

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\(^{18}\) The methods of building or creating credibility put forth by Blinder (2000), were: (1) A history of living up to its word; (2) Central bank independence; (3) A history of fighting inflation; (4) Openness and transparency; (5) Fiscal discipline by the government; (6) Precommitment; and (7) Incentive-compatible contracts. The methods appear in the order of ranking by the two groups.

\(^{19}\) See Barro and Gordon (1983a) and Blinder (2000).

\(^{20}\) Some of the countries that have adopted this regime in recent years are New Zealand (1990), Canada (1991), the United Kingdom (1992), Sweden (1993), Finland (1993), Australia (1994), and Spain (1994).

\(^{21}\) See Blinder (2000).
period being significantly different from the corresponding estimate of the original series. Thus, there is evidence that the pre-independent period does contain a long memory-dependent structure but not so for the post-independent period. Using data for Germany, there is evidence that the long memory parameter for similar periods is insignificant, i.e. the credibility for the monetary policy objective of the Bundesbank in the pre- and the post-period has not changed significantly as is the case for its monetary policy objective and relative independence. These results further strengthen the conclusion that the credibility of the monetary policy objective of the Swedish Central Bank has increased from the pre- to the post-independent period.
References.


The predictive power of historical based volatility measures: Evidence from Sweden.

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ABSTRACT

The behaviour of the volatility of eight Swedish exchange rates over the recent floating period is analysed. Various econometric tests are performed in an attempt to identify the presence of ARCH effects using data for the period November 1992 to March 1998. Furthermore, the presence of asymmetry in the ARCH effects is tested by the use of the sign bias test and formal ARCH models that incorporate the possibility of asymmetric effects. The sign bias test reveals no significant evidence of asymmetries in the data, which then suggests that it is appropriate to fit the standard ARCH models to the data. However, the less restrictive ARCH models, which allow for asymmetric effects, do contradict the sign bias test for three of the included exchange rates. This then indicates the importance of combining different tests for examining the same type of effect in the conditional volatility models. Furthermore is the hypothesis that the ARCH effects diminish with less frequency in sampled data tested.

KEYWORDS: ARCH models, exchange rate, Sweden, volatility.
1. Introduction.

The family of ARCH-models has, since their introduction by Engle (1982), firmly established themselves as one of the foremost techniques for modelling volatility in financial markets. They have the ability to model the volatility of a series as both conditional and exhibiting periods of relative tranquillity, which are two of the most important stylised facts of financial time series suggested by the current literature\(^1\). Applications of the ARCH-models dealing with its empirical properties have shown that they are applicable to a wide range of financial instruments, both the generalisations of the original model\(^2\) and applications of the ARCH-models\(^3\).

It is important for market participants to make accurate predictions of future volatility since expectations about future volatility play a crucial role as an important input in e.g. asset pricing and portfolio management. The ability to predict future volatility in e.g. asset prices and exchange rates is also important for a central bank as inputs in the process of conducting its monetary policy. It is especially so for a central bank like the Swedish central bank operating an inflation target under a floating exchange rate regime.

Considerable attention in the empirical research conducted on volatility models has been given to the foreign exchange-, stock-, and derivative markets in the US. The experience from non-US countries remains largely unknown, especially from smaller economies such as the Swedish economy with less frequently traded assets. For example with the use of Swedish data, these models have been applied to high frequency stock returns by Lyhagen (1997), volatility forecasting and efficiency in the Swedish call options market by Andersson (1995), and to daily SEKUSD and SEKDEM exchange rate data by Augilar (1999). Although the study by Augilar (1999) does specify ARCH-models in the analysis, the emphasis of the paper is to evaluate volatility implied by option contracts versus volatility models based solely on the past behaviour of the exchange rate under consideration. The volatility models

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\(^1\) See Campbell et al (1997) among others.


\(^3\) See Bollerslev et al (1992) among others.
based on the past behaviour of the exchange rate that are evaluated in the paper are the GARCH(1,1)- and EGARCH(1,1)-models. However, the paper does not conduct a comprehensive study to model Swedish bilateral exchange rates using a more wide range of the family of the ARCH-models or for the frequency of the data.

The Swedish exchange rate and economy has some interesting features which makes it of some interest to researchers as well as to practitioners, e.g. the central bank, to gain some additional insight into exchange rate behaviour and the ARCH-modelling of exchange rates beyond the US experience. The features of interest in the Swedish economy are that it is (1) a highly open economy with a large proportion of international trade, and has (2) a highly liberalised and deregulated financial market. Furthermore, Sweden has (3) a relatively independent central bank operating an inflation target under a floating exchange rate regime, and (4) a relatively high degree of international capital mobility. These features imply that the markets in the Swedish economy are highly deregulated without a significant amount of constraints imposed upon the markets. This is also, to a significant degree, the case for the underlying markets for e.g. goods and services, which implies currency exchange to a high degree. As these features imply a relative low degree of regulation and intervention in the currency market and related markets, it supports a relatively more efficient allocation of relevant information of preferences and valuation of resources to the market participants to be used in the allocation process of resources and rationing of goods and services to the consumers with the highest willingness to pay.

Although that the currency market and related markets that implies a currency exchange are (1) highly deregulated, (2) relatively informationally efficient, and (3) relatively highly competitive; the Swedish currency is not as frequently traded as major currencies such as the US dollar. Although operating in a highly deregulated and relative efficient market, an implication for the behaviour of the volatility of a less frequently traded currency might be that it is to a significant degree affected by a volatility structure that is not present in a relatively more frequently traded and liquid currency such as the US dollar. It has, in fact, been observed that the volatility of the Swedish krona is asymmetrically affected in the sense that the volatility tends to be
higher when the krona depreciates relative to when it appreciates. This effect is generally not observed in the US dollar and it might be linked to a less frequently traded currency as a country-specific (risk) component. Due to these interesting features, it might be of interest to analyse the Swedish krona to gain some additional insight into exchange rate behaviour and the ARCH-modelling of exchange rates beyond the US experience. This study will undertake such a task by examining eight different bilateral exchange rates, for which Sweden has its overwhelmingly majority of international exchange, and over a wide range of the frequency of the data.

The rest of this paper is structured as follows. Section two formally sets out the methodology used in the empirical part of the paper. Section three presents the empirical results of the ARCH-models fitted to the data. Section four examines the issue of asymmetrical ARCH responses to innovations in the market by the sign bias test and section five examines the asymmetric effect by the use of the EGARCH- and TGARCH-models. Section six concludes and briefly summarises the results.

2. Background and methodology.

The exchange rate data has been received from the central bank of Sweden (Sveriges Riksbank). Eight bilateral exchange rates are considered at the daily (end of day), weekly (end of week) and monthly (monthly average) frequency. The series range from the end of November 1992, just after the abandonment of the fixed exchange rate regime, to the end of March 1998.

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4 See Augilar (1999) among others.

5 The spot nominal bilateral exchange rates are the Swedish krona versus the US dollar (SEKUSD), Japanese yen (SEKJPY), Norwegian krona (SEKNOK), [the non EU-currencies]; Deutsche mark (SEKDEM), British pound (SEKBGP), French franc (SEKFRF), Finnish mark (SEKFIM), and Danish krona (SEKDKK) [the EU-currencies]. The above countries are those with which Sweden has its overwhelmingly majority of international exchange. The relative high frequency and liquidity of the trade in the above exchange rates makes them of special interest in this study.
The exchange rates under consideration were not totally independent during this period because the EU-currencies were involved in or linked to the currencies in the Exchange Rate Mechanism (ERM) to various degrees. ERM was a system of fixed but adjustable rates, where the fixity was defined as an official central parity between any pair of member-currencies. This implied a central rate with a band of fluctuation within which the exchange rate is allowed to move around the fixed parity. The normal margins were at first at $\pm 2.25$ percent but they were widened in the end of August 1993 to margins of fluctuation of $\pm 15$ percent as a result of an exchange rate crisis. This widening was to the point where the fixed exchange rate differed little from a free-floating exchange rate regime.

The EU-countries which do not have an EMU-membership, i.e. Denmark, Sweden, and the U.K., can choose to participate in the ERM or not. Denmark is a member of the ERM but not Sweden and the U.K., which instead has a free float with an inflation target in line with the ESCB for its monetary policy. This and the ERM exchange rate arrangement makes it of some interest to make a distinction in the analysis between the EU-currencies and the currencies outside the EU to find distinctions or similarities concerning the volatility between the two groups of currencies.

Due to the ERM and the band for exchange rate fluctuation we might expect that the EU-currencies would exhibit the same type of volatility but not necessarily so for the non EU-currencies. The reason is that the ERM-arrangement implies exchange rate interventions to keep the currencies within the relatively wide band but not so for the non EU-currencies. The relative wide band for the ERM, which was imposed as a result of the currency crisis of August 1993 and which more or less implied a free float for the EU-currencies, provided the EU-currencies with the possibility to divert from the volatility-path of each currency linked to the ERM. This feature of the ERM will then imply that there might be a country-specific (risk) source into the ARCH-model of each EU-currency as well as that there is a common component as a result of the ERM-arrangement despite that it has a relative wide band of fluctuation. One effect of this country-specific component might be that some EU-currencies might exhibit a volatility structure that is not exhibited in the other EU-currencies. Such a country-specific risk adds a more dynamic structure to the exchange rate of the
country and it will also build in a more complicated structure when analysing the economy and when conducting the monetary policy.

The log of each bilateral exchange rate \( S_t \) at all frequencies was generated because the natural logarithm for time series analysis has many advantages both in regression and forecasting\(^6\). Evidence suggests that financial time series data, such as exchange rates, are typically characterised by non-stationarity. It is then appropriate to test for the presence of a unit root in the data to establish whether or not \( S_t \) is mean reverting.

One such test is to examine the autocorrelation function (ACF) and partial autocorrelation function (PACF) for \( S_t \). A slow rate of decay in the ACF, a spike at \( t=1 \) in the PACF, and a significant \( t \)-statistic for \( \rho_1 \) are highly suggestive of a unit root. Additional tests are the augmented Dickey-Fuller\(^7\) (ADF) statistic (H0:I(1)) and the Kwiatkowski et al (1992) (KPSS) statistic (H0:I(0))\(^8\). Where the presence of a unit root is established, convention dictates the use of first-differenced data. As such, the log of the first difference of each exchange rate \( R_t = \log \left( \frac{S_t}{S_{t-1}} \right) \) may be generated and retested for the presence of a unit root.

The mean-reverting series \( R_t \) may contain higher-order autocorrelation in the form of structures such as day of the week\(^9\), weekly and monthly effects\(^10\). This may

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\(^6\) For a detailed discussion see Hodrick (1987) among others.

\(^7\) The ADF-test may be sensitive to the chosen period of lags in the test. It is then appropriate to test for unit roots with varying lag structures. In this paper the lag structure, to be used in the ADF-test, will be chosen according to the Akaike information criterion (AIC), i.e. the lag length that produces the lowest AIC. Furthermore, the statistic can be calculated with or without an intercept term. Since the superiority of one over the other cannot be established \textit{a priori}, it is appropriate to calculate both versions of the statistic.

\(^8\) For a discussion of the interpretation of the combined use of the ADF- and KPSS-test see Maneschiöld (2000).

\(^9\) Because of the global nature of capital markets, a weekly effect is deemed not only to include a five-period lag but also a six-period lag.

\(^10\) Any other structure is assumed to be of a random occurrence.
be verified by a visual inspection of the ACF and PACF, and a check of the relevant $Q$- and $t$-statistics for each lag to their critical values\textsuperscript{11}.

Where higher-order autocorrelation is present, it is appropriate to model this autoregressive structure by the use of a Box-Jenkins approach\textsuperscript{12}. If successful, the residuals from this AR(p)-process should largely approximate white noise\textsuperscript{13}. The application of ARCH-models to $R_t$ or the residuals from the AR(p)-model is only appropriate where one may identify the existence of ARCH-processes within the data\textsuperscript{14}. When testing for the presence of ARCH-effects, one may consider the $(R_t)^2$-series were highly significant autocorrelations for the squared returns ($Q_{sq}$) is evidence for that the conditional distributions of the nominal returns are changing through time\textsuperscript{15}. Thus, where the $Q_{sq}$-statistic for $(R_t)^2$ is significant, this is deemed as further evidence of a time-varying conditional variance.

\textsuperscript{11} The $Q$-statistics are calculated according to the Ljung-Box specification. The $t$-statistics are calculated as \( \left( \frac{\alpha_i}{SE} \right) \) where the standard error of the estimate is approximated by using the formula \( \frac{1}{\sqrt{n}} \).

\textsuperscript{12} Nelson (1990a,b), and Gannon (1996) show that the order of the specified AR(p)-model does not impact significantly on the fitted ARCH-models.

\textsuperscript{13} A visual inspection of the ACF and PACF on the residuals and a critical evaluation of the $Q$-statistics can verify this.

\textsuperscript{14} It is likely that several ARCH-models will be estimated from the data. In this paper I will select the optimal model that produces the lowest Akaike information criterion (AIC) or Schwartz Bayesian criterion (SBC) statistic in the case of a large sample from the models were the estimated parameters are statistically significant according to the calculated $t$-statistics. To choose the most parsimonious model I will not use the Pagan and Schwert $R^2$-criterion, as it does not place any penalty upon the inclusion of additional parameters in a model. Furthermore, the joint estimation of the AR-process and the ARCH-model might be important to avoid a loss in estimating power.

\textsuperscript{15} For details see Hsieh (1988) among others.
3. Empirical results.

Graphs of the $S_t$ and $R_t$ monthly nominal exchange rate series are presented in Appendix A. The $R_t$-series appear to exhibit random fluctuations around zero and have no obvious structural breaks during the sample period; that is, the exchange rate data seems to be difference stationary. This is especially so after the first three months, which might be interpreted as a stabilisation period for the Swedish krona just after the transition from a fixed to a floating exchange rate regime.

Table I in Appendix B presents the results of applying the ADF and KPSS unit root tests to each monthly exchange rate series of $S_t$ and table II presents the corresponding unit root tests for $R_t$. The combined use of the ADF- and KPSS-statistic indicates a unit root in $S_t$ but not in $R_t$ for each exchange rate. Thus, the creation of the $R_t$-series appears to satisfactorily remove the unit root present in the $S_t$-series.

Table III in Appendix B provides summarised information for each monthly $R_t$ exchange rate series. The data reveals, with one exception, that they do not match the

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16 The data range here from July 1991 to March 1998. The purpose is to see the effect just after the Swedish krona was floating in November 1992 compared to the immediate fixed regime period just before the regime shift. The data range from November 1992 to March 1998 in the empirical analysis part of this paper.

17 The pattern is, though, less clear for the SEKJPY, which might be due to the disturbances on the Asian financial markets during this period.

18 To conserve space I will only present, except in table IV, the data for the monthly (average) exchange rate series as the test statistics and estimations throughout this paper for the daily and weekly data was similar to those reported for the monthly (average) series. Therefore, the frequency of the data seems not to significantly affect the outcome of the results during this period.

19 The $S_t$-series also exhibit a slow decay in their ACF, a spike at $t=1$ in the PACF, and a significant $t$-statistic for $\rho_1$, which are all suggestive of a unit root. The $R_t$-series does not exhibit the characteristics mentioned above. Support to the unit root test was also given by a significant negative autocorrelation at lag one of the first difference of the $R_t$-series of all exchange rates, i.e. of the second difference of the log nominal exchange rate (The results are not reported here but can be obtained from the author). This is an indication of an overdifferenced series. Furthermore, the $R_t$-series does not show any systematic significant effect of a day of the week-, weekly-, or monthly characteristic, as there were no systematic significant spikes in the ACF at any regular frequency. For details of the characteristics mentioned above see Maneschiöld, 2000.
normal distribution assumption as indicated by the skewness, kurtosis and the Jarque-Bera test statistic for normality. The exception is the SEKFIM exchange rate where the hypothesis of a normal distribution is not rejected. The Ljung-Box test statistic for serial correlation in levels \([Q(n)]\) and squares \([Qsq(n)]\) of the \(R_t\)-series reveals the possibility of dependence in the return distributions. The first test is an ordinary test for serial correlation while the test based on \((R_t)^2\) can give an indication about heteroscedasticity in the data series. The \(R_t\) exchange rate series does then show sign of serial correlation as the Ljung-Box \(Q\)-statistic is significant at the 1% level for all exchange rates. The test for the presence of ARCH-effects, the \(Qsq\)-statistic, is significant for all exchange rates. This then indicates that the \(R_t\) exchange rate series exhibit a time-varying conditional variance. We can then conclude, from the descriptive statistics, that the \(R_t\) exchange rate series are generally not normally distributed\(^{20}\) and that they are autocorrelated in both levels and squares of returns. The latter test indicates ARCH-effects in the data.

A recent trend in empirical studies has been to consider temporal issues and the presence of ARCH-effects. The reliable ARCH-models do not depend on the frequency with which the data is sampled for most types of domestic assets\(^{21}\). Research\(^{22}\) for exchange rate data indicate however that ARCH-effects tend to diminish as the periodicity of the sampling frequency decreases\(^{23}\). By the use of daily (end of day), weekly (end of week) and monthly (monthly average) data various ARCH-models were fitted and an optimal model was selected subject to the same analytical procedure outlined in section two. Most of the exchange rates under consideration provided the hypothesised result, i.e. that the Ljung-Box \(Q\)-statistic \([Qsq(n)]\) declines as the sample frequency declines (see table IV in Appendix B). The

\(^{20}\) The exception is the SEKFIM where the Jarque-Bera normality test indicates a normal distribution.

\(^{21}\) See for example Engle et al (1987) (who fitted significant ARCH-models to quarterly interest rate data), Morgan et al (1987), and Chou (1988) (who both found low frequency stock returns (monthly and less) exhibited reliable ARCH-effects).

\(^{22}\) See Diebold (1988), and Baillie et al (1989) among others.

\(^{23}\) As the Ljung-Box \(Q\)-statistic for \((R_t)^2 [Qsq(n)]\) may potentially indicate the presence of ARCH-effects, one might expect that the \(Qsq\)-statistic, calculated for a given lag length \((n)\), should decline as the sampling frequency of the exchange rate series declines.
exceptions from the anticipated hypothesis are the SEKUSD, SEKDEM and SEKJPY exchange rates were the $Q_{sq}$-statistic decreases from the daily- to the weekly frequency but then increases from the weekly- to the monthly frequency. Two interesting features to note from those exceptions from the anticipated hypothesis are; firstly, the exceptions are linked to the major currencies in the sample; secondly, the lower frequency of the data (weekly and/or monthly) appears to be the cause of the failure of the hypothesis.

More formal evidence on short-term dependence and conditional heteroscedasticity is obtained by modelling the data series directly in a Box-Jenkins time series framework fitting ARCH-models to the exchange rates\textsuperscript{24}. The optimal models are summarised in table V in Appendix B\textsuperscript{25}. The results in table V indicate that the exchange rate series are short term dependent because the AR(1)-term is significantly different from zero. The significance level is the 1 percent level for all exchange rates but SEKDEM and SEKDKK, which are significant at the 5 percent level. However, the short-term dependency found in the exchange rates series is moderate since the span for the estimated AR(1)-parameters runs from 0,42, for the SEKUSD exchange rate, to 0,25, for the SEKDKK exchange rate. The estimation of the volatility models is significant between the 1 percent to the 5 percent level and they are of an ARCH(1) form for all exchange rates but for SEKGBP and SEKFIM, which are of the GARCH(1,1) form\textsuperscript{26}.

The Jarque-Bera test for normality indicates that the null of normally distributed residuals is rejected for the SEKJPY at the 1 percent level and for SEKUSD, SEKDEM and SEKGBP at the 5 percent level, respectively. The Jarque-Bera test for

\textsuperscript{24} The selection of the most optimal model for each exchange rate is based on the selection criteria detailed in Section two. Moreover, Nelson (1990a,b), and Gannon (1996) has shown that the order of an AR(p)-model does not significantly impact the fitted ARCH-models for financial data and Leander (1996) indicates that the same framework does not significantly impact the conditional heteroscedastic volatility for the Swedish krona.

\textsuperscript{25} To conserve space I will only present the data for the monthly (average) exchange rate series in this paper as the test statistics and estimations throughout this paper for the daily and weekly data was similar to those reported on the monthly (average) series. Therefore, the frequency of the data seems not to significantly affect the outcome of the results during this period.

\textsuperscript{26} There is then no evidence of a significant distinction between the EU-currencies and the non EU-currencies concerning the fitted volatility model due to that the exception from the general ARCH(1)-model are linked to some but not all EU-currencies, i.e. the SEKGBP and SEKFIM exchange rates.
the other exchange rates is not rejected, i.e. the hypothesis of a normal distribution is not rejected. The Ljung-Box test statistic for standardised residuals and squared standardised residuals show no presence of serial correlation up to the 40:th order for the return series. By modelling conditional heteroscedasticity in the exchange rate return series I found that the serial correlation found in the descriptive statistics of the \( R_t \)-series is not present (see table III). Furthermore, the data reveals that the fit of the normal distribution is improved.

4. Sign bias testing.

Engle et al (1993) make the point that ARCH-models only allow for a quadratic response to innovations irrespective of the size of any given innovation. Hence, large and small innovations are assumed to impact uniformly on volatility. They also mention that ARCH-models are symmetric in their response to past innovations, i.e. positive and negative innovations of cognate magnitude have a similar impact on the volatility. When the exchange rate data conforms to these restrictive assumptions, the estimation of the standard ARCH class of models is entirely appropriate. However, when the data exhibits asymmetrical responses, these assumptions may prove too restrictive and limit the estimating power of the fitted model. The standard ARCH-models are inappropriate when such asymmetries are present. Thus, one must look to models exhibiting less restrictive characteristics. For example, the model developed by Glosten et al (1993) explicitly incorporates the potential for asymmetry in its specification of the conditional variance equation\(^\text{27}\).

It might be possible that the exchange rate volatility of one currency, such as the Swedish krona, can be affected asymmetrically by positive (interpreted as “good” news for Sweden) and negative shocks (“bad” news) to the exchange rate or the

\(^{27}\) For a further discussion see Engle et al (1993), and Glosten et al (1993).
economy\textsuperscript{28}. It has, in fact, been observed that the volatility tends to be higher when the Swedish krona depreciates ("bad" news) compared to when it appreciates\textsuperscript{29}. Furthermore, empirical evidence demonstrates relatively often that there is a negative relationship between stock returns and volatility but it is not all that clear if this type of relationship is also present in the case of exchange rate volatility\textsuperscript{30}. It is then appropriate to test for asymmetries in the ARCH-models fitted to the exchange rate data.

Engle \textit{et al} (1993) suggest a sign bias test, a negative size bias test and a positive size bias test to test for asymmetric effects in the data. These tests are to be conducted jointly by an OLS-regression on the following equation

\begin{equation}
 z_t^2 = a + b_1(S_t^{-1}) + b_2(S_t^{-1}e_{t-1}) + b_3(S_t^{+1}e_{t-1}) + \nu_t
\end{equation}

where $z_t$ are the standardised residuals (i.e. $z_t = \frac{\hat{e}_t}{\sqrt{h_t}}$) and $S_t^{-1}(S_t^{+1})$ is a dummy variable that takes on a value of unity if $e_{t-1}$ is negative (positive) and zero otherwise. The sign bias test relates to the statistical significance of $b_1$. Where the test statistics are insignificant, then positive and negative shocks do not have an appreciably different impact on the volatility. The negative (positive) size test relates to the statistical test on $b_2$ ($b_3$). If $b_2$ ($b_3$) is statistically different from zero, then it is likely that large and small negative (positive) innovations have a different impact on the volatility. The standard ARCH-models require that $b_1$, $b_2$ and $b_3$ are jointly equal to zero, which may be tested with the standard $F$-statistic.

\footnotesize
\textsuperscript{28} It might be the case that it is the shocks that are asymmetrical in the sense that they affect economies differently e.g. due to a difference in the industry structure of the two countries. The effect will then be that the volatility of the exchange rates does in general behave in a different and opposite way for the two countries. However, the possible asymmetry in the exchange rate still has to be accounted for by e.g. a central bank using the exchange rate as an indicator for future inflation whether this asymmetry is related to the exchange rate \textit{per se} or to shocks.

\textsuperscript{29} See Aguilar (1999) among others.

\textsuperscript{30} There is however a difference between stocks and exchange rates in the sense that all exchange rates cannot be affected by the same type of asymmetry to its volatility. The reversed exchange rate should in general exhibit the opposite type of asymmetry found in the original exchange rate as the exchange rate is a relative price between two currencies. If the volatility for e.g. SEKDEM tends to be higher when the Swedish krona is depreciating ("bad" news) then when it is appreciating then should the reversed exchange rate, i.e. DEMSEK, in general exhibit the opposite asymmetric structure.
The OLS-regression specified in equation 1 and the subsequent sign bias testing was applied to each of the optimal ARCH-models selected in Section three. The $t$- and $F$-statistic and probability for each of the test coefficients are presented in table VI in Appendix B. The results show no conclusive evidence of asymmetrical ARCH-effects in the data by the use of the sign bias test. There are some estimates that do have relative low probabilities of rejection but they are still above the 10% level. In general, these results suggest that it is appropriate to fit the standard ARCH- and GARCH-models to the exchange rate data irrespective of the restrictive assumptions regarding the impact of innovations on volatility.

5. Asymmetric volatility models.

To confirm the sign bias test and possible asymmetries of bad and good news in the conditional volatility of the exchange rate series I will use the exponential GARCH- (EGARCH)-model to formally model asymmetries in the data. Nelson (1991) developed an exponential GARCH-model that allows for such asymmetric effects on the conditional volatility. Furthermore, Heynen et al (1994) found that the GARCH- and EGARCH-models performs equally well in forecasting exchange rate

\[ \ln(\sigma^2) = c + p\frac{\epsilon_i}{\sqrt{\sigma^2_{i-1}}} + a\left[\frac{|\epsilon_i|}{\sqrt{\sigma^2_{i-1}}} - \frac{1}{\sqrt{\pi}}\right] + q\ln(\sigma^2_{i-1}) \] 

which can be compared to the GARCH(1,1)-model specified as

\[ \sigma^2 = c + p\epsilon^2_i + q\sigma^2_{i-1} \]

No restrictions are necessary on the parameters of the EGARCH-model to ensure non-negativity since the model is specified for the logarithm of the conditional variance. The second term of equation 2 allows the conditional variance to respond asymmetrically to positive and negative changes in the exchange rate.
volatility. I will also use the threshold GARCH- (TGARCH)-model to confirm the findings from the EGARCH-estimates.

Table VII in Appendix B summarises the significant EGARCH-models subject to the same methodology outlined in section two. The significant exchange rates are the SEKDEM, SEKFRF and SEKDKK\(^{33}\). This implies that there are asymmetries in the way that positive and negative shocks significantly affect the volatility for those exchange rates differently, i.e. the negative sign of the asymmetric volatility parameter in table VII suggest that bad news or a negative return has a larger impact on the volatility than good news or a positive return. Thus, the exchange rates are asymmetrically affected by past innovations and the volatility increases more when the Swedish krona suddenly depreciates than when it appreciates. The asymmetric news parameter is not significant for the other exchange rates. This implies that there are no asymmetries such that positive and negative shocks significantly affects the volatility of those exchange rates. The ARCH/GARCH-models, that were estimated and described in table V, therefore best describe the conditional volatility for those exchange rates.

An interesting feature, for the exchange rates with a significant asymmetric news parameter, is that the parameter for the lagged variance (q) is insignificant in table V, when estimating its conditional volatility without the asymmetric news structure included. When I allow for the possibility of an asymmetric news structure in the conditional volatility model, the parameter (q) turns up to be significant for SEKDEM, SEKFRF and SEKDKK. This difference in the result suggests that the conditional variance for the Swedish exchange rate for the recent float is more complicated than described by a homoscedastic variance. In addition to those differences, the sign bias test in section four did not indicate evidence of an asymmetric ARCH-effect in those exchange rates. These different results indicate the

\(^{33}\) The significant asymmetries found in the Swedish krona seems, in general, to be linked to the exchange rates that have participated in the ERM during the period. Exceptions from the general conclusion are the SEKFIM and SEKGBP exchange rates, where the U.K. has not been in the ERM for the full period under consideration. This outcome might be a result of the currency crisis in August 1993 as well as of a currency that was struggling in the “periphery” to resist the attacks as well as building a reputational record for its monetary policy of a strict inflation target, i.e. the Swedish krona and its central bank. The crisis implied a widening of the currency band for the ERM, which in turn implied a more or less free float for the ERM-currencies.
importance of combining different tests for examining the same type of effect in the exchange rate and the conditional volatility models because it is important to make accurate predictions of the future volatility as expectations about the future volatility play a crucial role as an important input in e.g. asset pricing, portfolio management and in the monetary policy analysis of the central bank. It is especially so for a central bank like the Swedish central bank with an inflation target regime and a floating exchange rate were the volatility of the free floating Swedish krona signals expectations of future exchange rate changes. Together with other indicators and together with an analysis of the pass-through effect into domestic prices, it will then be of use for the monetary policy of the central bank as an indicator of future inflation tendencies in the economy.

To confirm the significant asymmetric effects in the EGARCH-model I also estimated the conditional volatility by the TGARCH-model (T for threshold) for the included exchange rates. The empirical results using the TGARCH-model indicate the same conclusion as the EGARCH-model, i.e. that there exists significant evidence that bad news has a larger impact on the conditional volatility than good news for SEKDEM, SEKFRF and SEKDKK but not significantly so for the other exchange rates.35

34 The TGARCH-model as well as the EGARCH-model tests if downward movements in the market (bad news) are followed by a higher volatility compared to upward movements (good news) of the same magnitude. The conditional variance equation is now specified as

$$\sigma_i^2 = c + p * \epsilon_{i-1}^2 + T * \epsilon_{i-1}^2 * k_{i-1} + q * \sigma_{i-1}^2$$  \hspace{1cm} (4)

where $k_i = 1$ if $\epsilon_i < 0$ and zero otherwise. Bad news has a larger impact on the volatility than good news if the threshold parameter $T$ is significantly different from and larger than zero, i.e. if $T > 0$. The threshold model indicates that good news has a significant impact of $p$ while bad news has a significant impact of $p+T$ on the conditional volatility. For a further discussion on EGARCH- and TGARCH-models see Lyhagen (1997), and Aguilar (1999) among others.

35 The results are not reported here but can be obtained from the author.
6. Conclusions.

The volatility behaviour of eight Swedish bilateral exchange rates is, in this paper, analysed by the use of the ARCH-/GARCH-models. The ARCH-models were fitted to the data by the use of the standard model selection criterion, i.e. the AIC and SBC. These standard ARCH-models assume symmetrical responses by the volatility to innovations in the market. Where asymmetric responses are evident it may be appropriate to fit less restrictive ARCH-models. The sign bias test for such asymmetries showed that asymmetries were insignificant for the included exchange rates. By formally testing for asymmetries by less restrictive ARCH-models, significant evidence of asymmetries occurred in three of the included exchange rates. These different results indicate the importance of combining different tests for examining the same type of effect in the conditional volatility models.

There is evidence in the literature that indicates that the ARCH-effects tend to diminish as the sampling frequency decreases. The evidence for the Swedish exchange rate in general supports this hypothesis because there were three out of eight exchange rates that contradicted and thereby challenged the stated hypothesis.
APPENDIX A.

**Figure A1-A8:** Graphs of the log nominal monthly exchange rate and first difference log nominal monthly exchange rate. The first difference seems to exhibit a random fluctuation around zero for many of the exchange rates. Furthermore, it seems that there are no obvious breaks during the sample period; that is, exchange rate data seem to be difference stationary for the majority of the exchange rates. This is especially so after the first three months, which might be interpreted as a stabilisation period for the Swedish krona just after the transition from a fixed to a floating exchange rate.

**Figure A1:** Graphs of the log nominal (to the left) and first difference log nominal (to the right) monthly exchange rate SEKUSD.

**Figure A2:** Graphs of the log nominal (to the left) and first difference log nominal (to the right) monthly exchange rate SEKOK.
Figure A3: Graphs of the log nominal (to the left) and first difference log nominal (to the right) monthly exchange rate SEKJPY.

Figure A4: Graphs of the log nominal (to the left) and first difference log nominal (to the right) monthly exchange rate SEKGBP.
Figure A5: Graphs of the log nominal (to the left) and first difference log nominal (to the right) monthly exchange rate SEKFRF.

Figure A6: Graphs of the log nominal (to the left) and first difference log nominal (to the right) monthly exchange rate SEKFIM.
**Figure A7:** Graphs of the log nominal (to the left) and first difference log nominal (to the right) monthly exchange rate SEKDEM.

**Figure A8:** Graphs of the log nominal (to the left) and first difference log nominal (to the right) monthly exchange rate SEKDKK.
Table I: Test for order of integration of the $S_t$ (log nominal exchange rate series). The ADF test is not significant for any of the exchange rates both with and without a constant included, i.e. the $H_0$:I(1) is not rejected by the use of the ADF-test. The critical values for $\hat{\eta}_c$ (KPSS with an intercept included) and $\hat{\eta}_c$ (KPSS with an intercept and time trend included) are 0.739 and 0.216 at the 1 percent significance level and 0.463 and 0.146 at the 5 percent significance level, respectively. The test is an upper tail test, i.e. a test statistic over the critical value implies rejection of $H_0$:I(0). The results from the KPSS test are all significant at the 1 percent level (marked with *), i.e. a rejection of $H_0$:I(0). Thus, the test statistics indicates a unit root in the $S_t$ (log nominal exchange rate series).

<table>
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<th>Exchange rate:</th>
<th>ADF; $H(0)$:I(1)</th>
<th>Lag</th>
<th>KPSS $\hat{\eta}_c$; $H0$:I(0)</th>
<th>Lag</th>
<th>KPSS $\hat{\eta}_c$; $H0$:I(0)</th>
<th>Lag</th>
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<td>SEKUSD</td>
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<td>0.987*</td>
<td>4</td>
<td>0.233*</td>
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<td>0.815*</td>
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<td></td>
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</table>
Table II: Test for order of integration of the \( R_t = \log \left[ \frac{S_t}{S_{t-1}} \right] \) exchange rate series. The ADF test is significant for all exchange rates at the 1 percent level (marked with *) both with and without a constant included, i.e. the H0:I(1) is rejected by the use of the ADF-test. The KPSS test statistics are, for all exchange rates, below the critical value, i.e. the H0:I(0) is not rejected by the use of the KPSS tests. Thus, the test statistics does not indicate a unit root in the \( R_t = \log \left[ \frac{S_t}{S_{t-1}} \right] \) exchange rate series.

<table>
<thead>
<tr>
<th>Exchange rate:</th>
<th>ADF; H(0):I(1)</th>
<th>Lag</th>
<th>KPSS ( \hat{\eta}_u ; ) H0:I(0)</th>
<th>Lag</th>
<th>KPSS ( \hat{\eta}_c ; ) H0:I(0)</th>
<th>Lag</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEKUSD</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No constant</td>
<td>-4.22*</td>
<td>1</td>
<td>0.079</td>
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<td>0.082</td>
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<tr>
<td>Constant</td>
<td>-4.41*</td>
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</tr>
<tr>
<td>SEKDEM</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>No constant</td>
<td>-5.13*</td>
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<td>0.059</td>
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<tr>
<td>Constant</td>
<td>-5.56*</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>SEKGBP</td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>No constant</td>
<td>-6.32*</td>
<td>2</td>
<td>0.109</td>
<td>4</td>
<td>0.091</td>
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</tr>
<tr>
<td>Constant</td>
<td>-6.65*</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SEKFRF</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No constant</td>
<td>-5.89*</td>
<td>1</td>
<td>0.272</td>
<td>4</td>
<td>0.061</td>
<td>4</td>
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<tr>
<td>Constant</td>
<td>-6.01*</td>
<td>1</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>SEKJPY</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No constant</td>
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<td></td>
</tr>
<tr>
<td>SEKNYK</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No constant</td>
<td>-5.49*</td>
<td>1</td>
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</tr>
<tr>
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<td>-5.76*</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SEKDKK</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No constant</td>
<td>-5.67*</td>
<td>1</td>
<td>0.253</td>
<td>4</td>
<td>0.054</td>
<td>4</td>
</tr>
<tr>
<td>Constant</td>
<td>-5.89*</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SEKFIM</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No constant</td>
<td>-6.65*</td>
<td>2</td>
<td>0.144</td>
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<td>0.141</td>
<td>4</td>
</tr>
<tr>
<td>Constant</td>
<td>-6.87*</td>
<td>2</td>
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<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table III: Descriptive statistics for the \( R_t = \log \left( \frac{S_t}{S_{t-1}} \right) \) of the different exchange rate series. The standard error for the sample mean are given in parenthesis, while the standard deviation for the sample of the different exchange rates are given separately in the table. The \( t \)-test for testing that the sample mean of the series is different from zero is insignificant for all exchange rates. The Jarque-Bera (J-B) statistic for normality and the Ljung-Box Q-statistic for serial correlation in level (Q) and squared returns (Qsq) are given in the table. The distribution of these statistics is \( \chi^2(2) \) under the null of normality and \( \chi^2(n) \) under the null of no serial correlation, respectively. The test for all exchange rates are statistically significant at the 1 percent level except for those marked with a *, which indicates normality. The rows for Q(n) and Qsq(n) give respectively the Ljung-Box Q-statistic for the series and squared series up to the \( n \)-th order of serial correlation. The \( Q(n) \)- and \( Qsq(n) \)-test indicates serial correlation and a time-varying conditional variance, respectively.

<table>
<thead>
<tr>
<th>Exchange rate:</th>
<th>SEKUSD</th>
<th>SEKDEM</th>
<th>SEKGBP</th>
<th>SEKFRF</th>
<th>SEKJPY</th>
<th>SEKOK</th>
<th>SEKDKK</th>
<th>SEKFIM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.002</td>
<td>0.0024</td>
<td>0.0024</td>
<td>0.0026</td>
<td>0.0033</td>
<td>0.0019</td>
<td>0.0027</td>
<td>-0.0005</td>
</tr>
<tr>
<td>(0.0033)</td>
<td>(0.0025)</td>
<td>(0.0029)</td>
<td>(0.0023)</td>
<td>(0.0045)</td>
<td>(0.0021)</td>
<td>(0.0024)</td>
<td>(0.0026)</td>
<td></td>
</tr>
<tr>
<td>( t )-test</td>
<td>0.609</td>
<td>0.979</td>
<td>0.808</td>
<td>1.132</td>
<td>0.743</td>
<td>0.890</td>
<td>1.114</td>
<td>-0.205</td>
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<tr>
<td>H0:mean=0</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.029</td>
<td>0.022</td>
<td>0.025</td>
<td>0.020</td>
<td>0.039</td>
<td>0.019</td>
<td>0.021</td>
<td>0.023</td>
</tr>
<tr>
<td>Min</td>
<td>-0.052</td>
<td>-0.047</td>
<td>-0.070</td>
<td>-0.040</td>
<td>-0.101</td>
<td>-0.040</td>
<td>-0.038</td>
<td>-0.083</td>
</tr>
<tr>
<td>Max</td>
<td>0.109</td>
<td>0.104</td>
<td>0.116</td>
<td>0.096</td>
<td>0.099</td>
<td>0.073</td>
<td>0.100</td>
<td>0.084</td>
</tr>
<tr>
<td>Skewness</td>
<td>1.212</td>
<td>1.294</td>
<td>1.020</td>
<td>1.329</td>
<td>0.206</td>
<td>0.761</td>
<td>1.373</td>
<td>-0.131</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>2.772</td>
<td>5.181</td>
<td>4.816</td>
<td>5.020</td>
<td>0.255</td>
<td>1.995</td>
<td>5.252</td>
<td>3.746</td>
</tr>
<tr>
<td>J-B</td>
<td>18.77</td>
<td>36.27</td>
<td>23.62</td>
<td>35.29</td>
<td>24.40</td>
<td>10.53</td>
<td>39.94</td>
<td>1.98*</td>
</tr>
<tr>
<td>Q(5)</td>
<td>31.4</td>
<td>21.6</td>
<td>18.6</td>
<td>19.0</td>
<td>17.7</td>
<td>20.1</td>
<td>20.4</td>
<td>18.9</td>
</tr>
<tr>
<td>Q(10)</td>
<td>40.2</td>
<td>28.4</td>
<td>22.3</td>
<td>23.7</td>
<td>24.9</td>
<td>23.5</td>
<td>23.8</td>
<td>26.6</td>
</tr>
<tr>
<td>Q(20)</td>
<td>64.0</td>
<td>49.8</td>
<td>47.1</td>
<td>40.4</td>
<td>46.3</td>
<td>49.3</td>
<td>37.6</td>
<td>45.5</td>
</tr>
<tr>
<td>Q(40)</td>
<td>109.2</td>
<td>102.9</td>
<td>109.0</td>
<td>100.0</td>
<td>106.6</td>
<td>101.4</td>
<td>103.1</td>
<td>105.8</td>
</tr>
<tr>
<td>Qsq(5)</td>
<td>24.2</td>
<td>22.2</td>
<td>28.4</td>
<td>26.1</td>
<td>23.4</td>
<td>23.8</td>
<td>24.5</td>
<td>30.3</td>
</tr>
<tr>
<td>Qsq(10)</td>
<td>47.9</td>
<td>42.8</td>
<td>50.8</td>
<td>47.4</td>
<td>31.9</td>
<td>44.7</td>
<td>45.1</td>
<td>53.2</td>
</tr>
<tr>
<td>Qsq(20)</td>
<td>59.8</td>
<td>56.6</td>
<td>64.0</td>
<td>60.1</td>
<td>53.8</td>
<td>60.0</td>
<td>57.8</td>
<td>67.4</td>
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<tr>
<td>Qsq(40)</td>
<td>94.5</td>
<td>93.1</td>
<td>97.0</td>
<td>95.8</td>
<td>95.1</td>
<td>97.9</td>
<td>95.5</td>
<td>101.3</td>
</tr>
</tbody>
</table>
Table IV: The Ljung-Box Q-statistic for the \( \{ R_t \} \) exchange rate series at lag 10 and 20 for the daily (end of day)-, weekly (end of week)-, and monthly (monthly average) frequency.

<table>
<thead>
<tr>
<th>Exchange rate</th>
<th>Frequency</th>
<th>Qsq(10)</th>
<th>Qsq(20)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEKUSD</td>
<td>daily</td>
<td>143.2</td>
<td>162.5</td>
</tr>
<tr>
<td></td>
<td>weekly</td>
<td>35.1</td>
<td>43.1</td>
</tr>
<tr>
<td></td>
<td>monthly</td>
<td>47.9</td>
<td>59.8</td>
</tr>
<tr>
<td></td>
<td>daily</td>
<td>131.7</td>
<td>159.1</td>
</tr>
<tr>
<td></td>
<td>weekly</td>
<td>31.3</td>
<td>45.8</td>
</tr>
<tr>
<td></td>
<td>monthly</td>
<td>42.8</td>
<td>56.6</td>
</tr>
<tr>
<td>SEKDEM</td>
<td>daily</td>
<td>147.9</td>
<td>171.2</td>
</tr>
<tr>
<td></td>
<td>weekly</td>
<td>61.3</td>
<td>71.4</td>
</tr>
<tr>
<td></td>
<td>monthly</td>
<td>50.8</td>
<td>64.0</td>
</tr>
<tr>
<td>SEKGBP</td>
<td>daily</td>
<td>157.1</td>
<td>175.3</td>
</tr>
<tr>
<td></td>
<td>weekly</td>
<td>52.9</td>
<td>70.6</td>
</tr>
<tr>
<td></td>
<td>monthly</td>
<td>47.4</td>
<td>60.1</td>
</tr>
<tr>
<td>SEKFRF</td>
<td>daily</td>
<td>195.3</td>
<td>235.1</td>
</tr>
<tr>
<td></td>
<td>weekly</td>
<td>23.1</td>
<td>34.9</td>
</tr>
<tr>
<td></td>
<td>monthly</td>
<td>31.9</td>
<td>53.8</td>
</tr>
<tr>
<td>SEKJPY</td>
<td>daily</td>
<td>122.1</td>
<td>151.2</td>
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<td></td>
<td>weekly</td>
<td>53.2</td>
<td>75.6</td>
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<td></td>
<td>monthly</td>
<td>44.7</td>
<td>60.0</td>
</tr>
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<td>SEKNOK</td>
<td>daily</td>
<td>131.7</td>
<td>159.2</td>
</tr>
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<td></td>
<td>weekly</td>
<td>57.1</td>
<td>74.1</td>
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<tr>
<td></td>
<td>monthly</td>
<td>45.1</td>
<td>57.8</td>
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<tr>
<td>SEKDKK</td>
<td>daily</td>
<td>167.1</td>
<td>190.1</td>
</tr>
<tr>
<td></td>
<td>weekly</td>
<td>71.2</td>
<td>87.2</td>
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<tr>
<td></td>
<td>monthly</td>
<td>53.2</td>
<td>67.4</td>
</tr>
</tbody>
</table>
Table V: ARCH- and GARCH-estimates for the $R_t = \log \left( \frac{S_t}{S_{t-1}} \right)$ exchange rate series with $t$-values for H0:parameter=0 inside the brackets. The * and ** indicate significance at the 1 percent and 5 percent level respectively. J-B indicate the Jarque-Bera test for normality and $Q(n)$ and $Qsq(n)$ give, respectively, the Ljung-Box statistic for standardised residuals and squared standardised residuals up to the $n$:th order of serial correlation. The distribution of these statistics is $\chi^2(2)$ under the null of normality and $\chi^2(n)$ under the null of no serial correlation, respectively. SSR, AIC and SBC indicates the sum of squared residuals, Akaike and Schwartz information criterion, respectively.

<table>
<thead>
<tr>
<th>Exchange rate</th>
<th>SEK/USD</th>
<th>SEKDEM</th>
<th>SEKGBP</th>
<th>SEKFRF</th>
<th>SEKJPY</th>
<th>SEKOK</th>
<th>SEKDKK</th>
<th>SEKFIM</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1)</td>
<td>0.42 (3.00)*</td>
<td>0.34 (2.00)**</td>
<td>0.41 (3.09)*</td>
<td>0.29 (2.42)*</td>
<td>0.36 (2.40)*</td>
<td>0.38 (3.13)*</td>
<td>0.25 (1.92)**</td>
<td>0.36 (2.71)*</td>
</tr>
<tr>
<td>c</td>
<td>0.0004 (1.54)</td>
<td>0.0004 (1.42)</td>
<td>0.0001 (1.85)</td>
<td>0.0002 (1.58)</td>
<td>0.0009 (1.88)</td>
<td>0.0003 (1.75)</td>
<td>0.0002 (1.65)</td>
<td>0.0002 (1.32)</td>
</tr>
<tr>
<td>p</td>
<td>0.43 (2.26)**</td>
<td>0.25 (2.27)**</td>
<td>0.34 (2.27)**</td>
<td>0.70 (2.19)**</td>
<td>0.30 (2.14)**</td>
<td>0.13 (2.17)**</td>
<td>0.69 (2.45)*</td>
<td>0.14 (2.02)**</td>
</tr>
<tr>
<td>q</td>
<td>- - 0.55 (3.67)*</td>
<td>- - - -</td>
<td>- - - -</td>
<td>- - - -</td>
<td>- - - -</td>
<td>0.52 (2.68)*</td>
<td>- - - -</td>
<td></td>
</tr>
</tbody>
</table>

Residual test.

| Skewness       | 0.45 | 0.71 | 0.41 | 0.50 | -0.38 | 0.25 | -0.02 | -0.38 |
| Kurtosis       | 1.73 | 3.84 | 4.30 | 3.76 | 0.90  | 3.87 | 3.56  | 3.75  |
| J-B            | 7.67** | 8.62** | 7.37** | 4.93 | 15.79* | 3.15 | 0.97  | 3.56  |
| Q(5)           | 4.35 | 7.02 | 6.33 | 8.23 | 1.89  | 5.29 | 7.78  | 8.99  |
| Q(10)          | 15.95 | 11.12 | 10.31 | 11.39 | 5.75  | 7.87 | 11.18 | 15.65 |
| Q(20)          | 32.79 | 25.64 | 15.08 | 29.35 | 15.45 | 28.15 | 21.95 | 27.56 |
| Q(40)          | 46.72 | 38.26 | 27.20 | 40.07 | 35.64 | 41.83 | 35.53 | 42.03 |
| Qsq(5)         | 3.03 | 1.27 | 1.89 | 1.09 | 4.21  | 1.46 | 1.22  | 6.47  |
| Qsq(10)        | 9.16 | 4.32 | 3.98 | 3.57 | 7.41  | 3.41 | 3.39  | 12.99 |
| Qsq(20)        | 12.58 | 7.83 | 7.09 | 7.54 | 10.89 | 9.89 | 9.30  | 16.46 |
| Qsq(40)        | 17.09 | 16.38 | 17.36 | 20.66 | 26.32 | 24.73 | 30.00 | 20.73 |
| SSR            | 0.042 | 0.030 | 0.044 | 0.027 | 0.097 | 0.022 | 0.028 | 0.034 |
| AIC            | -235.43 | -257.30 | -225.63 | -263.76 | -171.29 | -279.41 | -260.79 | -245.45 |
| SBC            | -228.44 | -250.34 | -216.36 | -256.81 | -164.30 | -272.46 | -253.84 | -236.18 |
**Table VI:** Optimal ARCH-model sign bias test result. The *t*-statistics (probabilities of rejection) for each of the test coefficients in equation 1.

<table>
<thead>
<tr>
<th>Exchange rate</th>
<th>ARCH(q) or GARCH(p,q)</th>
<th>Sign bias test</th>
<th>Negative size bias test</th>
<th>Positive size bias test</th>
<th>Joint test for the three effects</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEKUSD</td>
<td>(1)</td>
<td>-0.67 (0.50)</td>
<td>0.38 (0.71)</td>
<td>0.77 (0.44)</td>
<td>1.55 (0.21)</td>
</tr>
<tr>
<td>SEKDEM</td>
<td>(1)</td>
<td>-0.18 (0.86)</td>
<td>0.95 (0.35)</td>
<td>0.43 (0.67)</td>
<td>0.92 (0.44)</td>
</tr>
<tr>
<td>SEKGBP</td>
<td>(1,1)</td>
<td>-0.23 (0.82)</td>
<td>-0.36 (0.72)</td>
<td>-0.16 (0.87)</td>
<td>0.05 (0.98)</td>
</tr>
<tr>
<td>SEKFRF</td>
<td>(1)</td>
<td>1.34 (0.18)</td>
<td>1.59 (0.12)</td>
<td>0.78 (0.44)</td>
<td>1.06 (0.37)</td>
</tr>
<tr>
<td>SEKJPY</td>
<td>(1)</td>
<td>-1.32 (0.19)</td>
<td>0.49 (0.62)</td>
<td>-0.10 (0.92)</td>
<td>2.03 (0.14)</td>
</tr>
<tr>
<td>SEKDKK</td>
<td>(1)</td>
<td>1.06 (0.29)</td>
<td>1.50 (0.14)</td>
<td>1.17 (0.25)</td>
<td>1.25 (0.30)</td>
</tr>
<tr>
<td>SEKFIM</td>
<td>(1,1)</td>
<td>-1.01 (0.32)</td>
<td>0.03 (0.98)</td>
<td>-0.11 (0.92)</td>
<td>0.58 (0.63)</td>
</tr>
</tbody>
</table>

**Table VII:** Estimates of the EGARCH conditional volatility model with *t*-statistics in parenthesis for SEKDEM, SEKFRF and SEKDKK.

<table>
<thead>
<tr>
<th>Exchange rate:</th>
<th>SEKDEM</th>
<th>SEKFRF</th>
<th>SEKDKK</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1)</td>
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<td>0.31 (2.46)</td>
<td>0.29 (2.07)</td>
</tr>
<tr>
<td>c</td>
<td>-0.59 (2.89)</td>
<td>-1.73 (2.25)</td>
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<tr>
<td>p</td>
<td>0.93 (15.8)</td>
<td>0.79 (7.12)</td>
<td>0.82 (6.99)</td>
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<tr>
<td>q</td>
<td>0.56 (2.88)</td>
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<td>a</td>
<td>-0.78 (2.35)</td>
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References.


Long memory dynamics in the Swedish exchange rate.

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ABSTRACT

The behaviour of eight Swedish exchange rates over short and long run horizons are analysed by the use of the ARFIMA-GARCH model and the Geweke and Porter-Hudak (GPH) estimator. The ARFIMA-GARCH test provides no evidence of long memory in the Swedish exchange rate. The GPH-estimator provided a significant result in two cases; the SEKFIM and the SEKGBP exchange rates. The significance is at the 5 percent level but at different frequency levels for the GPH-test. There is thus no systematic significant evidence of a long memory property in the Swedish exchange rate. The result was also supported by the test for a memory structure in the randomly rearranged data series evaluated against the memory structure in the original series. This result indicates the importance of combining different tests for examining the same type of effect in financial data. Some possible sources of a long memory structure in exchange rates are briefly discussed.

KEYWORDS: ARCH-models, exchange rate, long memory, Sweden.
1. Introduction.

The empirical behaviour of exchange rates provides useful information for constructing models for exchange rate determination and can, furthermore, be used to evaluate the performance of exchange rate models. The development of exchange rates has consequences for e.g. the inflation rate through the exchange rate pass-through mechanism, interest rates and prices of tradable goods, and indirectly on wages, production levels and employment opportunities. The ability to predict the future development of exchange rates is therefore important, not only for participants in the exchange rate market, but also for economic policy makers (such as a central bank) as an input in the (monetary) policy making process\(^1\).

In the last decade, there has been much research and discussion that exchange rates might exhibit a rather special and possibly important property of a long memory process\(^2\). The long memory process has a dynamic property, which is not incorporated in standard time series models such as the ARIMA-model. Many previous studies on exchange rates conclude that spot (log) nominal exchange rates contain a unit root, i.e. that they are well described as a martingale difference\(^3\). The first difference of the spot (log) nominal exchange rate is, on the other hand, often concluded to be a stationary process. The standard tests for a unit root discriminates between \(I(0)\) and \(I(1)\) processes but do not incorporate the possibility for the more general process of fractional integration \([I(d)]\), i.e. a process where \(0<d<1\). The fractionally integrated ARMA-model, the ARFIMA-model, incorporates this generalisation and makes it a parsimonious and more flexible model for the simultaneous study of both the long memory and short-run dynamics. Moreover, fractional integration is a more general way to describe long-range dependence than the unit-root specification and provides an alternative perspective to examine the hypothesis of a unit root.

\(^1\) It is of a relative importance for a central bank like the Swedish Central Bank with an inflation target under a floating exchange rate regime.

\(^2\) A covariance stationary time series exhibits a long memory property if \(\sum_{k=1}^n |\rho(k)| \to \infty \text{ as } n \to \infty\) where \(\rho(k)\) is the autocorrelation at lag \(k\). This infinite-sum condition suggests that correlations at long lags are not negligible. For a further discussion on long memory models see Maneschiöld (2000) among others. For a discussion about applications of long memory properties in exchange rates see Cheung (1993) and Maneschiöld (2000) among others.

\(^3\) See Brockwell et al (1991) among others.
The research on long memory dependence in exchange rates has mainly been conducted on the major traded currencies such as the US dollar, Deutsche mark, and Japanese yen.\(^4\) The results are mixed in the sense that a long memory effect is not in general found in the exchange rates but only for some exchange rates and periods. However, there has been no research on the long memory effect conducted on a less frequently traded currency such as the Swedish krona. I will in this paper analyse and test for long memory properties in the Swedish exchange rate as the appropriate statistical tool for studying the relationship between exchange rates and other economic variables can depend on the time series properties of the exchange rate. I will use the fractional differencing test for long memory devised by Geweke et al (1983), GPH, and the ARFIMA(p,d,q)-GARCH(P,Q)-model devised by Baillie et al (1996). This latter model is an ARMA(p,q)-model generalised to include a long memory component (fractional integration) and conditional heteroscedasticity of the ARCH/GARCH type in the error term. The fractional differencing in the GPH-model involves differencing a time series by a fractional nonintegral exponent, \(d\). The long run dependence in these two models is captured by this \(d\)-parameter, the fractional differencing parameter, which describes the higher order correlation structure in the series.

The paper is organised as follows. Section two introduces the fractionally differenced time series models and the Geweke and Porter-Hudak estimator. Section three presents the empirical results. Section four presents a brief discussion about sources of long memory in exchange rates and section five concludes.

\(^4\) See Cheung (1993) for a further discussion.
2. Fractionally differenced time series models.

Granger et al (1980) and Hosking (1981) introduced the ARFIMA-model (the method of fractional differencing), which incorporate long memory dynamics into time series models. Whether a data series displays a structure of long-term dependency or not is in this model conditional on a fractional differencing parameter, which can be estimated.

A general class of long memory processes can be described by

\[
\phi(L)(1-L)^d y_t = \theta(L)e_t
\]

where \(\phi(L)\) and \(\theta(L)\) are polynomials in the lag operator (L) with p and q lags, respectively. The \(d\)-parameter is the difference parameter, which is allowed to take fractional values indicating the dynamics of the memory process in the series. All roots of \(\phi(L)\) and \(\theta(L)\) are stable and \(e_t\) is a white noise disturbance term. The fractional parameter, \(d\), can assume any real value. This fractional model includes the usual autoregressive moving average model when \(d=0\) as a special case, i.e. the ARMA(p,q)-model. Upon using a return series as the dependent variable\(^5\), a value of \(d\) greater than zero indicates a mean diverting property while a value less than zero indicates a mean reverting property in the original series.

The extension to allow for non-integer values of \(d\) raises the flexibility in modelling long-term dynamics. This extension allows for a more rich class of spectral behaviour at low frequencies. Hosking (1984) has further developed this approach in proposing the sum of squares estimator for the ARFIMA-model, which is asymptotically equivalent to the maximum likelihood estimator. Baillie et al (1996) has extended Hosking’s approach to incorporate ARCH-errors for the white noise disturbance term. This ARFIMA(p,d,q)-GARCH(P,Q)-process can be written as

\(^5\) As will be done in this paper as the log nominal exchange rate series contains a unit root but not the first difference of the log nominal exchange rate series. For a further discussion of the data see Maneschiöld (2001) in this thesis.
\[ \phi(L)(1 - L)^d (y_t - \mu - \lambda \sigma_t) = \theta(L)e_t \quad (2) \]

\[ \epsilon_t/\Omega_{t-1} \approx D\left(0, \sigma_t^2\right) \quad (3) \]

\[ \beta(L)\sigma_t^2 = \omega + \alpha(L)e_t^2 \quad (4) \]

where \( \mu \) is the mean of the process, \( \phi(L) \), \( \theta(L) \), \( \beta(L) \) and \( \alpha(L) \) are the lag polynomials of order \( p \), \( q \), \( P \) and \( Q \), respectively. The distribution of the residuals conditioned upon information at \( t-1 \), \( \Omega_{t-1} \), is normally or \( t \)-distributed.

Geweke and Porter-Hudak (1983) proposed a frequency domain estimator for the long memory parameter \( d \) (hereafter abbreviated as the GPH-estimator). The GPH-estimator is a semi non-parametric test for fractional processes that does not require any specification of the short memory process, i.e. the ARMA-part. This is so since only a fraction of the first \( T^a \) frequencies are used. The fractional differencing test serves to uncover a fractal structure in a time series based on spectral analysis of its low frequency dynamics. The spectral regression is specified as

\[ \ln[\hat{f}(w_j)] = \phi_0 - \phi_j \ln[4\sin^2(w_j/2)] + \epsilon_{\tau}, \quad j = 1, ..., n \quad (5) \]

where \( \ln[\hat{f}(w_j)] \) is the periodogram at the frequency \( w_j = 2\pi j/T \). The asymptotic variance of the error term \( \epsilon_{\tau} \) is \( \pi^2/6 \), which is imposed in the estimation to raise the efficiency of the estimator. Geweke et al (1983) also show that the ordinary least square estimate of \( \phi_j \) provides a consistent estimate of the long memory parameter, \( d \), and hypothesis testing concerning the value of the long memory parameter can be based on the usual \( t \)-statistics\(^6\). There is evidence of a long memory process in the data

\(^6\) The estimation of the long memory parameter, using the GPH-estimator, does not make any assumptions about the shape of the underlying distribution, e.g. that it must be Gaussian, more than that the underlying process is independent. Furthermore, the estimation of the long memory parameter is asymptotically standard normally distributed, which makes the hypothesis tests that are based upon the normal distribution applicable. Using synthetic data, Geweke et al (1983) shows that the asymptotic theory proves to be reliable in samples of 50 observations or more. For a more detailed derivation and discussion of the GPH-estimator see Geweke et al (1983) and Maneschiöld (2000) among others.
if the ordinary least square estimate of the long memory parameter is significantly larger than zero.

With a proper choice of \( n \) in equation (5) then will the asymptotic distribution of the long memory parameter, \( d \), neither depend on the order of the ARMA-components nor on the distribution of the error term of the ARFIMA-process. Cheung (1993) and Cheung et al (1995) suggest to set \( n = T^{\alpha} \) with \( \alpha \)-values of 0.50 and 0.55 as a fair compromise between low and high frequencies. The GPH-estimator can also be used as a unit root test by determining whether the long memory parameter from the first-differenced data is significantly different from zero.

An alternative method to detect long run dependence in highly non-Gaussian time series is the modified R/S-analysis proposed by Lo (1991). This test for a long memory process can be regarded as a robust non-parametric test and examines the null hypothesis even if the data generating process generates short-term dependence and heteroscedasticity. The modified R/S-statistic is given by the range of cumulative sums of the deviations of the time series (with \( T \) observations) from its mean rescaled

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7 The \( k:th \) autocorrelation of a stochastic process is denoted by \( \rho(k) \), which can be shown to be defined by \( \rho(k) = A k^{-\alpha} \) where \( A \) is a suitable constant and \( d \) is the memory parameter. A stationary short memory process is characterised by \( \sum_{k=-\infty}^{\infty} |\rho(k)| < \infty \) and a significant parameter estimate within the bounds \(-0.50<d<0\). A stationary long memory process is characterised by \( \sum_{k=-\infty}^{\infty} |\rho(k)| = \infty \) and a significant parameter estimate within the bounds \( 0<d<0.50 \). When \( 0.50<d<1 \), then does the process possess a long memory but it is non-stationary and mean-reverting and for \( d>1 \) it is non-stationary but mean-diverting. For a further discussion see Brockwell et al (1991), Hosking (1981) and Maneschiöld (2000) among others.

8 The \( \alpha \)-value represents the proportion of the observations included in the GPH-estimator. The number of low frequency ordinates, \( T^\alpha \), used in the spectral regression is a choice variable in the GPH-estimator. A too large number of \( T^\alpha \) will cause contamination of the estimate of the long memory parameter, \( d \), due to that medium- or high frequency components and thus more short-term influences are included in the analysis. A too small value of \( T^\alpha \) will lead to imprecise estimates due to limited degrees of freedom in the estimation. For a further discussion see Cheung et al (1995) and Maneschiöld (2000) among others.

9 This has a particular interest in the examination of the dynamics of exchange rates as standard unit root tests, which have a stationary ARMA-process as their alternative hypothesis (e.g. the ADF-test), usually cannot reject the hypothesis of a unit root in exchange rate data. For a further discussion see Cheung (1993) among others.
by a consistent estimate of its standard deviation\textsuperscript{10}. An evaluation of the relative efficiency between the GPH- and the modified R/S-estimator is conducted with the use of the exchange rate data used in this paper\textsuperscript{11}. The result of the estimation of the long memory parameter using the GPH- and the modified R/S-estimator are not significantly different in magnitude but the GPH-estimator seems to be the relative more efficient estimator\textsuperscript{12}.

3. Empirical results.

In this section we will examine the eight most important bilateral exchange rates for Sweden\textsuperscript{13} using the method of fractional differencing. The results of the ARFIMA-ARCH/GARCH and the GPH-estimator are displayed in table I and table II in Appendix A, respectively\textsuperscript{14}. From table I for the ARFIMA-ARCH/GARCH estimation, two facts are to be mentioned. The first fact is that the long memory parameter, \(d\), is not significantly different from zero, i.e. there is no indication of a long memory process in any of the exchange rates. The second fact to be mentioned is that the other parameter estimates in the table are more or less of the same magnitude as those reported in Maneschiöld (2001) table V. In other words, the exchange rate

\textsuperscript{10} For a more detailed derivation and discussion of the GPH- and modified R/S-estimator see Maneschiöld (2000).

\textsuperscript{11} See Maneschiöld (2000) for results.

\textsuperscript{12} Agiakloglou \textit{et al} (1992) has shown that there are situations where the GPH-estimator has poor small sample properties. I will use the GPH-estimator upon estimating the long memory parameter in the empirical part of this paper as both the GPH- and the modified R/S-estimator produced similar estimates of the long memory parameter and that the variance of the GPH-estimator was relative smaller than the modified R/S-estimator. For a further discussion see Baillie \textit{et al} (1996) and Maneschiöld (2000) among others.

\textsuperscript{13} For a description of the characteristics of the data see Maneschiöld (2001) in this thesis. The examination of the long memory properties is conducted on the first difference of the log monthly (average), weekly (end), and daily (end) exchange rate. Using the different frequencies of the data, the empirical results seem not to significantly affect the outcome of the result during this period as the test statistics and estimates were similar over the different frequencies. I will therefore only present the data for the monthly (average) exchange rate series to conserve space.

\textsuperscript{14} The methodology for selecting the optimal model was as described in Maneschiöld (2001).
return series are short-term dependent in all models since the AR(1)-parameter is significant. The exchange rate of SEKGBP and SEKFIM reveal also significant evidence of a conditional heteroscedasticity of the GARCH(1,1)-form, while the conditional heteroscedasticity for the other exchange rates are of the ARCH(1)-form\textsuperscript{15}.

The Jarque-Bera test for normality indicates that the null of normally distributed residuals is rejected for SEKJPY at the 1 percent level and for SEKUSD, SEKDEM and SEKGBP at the 5 percent level, respectively. The Jarque-Bera test for normality is not rejected for the other exchange rates\textsuperscript{16}. The Ljung-Box test statistic for standardised residuals and squared standardised residuals show no presence of serial correlation up to the 40:th order for the exchange rate return series.

The empirical estimates of the long memory parameter by the use of the GPH-estimator are displayed in table II in Appendix A. The long memory parameter is significant for the SEKFIM exchange rate. This is so for both the OLS and asymptotic \( t \)-statistic but only for the 0,50 frequency value. The SEKGBP exchange rate has a significant long memory parameter by the use of the OLS \( t \)-statistic and the frequency value of 0,55, i.e. when I include more periodogram ordinates and thus more short-term influences into the regression. The significance level for the two exchange rates is at the 5 percent level. The long memory parameter is not significant for the other exchange rates. This insignificance is also supported by the graphs of the spectral density of the exchange rate series, which are displayed in appendix B. The spectral density for the exchange rate series is not only concentrated around the lower frequencies but also around the relative higher frequencies\textsuperscript{17}. This is in contrast to the theoretical shape of the spectral density function of a long memory process with a

\textsuperscript{15} This was the same evidence as was found in Maneschiöld (2001).

\textsuperscript{16} The figures in Appendix B that compares the distribution function to the normal distribution function reveal, as do table I in Appendix A, that all exchange rates are to some degree skewed and leptokurtic and not perfectly normally distributed.

\textsuperscript{17} This gives an indication that the series do not belong to the long memory processes. Confirmation of the estimated long memory parameter was also obtained by estimating the first difference of the exchange rate return series, i.e. the second difference of the log nominal exchange rate. The estimates were close to those in table II after subtracting the long memory estimate by one to get the corresponding estimate to those in table II. The estimates are not reported here but can be obtained from the author.
density function that is concentrated around the lower frequencies. There is also an
indication that the exchange rates do belong to the stationary memory processes.\textsuperscript{18}
This indication is obtained by a significant long memory parameter for only SEKFIM
and SEKGBP but that the hypothesis $H_0:d=0.50$ is insignificant for those exchange
rates. Thus, it seems that the exchange rate processes do belong to the stationary
memory processes.

A series with an indication of a significant long memory process might be the
result of two possible structures. The first possibility is that there is actually a long
memory process in the series, which then should be encountered upon modelling the
series. The second possibility is that the series actually has an independent structure of
random variables but that the observations do scale according to a value of the long
memory parameter that is significantly different from zero. More precisely, an
estimate of the long memory parameter that is significantly different from zero has
these two possible explanations:

1. There is a long memory component in the time series, which implies that each
   observation is correlated to some degree with the observation that follows.

2. The analysis itself is flawed and an anomalous value of the long memory
   parameter does not mean that there is a long memory effect at work. Perhaps do
   we not have enough data for a valid test as the guidelines in the literature to the
correct amount is somewhat fuzzy. Still, the series being studied is an independent
series of random variables, which happens to (1) scale according to a value
significantly different from zero, or (2) to be an independent process with fat tails,
as suggested by Cootner (1964).

We can test the validity of the results by randomly re-arrange the order of the data
of the original time series into a scrambled data set and recalculate the long memory
parameter. To confirm the results of the estimates I will perform this informal test\textsuperscript{19} of
the long memory parameter in the exchange rate return series. The frequency

\textsuperscript{18} The stationary memory process implies a significant parameter estimate in the domain $-0.50<d<0.50$.

\textsuperscript{19} See Peters (1991) and Peters (1994) for a discussion about the so-called scramble test.
distribution of the observations, upon which the GPH-estimator hinges, will remain unchanged. The reason is that all observations are still incorporated in the re-arranged series.

The order of the data is important if there is a long memory effect in the series. The scrambling of the data would then destroy this structure and the recalculated long memory parameter should then be insignificantly different from zero but significantly different from the estimated long memory parameter of the original series. If the series instead is truly an independent series then should the recalculated long memory parameter be virtually unchanged. The reason is that there did not exist a long memory effect, or correlation, between the observations in the original series. Therefore, scrambling the data would have no effect on the qualitative aspect of the data.

The result of the scramble test for the exchange rate return series is reported in table III in Appendix A together with the corresponding long memory estimates of the original series estimated by the GPH-estimator at the 0.50 frequency value. The estimates of the long memory parameter for the scrambled series are not significantly different from zero by the use of the OLS t-statistics. Using the GPH-estimator, there is evidence that there is no long memory effect in the exchange rates except for the SEKFIM exchange rate, where the long memory parameter is significant for the unscrambled original series but insignificant for the scrambled randomly re-arranged series. Furthermore, some of the scrambled d-estimates have larger amplitude than the d-estimates of the unscrambled series. However, the long memory estimate of the scrambled series of the exchange rates is not significantly different from the d-estimates of the original series by the use of the t-test (see table III in Appendix A). Therefore, there is evidence that the original series do not in general contain a systematic long memory dependence structure but rather belong to the short memory processes\textsuperscript{20}.

\textsuperscript{20} For a discussion on short memory time series models see Brockwell et al (1991) and Maneschiöld (2000) among others.
One interesting result to note is that the SEKFIM exchange rate, which had a significant long memory parameter at the 0.50 frequency value for the OLS t-statistic, is now not significantly different from zero when using the randomly re-arranged series. By using the t-test, the unscrambled and scrambled memory estimates are furthermore not significantly different from each other. This indicates both a significant and insignificant memory parameter. In other words, the scrambled insignificant memory parameter is not significantly different from the unscrambled significant memory parameter at the 0.50 frequency value by the use of the OLS t-statistic.

The above results show the importance of combining different tests for examining the same type of effect in exchange rate data in order to construct and evaluate the performance of exchange rate models.

4. Sources of long memory.

The linkages between exchange rates and real and nominal variables provides different possible sources of a long memory structure in the exchange rate, i.e. the long memory behaviour of the exchange rate can be related to the dynamic properties of other economic variables. The relative purchasing power parity (PPP) hypothesis suggests that exchange rate fluctuations are related to changes in relative prices. The empirical evidence for the PPP hypothesis for the included countries provides in general no support for PPP, i.e. a cointegrating relationship between exchange rates and changes in relative prices is in general not found\textsuperscript{21}. However in the papers by Alexius (1995) and Jacobson et al (1998), for slightly different countries, periods and approach there is evidence of the necessary conditions for PPP, i.e. a cointegrating relationship is in general found. The sufficient conditions are however not satisfied, i.e. the coefficients in the cointegrating relations are in general far from what PPP predicts. Thus, there is only some mild evidence of the PPP hypothesis during this

\textsuperscript{21} The results are not reported here but can be obtained from the author.
period in the sense that exchange rates and changes in relative prices are cointegrated. The estimated results of the long memory parameter for the relative inflation series\textsuperscript{22} for the included countries are reported in Table IV. The GPH-test suggests that the relative inflation series do not indicate a long memory property, as were the results in general for the corresponding exchange rates. Furthermore, the results are independent of the choice of the parameter for the frequency value, which also in general was the case for the corresponding exchange rates.\textsuperscript{23} The weak support for the sufficient condition for the PPP hypothesis during the period can be a possible explanation for the empirical results of the GPH-estimate of the exchange rate and its corresponding relative inflation series as it might be expected to find a long memory in exchange rate series and relative inflation series if (1) PPP holds in the long run with a slow adjustment towards PPP, and (2) inflation differentials and exchange rates are cointegrated. Thus, a lack of a significant memory in both variables indicates support for that the real exchange rate is a random process in spite the evidence from other researchers that exchange rates and inflation differentials are cointegrated.

The expected change in the spot exchange rate is related to the differential of the nominal interest rate by the uncovered interest parity (UIP) between two countries on two similar financial instruments. The empirical results are, in a paper by Alexius (1998) where the results are in contrast to the typical finding, rather favourable to the UIP for the, in this paper, included countries. The long memory parameter for the nominal interest rate differential of five-year government bonds at the monthly frequency is in general not significant.\textsuperscript{24} By incorporating the Ex ante purchasing power parity (“Fisher open relationship”) into the UIP and assuming that the expected outcome is equal to the actual outcome, then will the change in the exchange rate depend on the inflation differentials\textsuperscript{25}. Table IV reports the results for the estimates of

\textsuperscript{22} The sample period is November 1992 to March 1998 and the frequency is monthly for the consumer price index (CPI). The data are collected from the Central Bank of Sweden (Sveriges Riksbank) as are the data in this part of the paper.

\textsuperscript{23} There is also an indication that the relative inflation series might belong to the stationary memory processes, i.e. with a significant parameter estimate inside the domain \(-0.50<d<0.50\), as was the case for the corresponding exchange rates.

\textsuperscript{24} The results are not reported here but can be obtained from the author.

\textsuperscript{25} UIP is defined as
the long memory parameter of the relative inflation series. There is no evidence of a long memory process in the inflation differentials by the use of the GPH-test.

There is, furthermore, a relative high degree of integration between the countries from the point of view of a continuous flow of financial risk capital between the countries reflecting investment opportunities. Recent empirical studies on the dynamic properties of stock returns and relative stock returns do not in general indicate a long memory property\(^{26}\). The long memory property of the relative money supply and the relative output is furthermore estimated with only some mild evidence of long memory in the relative money supply series for some of the countries but not

\[ \Delta s_{t+1} = R_{t} - R_{t}^{*} \]

and the “Fisher open relationship” (Fisher effect) is defined as

\[ R_{t} = r_{t} + \Delta p_{t+1} \]
\[ R_{t}^{*} = r_{t}^{*} + \Delta p_{t+1}^{*} \]

where \( s \) is the spot rate, \( R \) is the nominal interest rate, \( r \) is the real interest rate, \( p \) is the price index, and \( * \) indicates a foreign variable. Substitute the “Fisher open relationship” into UIP under the assumption that \( r_{t} = r_{t}^{*} \). The UIP will then be approximated as

\[ \Delta s_{t+1} = \Delta p_{t+1} - \Delta p_{t+1}^{*} \]

The “Fisher open relationship” then implies that the expected appreciation/depreciation in the exchange rate is linked to the difference in expected inflation. Suppose, furthermore, that

\[ \Delta p_{t+1} = \Delta p_{t+1} + \varepsilon_{t} \]
\[ \varepsilon_{t} \sim N\left(0, \sigma_{\varepsilon}^{2}\right) \]

Without loss of generality, with the simplifying assumption that

\[ \Delta p_{t+1} = \Delta p_{t+1} \]

i.e. the expected inflation is equal to actual inflation. If we assume that the expected outcome in each variable is equal to the actual outcome, then can the UIP be stated as

\[ \Delta s_{t+1} = \Delta p_{t+1}^{*} - \Delta p_{t+1} \]

i.e. the same equation as for the relative purchasing power parity (PPP) hypothesis. Table IV then reports the results of a linkage and a possible source for a long memory structure in the exchange rate from both the relative PPP hypothesis and the UIP hypothesis with the assumption that the expected outcome is equal to the actual outcome in the UIP and the “Fisher open relationship”.

\(^{26}\) See Cheung and Lai (1995), Lo (1991) and Lyhagen (1997) among others. The referred papers do however not include stock returns from all countries included in this paper. The general conclusion in the papers implies although an indication of an insignificant long memory structure in the stock returns over different frequencies, countries and periods.
in the other series. More work is, however, required to establish the relationship of  
the dynamics of exchange rates and its fundamentals and the possible sources and 
causalities of long memory structures in exchange rates and its fundamentals.

5. Conclusions.

The time series properties over short and long run horizons of eight Swedish  
exchange rates are examined in this paper. The ARFIMA-GARCH model, in which  
the differencing parameter can assume non-integer values, provides no support for a  
long memory structure or that a fractal structure is exhibited in the Swedish exchange  
rates. The ARFIMA-GARCH model, in which the differencing parameter can assume non-integer values, provides no support for a  
long memory structure or that a fractal structure is exhibited in the Swedish exchange  
rates. The GPH-estimator, the fractional differencing test, gave, on the other hand,  
two significant results. The SEK/FIM exchange rate is significant at the frequency  
value 0.50 and for both the OLS and asymptotic t-statistics but not for higher  
frequency levels, i.e. when more periodogram ordinates and thus more short-term  
influences are included. The SEK/GBP has a significant long memory parameter but  
only by the use of the OLS t-statistic and the frequency of 0.55. The long memory  
parameter is insignificant for the other exchange rate series. The empirical tests also  
indicate that the exchange rates are short term dependent. The result of the scramble  
test provides further support to the conclusion that the Swedish exchange rate does  
not in general include a systematic significant long memory property. The  
contradicting results between the ARFIMA-GARCH approach and the GPH-estimator  
indicates the importance of combining different tests for examining the same type of  
effect in financial data such as exchange rate data.

Empirical results from recent studies on the dynamic properties of real and  
nominal variables indicates that long memory properties found in exchange rates  
might be linked to the dynamics of the fundamentals that are e.g. cointegrated with  
the exchange rate and not only isolated to the exchange rate per se. The relative PPP  

27 The results are not reported here but can be obtained from the author.
hypothesis suggests\textsuperscript{28} that exchange rate fluctuations should be related to changes in relative prices. As for the corresponding exchange rates there was no evidence of a long memory structure in the relative inflation series, not even for SEKFIM. The general lack of a long memory process in exchange rates as well as in inflation differentials and the general lack of a cointegrating relationship between exchange rates and inflation differentials supports the hypothesis that the real exchange rate follows a random process. However, more work is required to establish the relationship of the dynamics of exchange rates and its fundamentals and the possible sources and causalities of long memory structures in exchange rates and its fundamentals.

\textsuperscript{28} As do the UIP with the assumption that the expected outcome is equal to the actual outcome in the UIP and the “Fisher open relationship”.
APPENDIX A.

Table 1: ARFIMA-ARCH/GARCH estimates for the exchange rate return series with t-values for $H_0$: parameter=0 inside brackets. The * and ** indicate significance at the 1 percent and 5 percent level respectively. The $d$-parameter indicates the estimate of the long memory parameter while J-B indicate the Jarque-Bera test for normality and Q(n) and Qsq(n) give, respectively, the Ljung-Box statistic for standardised residuals and squared standardised residuals up to the $n$:th order of serial correlation. The distribution of the J-B, Q(n) and Qsq(n) statistics is chi-square with $n$ degrees of freedom, $\chi^2_n$, under the null of normality or no serial correlation respectively. SSR, AIC and SBC indicate the sum of squared residuals, Akaike and Schwartz information criterion respectively.

<table>
<thead>
<tr>
<th>Exchange rate</th>
<th>SEKUSD</th>
<th>SEKDEM</th>
<th>SEKGBP</th>
<th>SEKFRF</th>
<th>SEKJPY</th>
<th>SEKNOK</th>
<th>SEKDKK</th>
<th>SEKFIM</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1)</td>
<td>0.45 (3.07)*</td>
<td>0.32 (2.05)**</td>
<td>0.42 (3.45)*</td>
<td>0.32 (2.65)*</td>
<td>0.32 (2.73)*</td>
<td>0.41 (3.73)*</td>
<td>0.20 (2.02)**</td>
<td>0.38 (3.09)*</td>
</tr>
<tr>
<td>c</td>
<td>0.0004 (3.50)*</td>
<td>0.0004 (3.47)*</td>
<td>0.0001 (3.02)*</td>
<td>0.0002 (4.25)*</td>
<td>0.0009 (4.12)*</td>
<td>0.0003 (4.63)*</td>
<td>0.0002 (3.63)*</td>
<td>0.0002 (2.63)*</td>
</tr>
<tr>
<td>p</td>
<td>0.40 (2.43)*</td>
<td>0.23 (2.25)**</td>
<td>0.38 (2.21)**</td>
<td>0.67 (2.15)**</td>
<td>0.28 (2.11)**</td>
<td>0.17 (2.06)**</td>
<td>0.71 (2.63)*</td>
<td>0.54 (2.93)*</td>
</tr>
<tr>
<td>q</td>
<td>- -</td>
<td>0.50 (3.82)*</td>
<td>- -</td>
<td>- -</td>
<td>- -</td>
<td>0.56 (2.53)*</td>
<td>0.36 (1.29)</td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>0.37 (0.95)</td>
<td>0.24 (0.83)</td>
<td>0.30 (1.07)</td>
<td>0.26 (0.90)</td>
<td>0.38 (0.66)</td>
<td>0.21 (0.78)</td>
<td>0.25 (0.64)</td>
<td>0.36 (1.29)</td>
</tr>
</tbody>
</table>

Residual test.

| Skewness | 0.55 | 0.63 | 0.51 | 0.45 | -0.45 | 0.21 | 0.13 | -0.25 |
| Kurtosis  | 1.91 | 3.93 | 4.36 | 3.63 | 0.81 | 3.63 | 3.37 | 3.83 |
| J-B       | 7.59** | 7.77* | 9.15** | 3.82 | 17.75* | 1.81 | 0.65 | 2.19 |
| Q(5)      | 4.15 | 6.93 | 6.45 | 8.05 | 2.13 | 5.63 | 7.53 | 8.63 |
| Q(10)     | 15.65 | 11.03 | 10.47 | 10.93 | 5.91 | 7.91 | 11.25 | 15.51 |
| Q(20)     | 32.19 | 25.32 | 15.35 | 29.47 | 16.01 | 28.37 | 22.07 | 27.11 |
| Q(40)     | 45.83 | 37.51 | 28.45 | 39.87 | 36.31 | 42.07 | 36.14 | 41.62 |
| Qsq(5)    | 2.83 | 1.15 | 1.95 | 1.29 | 4.10 | 1.53 | 1.35 | 5.63 |
| Qsq(10)   | 8.55 | 4.25 | 4.12 | 4.05 | 7.53 | 3.71 | 3.27 | 12.13 |
| Qsq(20)   | 12.03 | 7.51 | 7.25 | 7.73 | 10.51 | 10.06 | 9.51 | 15.93 |
| Qsq(40)   | 16.35 | 15.93 | 17.95 | 23.31 | 27.37 | 25.24 | 30.35 | 19.63 |
| SSR       | 0.038 | 0.027 | 0.041 | 0.023 | 0.091 | 0.027 | 0.023 | 0.031 |
| AIC       | -231.35 | -255.63 | -227.37 | -266.35 | -176.84 | -281.31 | -261.78 | -247.63 |
| SBC       | -225.38 | -249.72 | -218.92 | -259.15 | -169.15 | -275.51 | -255.01 | -239.43 |
Table II: GPH estimates, which are indicated by $d$, for the exchange rate return series at the frequency value of 0.50 and 0.55. OLS $t$ and asymptotic $t$ indicates the $t$-values for $H_0:d=0$ at the OLS standard error and the asymptotic standard error respectively. Using the OLS standard error, $H_0:d=0.50$ indicates the value of the $t$-test for testing if the GPH-estimate of the series does belong to the domain of stationary memory processes, i.e. a significant $-0.50<d<0.50$. The * indicate significance at the 5 percent level.

<table>
<thead>
<tr>
<th>Exchange rate:</th>
<th>Frequency value</th>
<th>d</th>
<th>OLS $t$</th>
<th>Asymptotic $t$</th>
<th>H0:d=0.50</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEKUSD</td>
<td>0.50</td>
<td>0.06</td>
<td>0.16 (0.38)</td>
<td>0.17 (0.35)</td>
<td>-1.16</td>
</tr>
<tr>
<td></td>
<td>0.55</td>
<td>-0.06</td>
<td>-0.21 (0.29)</td>
<td>-0.21 (0.28)</td>
<td>-1.93*</td>
</tr>
<tr>
<td>SEKDEM</td>
<td>0.50</td>
<td>0.01</td>
<td>0.03 (0.33)</td>
<td>0.02 (0.52)</td>
<td>-1.48</td>
</tr>
<tr>
<td></td>
<td>0.55</td>
<td>0.06</td>
<td>0.21 (0.29)</td>
<td>0.21 (0.28)</td>
<td>-1.52</td>
</tr>
<tr>
<td>SEKGBP</td>
<td>0.50</td>
<td>0.20</td>
<td>0.71 (0.28)</td>
<td>0.57 (0.35)</td>
<td>-1.07</td>
</tr>
<tr>
<td></td>
<td>0.55</td>
<td>0.42</td>
<td>1.91 (0.22)*</td>
<td>1.45 (0.29)</td>
<td>-0.36</td>
</tr>
<tr>
<td>SEKFRF</td>
<td>0.50</td>
<td>0.08</td>
<td>0.27 (0.30)</td>
<td>0.23 (0.35)</td>
<td>-1.40</td>
</tr>
<tr>
<td></td>
<td>0.55</td>
<td>0.19</td>
<td>0.61 (0.31)</td>
<td>0.66 (0.29)</td>
<td>-1.00</td>
</tr>
<tr>
<td>SEKJPY</td>
<td>0.50</td>
<td>0.41</td>
<td>0.71 (0.58)</td>
<td>1.17 (0.35)</td>
<td>-0.16</td>
</tr>
<tr>
<td></td>
<td>0.55</td>
<td>0.25</td>
<td>0.58 (0.43)</td>
<td>0.86 (0.29)</td>
<td>-0.58</td>
</tr>
<tr>
<td>SEKNOK</td>
<td>0.50</td>
<td>-0.15</td>
<td>-0.55 (0.27)</td>
<td>-0.43 (0.35)</td>
<td>-2.41*</td>
</tr>
<tr>
<td></td>
<td>0.55</td>
<td>-0.13</td>
<td>-0.65 (0.20)</td>
<td>-0.45 (0.29)</td>
<td>-3.15*</td>
</tr>
<tr>
<td>SEKDKK</td>
<td>0.50</td>
<td>0.05</td>
<td>0.01 (0.36)</td>
<td>0.14 (0.35)</td>
<td>-1.25</td>
</tr>
<tr>
<td></td>
<td>0.55</td>
<td>0.23</td>
<td>0.59 (0.39)</td>
<td>0.79 (0.29)</td>
<td>-0.69</td>
</tr>
<tr>
<td>SEKFIM</td>
<td>0.50</td>
<td>0.59</td>
<td>1.97 (0.30)*</td>
<td>1.69 (0.35)*</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>0.55</td>
<td>0.38</td>
<td>1.36 (0.28)</td>
<td>1.31 (0.29)</td>
<td>-0.43</td>
</tr>
</tbody>
</table>

Table III: The long memory estimates, which are indicated by $d$, of the unscrambled, original, series and the scrambled, randomly rearranged, series with the corresponding OLS $t$-test in parenthesis estimated at the frequency value of 0.50. The * indicates significance at the 5 percent level. The scrambled $d$-estimates are not significantly different from zero. The $t$-test is the significance test for the hypothesis that the long memory estimate of the unscrambled, original, series is not significantly different from the long memory estimate of the scrambled, randomly rearranged, series, i.e. $t = \frac{\hat{d}_{unscrambled} - \hat{d}_{scrambled}}{se_{unscrambled}}$. A $t$-test value above the critical value implies rejection of the hypothesis that the unscrambled and scrambled long memory estimates are statistically equal. The critical values, at the 5 percent level, for the $t$-test is 1.96. The test statistics are in general clearly below the critical values, i.e. there is no significant evidence that the long memory estimates of the unscrambled, original, series and the scrambled, randomly rearranged, series are statistically different.

<table>
<thead>
<tr>
<th>Exchange rate:</th>
<th>Unscrambled d-parameter</th>
<th>Scrambled d-parameter</th>
<th>t-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEKUSD</td>
<td>0.06 (0.16)</td>
<td>0.10 (0.21)</td>
<td>-0.11</td>
</tr>
<tr>
<td>SEKDEM</td>
<td>0.01 (0.03)</td>
<td>0.03 (0.11)</td>
<td>-0.06</td>
</tr>
<tr>
<td>SEKGBP</td>
<td>0.20 (0.71)</td>
<td>0.18 (0.57)</td>
<td>0.07</td>
</tr>
<tr>
<td>SEKFRF</td>
<td>0.08 (0.27)</td>
<td>0.05 (0.35)</td>
<td>0.10</td>
</tr>
<tr>
<td>SEKJPY</td>
<td>0.41 (0.71)</td>
<td>0.37 (0.97)</td>
<td>0.07</td>
</tr>
<tr>
<td>SEKNOK</td>
<td>-0.15 (-0.55)</td>
<td>-0.10 (-0.68)</td>
<td>-0.18</td>
</tr>
<tr>
<td>SEKDKK</td>
<td>0.05 (0.14)</td>
<td>0.07 (0.11)</td>
<td>-0.06</td>
</tr>
<tr>
<td>SEKFIM</td>
<td>0.59* (1.97)</td>
<td>0.51 (1.55)</td>
<td>0.27</td>
</tr>
</tbody>
</table>
Table IV: GPH estimates, which are indicated by $d$, for the relative inflation series at the frequency value of 0.50 and 0.55. OLS $t$ and asymptotic $t$ indicates the absolute value of the $t$-values for $H_0:d=0$ at the OLS standard error and the asymptotic standard error respectively. Using the OLS standard error, $H_0:d=0.50 \ (H_0:d=-0.50)$ indicates the value of the $t$-test for testing if the GPH-estimate of the series does belong to the domain of stationary memory processes, i.e. with a significant $-0.50<d<0.50$. The * indicate significance at the 5 percent level.

<table>
<thead>
<tr>
<th>Relative inflation series</th>
<th>Frequency value</th>
<th>$d$</th>
<th>OLS $t$</th>
<th>Asymptotic $t$</th>
<th>$H_0:d=0.50$</th>
<th>$H_0:d=-0.50$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEKUSD</td>
<td>0.50</td>
<td>- 0.07</td>
<td>0.29 (0.24)</td>
<td>0.20 (0.35)</td>
<td>-2.37*</td>
<td>1.79</td>
</tr>
<tr>
<td></td>
<td>0.55</td>
<td>- 0.12</td>
<td>0.57 (0.21)</td>
<td>0.38 (0.32)</td>
<td>-2.95*</td>
<td>1.81</td>
</tr>
<tr>
<td>SEKDEM</td>
<td>0.50</td>
<td>- 0.21</td>
<td>0.45 (0.47)</td>
<td>0.60 (0.35)</td>
<td>-1.51</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td>0.55</td>
<td>- 0.34</td>
<td>0.81 (0.42)</td>
<td>1.06 (0.32)</td>
<td>-2.00*</td>
<td>0.38</td>
</tr>
<tr>
<td>SEKGBP</td>
<td>0.50</td>
<td>- 0.28</td>
<td>0.93 (0.30)</td>
<td>0.80 (0.35)</td>
<td>-2.60*</td>
<td>0.73</td>
</tr>
<tr>
<td></td>
<td>0.55</td>
<td>- 0.26</td>
<td>1.00 (0.26)</td>
<td>0.81 (0.32)</td>
<td>-2.92*</td>
<td>0.92</td>
</tr>
<tr>
<td>SEKFRF</td>
<td>0.50</td>
<td>- 0.18</td>
<td>0.72 (0.25)</td>
<td>0.51 (0.35)</td>
<td>-2.72*</td>
<td>1.28</td>
</tr>
<tr>
<td></td>
<td>0.55</td>
<td>- 0.23</td>
<td>1.05 (0.22)</td>
<td>0.72 (0.32)</td>
<td>-3.32*</td>
<td>1.23</td>
</tr>
<tr>
<td>SEKJPY</td>
<td>0.50</td>
<td>- 0.23</td>
<td>0.62 (0.37)</td>
<td>0.66 (0.35)</td>
<td>-1.97*</td>
<td>0.73</td>
</tr>
<tr>
<td></td>
<td>0.55</td>
<td>- 0.12</td>
<td>0.36 (0.33)</td>
<td>0.38 (0.32)</td>
<td>-1.88*</td>
<td>1.15</td>
</tr>
<tr>
<td>SEKDKK</td>
<td>0.50</td>
<td>- 0.16</td>
<td>0.73 (0.22)</td>
<td>0.46 (0.35)</td>
<td>-3.00*</td>
<td>1.55</td>
</tr>
<tr>
<td></td>
<td>0.55</td>
<td>- 0.21</td>
<td>1.11 (0.19)</td>
<td>0.66 (0.32)</td>
<td>-3.74*</td>
<td>1.53</td>
</tr>
<tr>
<td>SEKJJPY</td>
<td>0.50</td>
<td>- 0.07</td>
<td>0.17 (0.42)</td>
<td>0.20 (0.35)</td>
<td>-1.36</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td>0.55</td>
<td>- 0.12</td>
<td>0.33 (0.36)</td>
<td>0.38 (0.32)</td>
<td>-1.72</td>
<td>1.06</td>
</tr>
</tbody>
</table>
APPENDIX B.

Figure B1-B5: Graphs of the spectral density of a theoretical long memory process, the spectral density of each exchange rate return series and its density function compared to the normal distribution. The spectral density for the exchange rates are not decaying with the shape of the theoretical long memory process indicating that the exchange rate return series do not exhibit a long memory process. The graph of the density function for each exchange rate return series compared with the shape of the normal distribution function indicates that the exchange rates are to some degree skewed and leptokurtic.

Figure B1: Graph of the theoretical shape of the spectral density function of the long memory process (lmsdf). The spectral density function of the long memory process is concentrated around the lower frequencies.

Figure B2: The spectral density (to the left) and the density function compared to the normal distribution (to the right) for the SEKUSD (on top) and SEKDEM (on the bottom) exchange rate. The spectral density reveals that the exchange rates might not contain a long memory process (compare with the theoretical spectral density function for the long memory process above). The density function compared to the function for the normal distribution reveals that the exchange rates are to some degree skewed and leptokurtic.
Figure B3: The spectral density (to the left) and the density function compared to the normal distribution (to the right) for the SEKGBP (on top) and SEKFRF (on the bottom) exchange rate. The spectral density reveals that the exchange rates might not contain a long memory process (compare with the theoretical spectral density function for the long memory process above). The density function compared to the function for the normal distribution reveals that the exchange rates are to some degree skewed and leptokurtic.

Figure B4: The spectral density (to the left) and the density function compared to the normal distribution (to the right) for the SEKJPY (on top) and SEKNOK (on the bottom) exchange rate. The spectral density reveals that the exchange rates might not contain a long memory process (compare with the theoretical spectral density function for the long memory process above). The density function compared to the function for the normal distribution reveals that the exchange rates are to some degree skewed and leptokurtic.
**Figure B5:** The spectral density (to the left) and the density function compared to the normal distribution (to the right) for the SEKDKK (on top) and SEKFIM (on the bottom) exchange rate. The spectral density reveals that the exchange rates might not contain a long memory process (compare with the theoretical spectral density function for the long memory process above). The density function compared to the function for the normal distribution reveals that the exchange rates are to some degree skewed and leptokurtic.
References.


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ABSTRACT

We present in this paper empirical evidence on the link between the real exchange rate and the real interest rate differential in a risk-adjusted uncovered interest parity model within a small open economy with a central bank operating an inflation target under a floating exchange rate regime. By the use of the Johansen cointegration technique, the results show that the real exchange rate and real interest rate are non-stationary but cointegrated. The empirical evidence indicates support for a long-run relationship between the real exchange rate and real interest differentials in Sweden. The favourable results supports the portfolio balance/risk premium model of exchange rate determination. In addition, empirical evidence is provided to show that the estimated error-correction framework leads to real exchange rate forecasts that are superior to those generated by a random-walk model. The Diebold and Mariano test also support this superiority in out-of-sample prediction.

Keywords: Cointegration, Error-correction models, Out-of-sample forecasts, Random walks, Real exchange rate, Real interest rate, Sweden, Uncovered Interest Parity.
1. Introduction.

The real interest rate differentials and real exchange rates are often cited as “the most robust relationship in empirical exchange rate models”; see e.g. Meese and Rogoff (1988). This relationship builds on the international parity conditions that links real interest rate differentials and real exchange rates together. Past research on exchange rate determination has only been partly successful in explaining exchange rate movements. Many earlier papers, which model exchange rate movements as a function of real interest differentials and other economic fundamentals, often indicate a statistically significant coefficient for the parameter of the real interest rate differential.\(^1\)

More recent work, which makes use of more sophisticated empirical techniques, has in general been unable to establish this long-run relationship between the real exchange rate and the real interest differential however.\(^2\) In Campbell and Clarida (1987) there was evidence that the real interest differentials only partly could explain the real exchange rate movement of the dollar. The reason given was that the real interest rate differential had not been persistent enough and that its variance innovation had not been large enough to account for much of the fluctuation in the real exchange rate. Meese and Rogoff (1988) test for cointegration between real exchange rates and long-term interest rate differentials and found that they cannot reject the null hypothesis of non-cointegration between the variables. They interpret this result as an indication of the possible omission of one or more variables from the exchange rate – interest rate differential relationship with a large variance. They suggest that this possible omitted variable from the relationship might be the expected value of some future real exchange rate. This suggestion of an important missing variable is also in line with the Campbell and Clarida result.

Hooper and Merton (1982) showed that the equilibrium real exchange rate could be posited to be a linear function of a constant and the cumulated current account. Blundell-Wignall and Browne (1991) used this relationship in a recent paper

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\(^1\) See Frankel (1979), Hooper and Merton (1982) and Shafer and Loopesko (1983) among others.

\(^2\) Two of the most well known recent papers that have been unable to establish this long-run relationship are Campbell and Clarida (1987) and Meese and Rogoff (1988).
providing evidence that there might be a cointegrating relationship between real exchange rates, real interest rate differentials and the cumulated current account relative to GDP. This later variable is seen as a proxy for the expected future real exchange rate.

In an environment with an increasing degree of openness and flexibility of international capital as well as a greater independence of the central bank with an inflation target, it is increasingly relevant to re-examine the link between the real exchange rate and the real interest rate differential. As Meese and Rogoff (1988) cited, in such an environment one might expect that this robust relationship would be even more robust. This paper applies the Johansen cointegration test and the model developed in Edison and Pauls (1993) and Wu (1999) to re-examine the relationship between the real exchange rate and the real interest rate differential with the inclusion of the proposed proxy for the expected real exchange rate to evaluate this relationship under this new economic environment in Sweden with the U.S. as the base country. The paper is organised as follows. In section two the theoretical model will be derived and in section three we will examine the data. The empirical results will be presented in section four and, finally, section five concludes. There will, furthermore in the appendix, be a data section, the data-appendix, and a figure and table section, the figure-appendix.
2. The theoretical model.

To derive the relationship between real exchange rates, real interest differentials and cumulated current accounts, we start with the uncovered interest parity\(^3\) in nominal terms

\[
s_t - E_t(s_{t+k}) = \delta(t_{t,k} - i^*_{t,k})
\]  

(1)

where \(s_t\) is the log of current nominal spot exchange rate (foreign currency per US dollar), \(E_t\) is the time-\(t\) expectation operator, \(\delta\) is the annualising factor for the holding period equal to the number of years from time \(t\) to time \(t+k\), \(i_{t,k}\) is the annualised nominal yield on a \(k\)-period bond issued at time \(t\) and that matures at time \(t+k\) and \(*\) denotes variables for the foreign country.

The expected change in the nominal exchange rate can be approximated as

\[
s_t - E_t(s_{t+k}) = \delta(q_t - E_t(q_{t+k}) + (p^*_{t} - E_t(p^*_{t+k})) - (p_t - E_t(p_{t+k}))
\]  

(2)

i.e. the change in the nominal exchange rate can be approximated by the expected change in the real exchange rate and expected inflation differentials.\(^4\)

The model we will investigate is a real version of nominal exchange rate models that are empirically derived in the literature and subsumed in, for example, Obstfeld and Rogoff (1984). A common assumption for those models is that goods market prices adjust slowly in response to anticipated disturbances and to excess demand. That is, less than perfectly anticipated monetary disturbances could cause temporary deviations in the real exchange rate from its long-run equilibrium value.

\(^3\) Equation (1) is only an approximation under uncertainty because of Jensen’s inequality, which implies \(\log E_t(s_{t+k}) > E_t(\log s_{t+k})\); the log-function is strictly concave. For a further discussion see Obstfeld and Rogoff (1996).

\(^4\) Despite unstable nominal exchange rates, there is a strong relationship between real exchange rates and real interest rates. For a further discussion see Obstfeld and Rogoff (1996) among others. The poor performance of the nominal exchange rate regressions was shown in Feldstein (1986) and Shafer and Loopesko (1983) to be primarily attributable to money demand disturbances.
Define the real exchange rate as

\[ q_t = s_t + p_t - p_t^* \quad (3) \]

where \( q_t \) is the log of the real exchange rate and \( p_t \) is the log of the domestic price level (here US). Embodied in the equations below are three assumptions\(^5\) for which the interpretation of the empirical tests in the paper depends. First, assume that any temporary deviation of the real exchange rate from its flexible-price equilibrium value is expected to revert towards this flexible-price equilibrium at a constant rate (in the absence of further shocks). Then

\[ E_t(q_{t+k} - \bar{q}_{t+k}) = \theta^k(q_t - \bar{q}_t), \quad 0 < \theta < 1, \quad (4) \]

where \( \bar{q}_t \) is the real exchange rate that would prevail at time \( t \) if all prices were fully flexible, and \( \theta \) is the speed-of-adjustment parameter.\(^6\)

The second assumption outlined in Meese and Rogoff (1988) is that

\[ E_t q_{t+k} = E_t \bar{q}_{t+k} = \bar{q}_t \quad (5) \]

which implies that the expected long-run real exchange rate is equal to the equilibrium real exchange rate. The equality in equation (5) will hold only if the shocks to the economy do not have real effects, i.e. if there are no real shocks in the economy, or that, if there are real shocks in the economy, they follow a random-walk process. Moreover, Hooper and Merton (1982) showed that the equilibrium real exchange rate, \( \bar{q}_t \), can be posited to be a linear function of a constant and cumulated current accounts. Substituting (5) into (4) and adding \( q_t \) to both sides implies

\(^5\) For a further discussion on these assumptions and their implications see Meese and Rogoff (1988).

\(^6\) \( \theta \) is a function of the structural parameters of the model, which is not affected by additive disturbances such as e.g. money market shocks. Equation (3) embodies a monotonic adjustment property, which is a feature of a fairly broad class of sticky-price rational expectations monetary models.
\[ q_t = \alpha (q_t - E_t q_{t+k}) + \bar{q}_t \]  

where \( \alpha = \frac{1}{(1-\theta^*)} > 1 \).

The third important assumption used in Meese and Rogoff (1988) is the uncovered interest-parity relationship (here extended with an exchange rate risk premium \( \rho_t \), which is assumed to be covariance stationary)

\[ s_t - E_t (s_{t+k}) = \delta (i_{t,k} - i_{t,k}^* - \rho_t) \]  

where \( \rho_t \) is a normalised risk premium\(^7\).

Substituting equation (3) into equation (7) implies

\[ s_t = E_t (q_{t+k}) + E_t (p_{t+k}^*) - E_t (p_{t+k}) + \delta (i_{t,k} - i_{t,k}^* - \rho_t) \]  

By the use of the approximation

\[ E_t (p_{t+k}) = p_t + E_t (\pi_{t,k}) \quad E_t (p_{t+k}^*) = p_t^* + E_t (\pi_{t,k}^*) \]  

then can the expected real interest rate, \( E_t (r_{t,k}) \), be obtained by the use of the Fisher equation, which is denoted as

\[ E_t (r_{t,k}) = \delta [i_{t,k} - E_t (\pi_{t,k}^*]) \quad E_t (r_{t,k}^*) = \delta [i_{t,k}^* - E_t (\pi_{t,k})] \]  

where \( \pi_{t,k}^* = \frac{\pi_{t,k}}{\delta} \) is the annualised expected inflation rate. By combining equation (3), (5) and (8)-(10), the following equation is derived

\(^7\) This normalised risk premium would be equal to zero if the uncovered interest parity holds.
\[ q_t = \delta [E_t(r_{t,k}) - E_t(r_{t,k}^*)] + \bar{\eta}_t - \eta_t, \]  

(11)

where \( \eta_t = \delta \rho_t \).

As the uncovered interest parity relationship implies

\[ E_t(q_t - q_{t+k}) = r_{t,k} - r_{t,k}^* \]  

(12)

then can \( \delta \) in (11) be replaced by \( \alpha \) in (6), which then implies that

\[ q_t = \alpha [E_t(r_{t,k}) - E_t(r_{t,k}^*)] + \bar{\eta}_t - \eta_t. \]  

(13)

As noted above, Hooper and Merton (1982) showed that the equilibrium real exchange rate could be assumed to follow a linear function of a constant and cumulated current account. Thus, equation (13) can be rewritten as

\[ q_t = \alpha^* r_d + k + \omega^* (ccdbal_t) - \eta_t, \]  

(14)

where \( r_d = E_t(r_{t,k}) - E_t(r_{t,k}^*) \), \( k \) is a constant and \( ccdbal_t = ccbal_t - ccbal_t^* \) is the difference in the share of the cumulated current account relative to GDP. The sign of \( \alpha \) and \( \omega \) are both positive, as an increase in the real rate of return of holding a domestic asset relative to that of holding a foreign asset results in a capital inflow, which, in turn, causes a real appreciation. An increase in the domestic cumulative current account relative to the foreign country implies, furthermore, an appreciation-expectation of the dollar. We can, furthermore, estimate equation (14) for the real interest rates of maturity. This is due to the monotonic-adjustment assumption of equation (4). The maturity of the interest rate, \( k \), affects the equation via the relation in equation (6). For a further discussion see Meese and Rogoff (1988).
3. The data.

Before the empirical investigation of the model, we will describe and visually inspect the data. This is done due to that some of the opposing results in the existing literature seems to be linked to the aspects of the selected data. The purpose is then to evaluate the variables from the point of view of the theoretical relationship and if (1) the selected time period and/or (2) the inflation measure that is used to construct the real interest rates significantly alters this relationship.\(^9\)

The frequency of the data is quarterly and runs from 1980:Q1 to 1999:Q3. The nominal exchange rate is Swedish krona per one U.S. dollar. Nominal interest rates are the ten-year government bond yield for Sweden and the United States.\(^10\) The price index is measured by the CPI in the two countries. The cumulated current account balances are created assuming that the cumulated current accounts of the two countries were in balance as of 1979:Q4. The current accounts were then accumulated as of 1980:Q1.\(^11\)

Following Edison and Pauls (1993), I consider three alternative measures of the expected inflation rate to be able to evaluate the theoretical relationship more rigorously. The three alternatives are a twelve-quarter centered moving average of CPI inflation rates, where forecasts are used when published data are not available, a four-quarter change and a quarterly change in the CPI index, respectively. A more detailed picture of the data and its sources is outlined in the data-appendix.

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\(^9\) It is possible for graphs to portray the data misleadingly; nevertheless we think that this method is useful to highlight the issues pointed out in the text.

\(^10\) To check for the importance for the empirical part of the time to maturity for the chosen interest rate during this period, I performed tests for correlation between different lengths of the bond. The correlation for US between the 10-year bond and 3-year bond is 0.98, and the 10-year bond and the T-bill (3 month) is 0.91. The standard deviation for the 10-year US bond is 2.62, 2.88 for the 3-year bond, and 2.93 for the T-bill. The correlation for Sweden between the 10-year bond and 1-year bond is 0.96, and the 10-year bond and the T-bill (3-month) is 0.89. The standard deviation for the 10-year Swedish bond is 2.68, 2.61 for the 1-year bond, and 1.97 for the T-bill. It seems that interest rates are closely correlated in both Sweden and the US during this period and that the variance in both countries between its different bonds are of a similar magnitude. The data-sources are IFS, EcoWin and the Central Bank of Sweden (Riksbanken).

\(^11\) As was noted by Edison and Pauls (1993), this assumption does not of course accord with the data, i.e. that the cumulated current accounts were in balance. However, this assumption only affects our initial condition and does not alter the dynamic results of the model.
Figure 1 in the figure-appendix presents the real exchange rate and the real interest rate differential calculated by the use of the twelve-quarter centered moving average measure of the expected inflation rate. We can see, from the figure, that the movements in the two series have at least been partly correlated over most of the period. There are, though, at least two periods were this relationship seems to be very vague. The first period is in the beginning of the 1980’s. One reason for the vague relationship during this period might be that capital controls to some extent were in place in Sweden during this period. The second period is in the beginning of the 1990’s. The reason for that the relationship was vague during this period might be due to the turbulent period in the Swedish domestic financial markets that were prior to the free float of the Swedish krona.\footnote{This period was a “Leaning against the wind”–period. Another reason why this relationship seems to be partly correlated during the period might be that the consumer price index is not the most appropriate index to use. The effect by e.g. changes in some type of indirect taxes motivated the Riksbank (the Swedish Central Bank) to evaluate the inflation from both the CPI and an index were such effects are excluded, i.e. the UND1X. Furthermore, might the weight of raw commodities such as oil prices bias the calculation in the Swedish CPI.}

The nominal and real bilateral exchange rate of the Swedish krona vs. one U.S. dollar is presented in figure 2 in the figure-appendix. As is well known in the literature, there is a close relationship between those two variables where most of the movement in the real exchange rate mirrors movements in the nominal exchange rate. Figure 3 in the figure-appendix displays the nominal exchange rate and the nominal long-term interest rate differential. From this figure we can see that there is little apparent relationship between the two nominal series. This lack of correlation might be caused by that the expected future nominal exchange rate does not even approximate a stable anchor but varies with changes in inflation expectations.

This explanation of a possible lack of relationship between the two nominal series raises the question whether the relationship in real terms is dependent on the measure of the inflation rate that is used to construct the real counterparts. Figure 4 in the figure-appendix presents three alternative real interest differentials based on the three different expected inflation measures. From this figure we can see that the generated real interest rate differentials do vary considerably with the different measures of inflation. In fact, the 12-quarter centered moving average inflation measure creates a real interest differential that is smoother than the alternative
measures. This relative smoothness is especially clear for the real long-term interest differential based upon the inflation rate of the 12-quarter compared to the 1-quarter centered moving average. Edison and Pauls (1993) shows similar results using US-data.

The real exchange rate and the difference in the share of the cumulated current account relative to GDP for the U.S. and Sweden are presented in figure 5 in the figure-appendix. There seems to be a relationship during some parts of the sample period but during other parts there seems to be a lack of this possible relationship. In summary from the visual inspection of the data, is seems that there is a relationship between the real exchange rate and the real interest rate differential but only a weak, if any, relationship between the real exchange rate and the difference in the cumulated current account for the U.S. and Sweden. Thus, the evidence of a risk premium in uncovered interest differentials seems to be weak.

4. Empirical investigation.

In Engle and Granger (1987) it is shown that if a set of variables is cointegrated, then there exists an error-correction formulation of the dynamic model. In addition, error-correction models (ECM) also provide information about the short-run dynamics besides the long-run dynamics of the data. This section presents the results leading up to an error-correction model. We start by checking for the order of integration in the data series. The Augmented Dickey-Fuller (ADF) unit root test, which are displayed in table I in the figure-appendix indicates a unit root in the original series but not so for the first differenced series, i.e. the original series are integrated of order one or I(1).

To test for the long-run relationship among the variables we use the Johansen (1988) cointegration procedure. In order to implement the Johansen procedure, one needs to determine the optimal lag length in the VAR-system and to determine whether a deterministic component should be included in the model. We begin by
specifying the data generating process of a vector of N variables, X, as a general 
vector autoregressive, VAR, model in the levels of the variables

\[ X_t = \Pi_1 X_{t-1} + \cdots + \Pi_k X_{t-k} + \mu + \Theta D_t + \epsilon_t \]  

(15)

where X is an N vector of the I(1) variables of interest, \( \mu \) represents deterministic 
variables such as a vector of constants and trends, \( D_t \) is a vector of centred seasonal 
dummies, \( \epsilon_t \) is an independently identically distributed N-dimensional vector with 
zero mean and covariance matrix \( \Omega \) and each of the \( \Pi_i \) is an \( N \times N \) matrix of 
parameters. The long-run, or cointegrating matrix, is, within this framework, given by

\[ \Pi = I - \Pi_1 - \Pi_2 - \cdots - \Pi_k \]

where I is the identity matrix. When equation (15) is reparameterised into a reduced 
form error-correction model we receive

\[ \Delta X_t = \Pi X_{t-k} + \Gamma_1 \Delta X_{t-1} + \cdots + \Gamma_k \Delta X_{t-k+1} + \mu + \Theta D_t + \epsilon_t \]  

(16)

where \( \Gamma_1 = \Pi_1 + \Pi_2 + \cdots + \Pi_k - I = 1 \cdots k - 1 \), \( \Pi = I - \Pi_1 - \Pi_2 - \cdots - \Pi_k \) and I is the 
identity matrix. Equation (16) is a stationary error-correction first-difference VAR 
where the term \( \Pi X_{t-k} \) contains information of the long-run relationship between the 
variables in the VAR. The rank of \( \Pi \), the long-run matrix, which is denoted by \( r \), 
gives the number of distinct cointegrating vectors that exists between the variables of 
X.

The optimal lag length of the VAR-model was estimated as a VAR(2)- 
model\(^{13}\). The test for cointegration by the Johansen procedure is reported in table II.

\(^{13}\) Our procedure for choosing the optimal lag length was to test up from a general VAR(1) system until 
increasing the order of the VAR by one lag could not be rejected using a likelihood-ratio statistic. We 
do not accommodate date prior to quarter one 1980 as lags in the VAR estimation. The residuals from 
the chosen VAR were then checked for whiteness. If the residuals in any equation proved to be non-
white, we sequentially chose a higher lag structure until they were whiten. The F-test for system 
reduction between a VAR(3)- and a VAR(2)-model is \( F(9, 151) = 4.1 \), which is significant at the 1 
percent level. The log-likelihood is 488.73 and the Schwartz and Akaike information criteria are -12.33 
and -13.39 respectively.
The test statistics indicates one cointegrating vector at the 5 percent level. We also imposed and tested general restrictions on the cointegrating vector. The statistics for testing for the presence of the variables in the cointegrating space indicates that the real interest rate differential is significant at the 1 percent level and that the real exchange rate and the cumulated current accounts difference are significantly different from zero at the 5 percent level. The test for stationarity of the variables, i.e. the test for whether the cointegrating vector is made up of only one variable, indicates support for the ADF unit root test reported in table I. In other words, the variables are integrated of order one.

The reduction of the VAR to a single equation ECM requires the assumption of weak exogeneity, which can be examined based on the likelihood-ratio test by Johansen and Juselius (1990). The likelihood-ratio statistic is 13.09, which is significant at the 1 percent level. This indicates that real interest differences and cumulated current account differences is weakly exogenous to the log real exchange rate. Thus, it seems that there exists a single error-correction formulation of the dynamic model\textsuperscript{14}.

We will use the Granger causality test as an informative indicator of structural relationships\textsuperscript{15}. An informative indication implies that added information will improve the predictability of a certain outcome given the relevant information. In practice, the initial information set is often made up of lagged observations of the dependent variable as indicated by

\[
Y_t = \sum_{i=1}^{n} \alpha_{1i} Y_{t-i} + \sum_{i=1}^{m} \alpha_{2i} X_{t-i} + \varepsilon_t \tag{17}
\]

\textsuperscript{14} We also tested the speed of adjustment parameters in the cointegrating vector, which proved to be significantly different from zero at the 1 percent significance level.

\textsuperscript{15} The reason is that a VAR-model, which is used in the Granger causality test, can be viewed as an unrestricted reduced form of a particular structural model. For a further discussion see Spanos (1990).
where the lagged observations of \( X_t \) represent the added information and \( \varepsilon_t \) is the error term. Table III reports the results from the Granger causality test. It shows that the log real exchange rate and the difference in the cumulated current accounts Granger-cause each other. Furthermore, the difference in the real interest rate Granger-causes the log real exchange rate. For the other variables there is no significant evidence of Granger causality. There seems to be a relationship between the difference in the real interest rate and the difference in the cumulated current account as they Granger-cause the real exchange rate.

4.1 Out-of-sample forecasting.

If a cointegrating relationship exists among a set of I(1) variables then there exists a dynamic error-correction representation of the data according to Granger’s representation theorem. This suggests that there should exist a real exchange rate function of the form

\[
\Delta q_t = k + \sum_{i=0}^{n} \beta_i \Delta q_{t-i} + \sum_{i=0}^{n} \alpha_i \Delta r_{t-i} + \sum_{i=0}^{n} \omega_i \Delta \text{ccdbal}_{t-i} + \lambda_k \phi_{t-i} - \eta_t
\]  

16 The significance of the \( \alpha \)’s are tested by an F-test. The reverse direction of the Granger causality test is investigated by estimating (17) with \( X_t \) as the dependent variable. One weakness of the Granger causality test is that the F-test that is used might be sensitive to the, in the test, included information. Since the F-test can be written as a function of \( R^2 \), then will changes in \( R^2 \) due to the inclusion of additional variables lead to different test values. This might be the outcome, as we normally do not know what all the significant relevant information is. An other weakness is that one variable is not necessarily causing the other simply because it precedes it in time. This phenomenon could occur, for example, if one variable reacts faster than the other and if this variable drives the two that are tested. The Granger causality test is, furthermore, often sensitive to the specification of the model that is estimated, e.g. non-stationarity, deterministic time trends, and number of lags used. It has, for example, been shown that the variable, which is tested for Granger causality, must be stationary or that it can be written as a stationary variable. If this is not the case then will the F-statistic not approximate the F-distribution, i.e. when adding a non-stationary variable to a regression it must be cointegrated with a variable that is already included in (17) for it to be stationary. It should also be noted that two variables that are cointegrated imply Granger causality in at least one direction. For a further discussion on Granger causality and its weaknesses and references see Durevall (1993).
where $\eta_i$ is a stationary random variable and $\phi_{t-1}$ denotes the equilibrium error where the cointegrating error is normalised on $q_i$. The error-correction parameter $\lambda_k$ is expected to be negative. It reflects the response of the dependent variable in each period to a departure from its long-run equilibrium.\footnote{The variables are denoted in the first difference of the level and first difference of log level for the real exchange rate in equation (18).}

The reduction of the general VAR-model into a single equation error-correction model requires that the real interest rate differential and the difference in the share of the cumulated current account relative to GDP is weakly exogenous to the real exchange rate. As reported in part four there was evidence of such weak exogeneity and therefore an indication that there exists an error-correction formulation of the dynamic model. This error-correction model will not only provide information about the long-run dynamics but also on the short-run dynamics of the data.

We impose a priori the cointegrating vector on the error-correction model using the coefficient values estimated from the Johansen method. In practice, all coefficients in equation (18) may not be statistically significant and greater efficiency may be gained by eliminating insignificant coefficients\footnote{We tested whether any of the excluded variables were different from zero by an F-test. There were no significant results. We, furthermore, tested for the significance of each variable, of each lag and of all lags up to lag ten in equation (19) by an F-test in the final parsimonious error-correction model. The test statistics are significant for the log real exchange rate, the difference in real interest rates and the difference in the cumulated current accounts and its lags. This supports the results in table IV.}. Our final preferred parsimonious equation, based on the Hendry-type general-to-specific rule, is

$$
\Delta q_t = \sum_{i=1}^{6} \beta_i \Delta q_{t-i-1} + \beta_7 \Delta q_{t-10} + \alpha_1 \Delta \text{rd}_{t-3} + \alpha_2 \Delta \text{rd}_{t-7} + \\
+ \alpha_3 \Delta \text{rd}_{t-9} + \omega_1 \Delta \text{cdbal}_{t-3} + \lambda_1 \phi_{t-1} - \eta_t.
$$

\text{(19)}

The empirical results are shown in table IV, where we can see that the coefficient of the error-correction term, $\lambda_1$, is negative and statistically significant. The negative sign of $\lambda_1$ implies that the log real exchange rate converges towards its long-run
equilibrium\textsuperscript{19}. The coefficient of the error-correction term suggests, furthermore, that approximately 0.9 percent of the difference between the actual and equilibrium level is corrected each quarter. The dynamics of equation (19) also indicates that the past log real exchange rate, the difference in real interest rates, and the difference in the cumulated current account have significant short-term effects on the log real exchange rate in addition to long-run effects\textsuperscript{20}. We also included and tested for a trend in the data and for seasonal dummy variables. However, there is no indication of either a trend or seasonal dummy variables in the log real exchange rate as the tests were insignificant. This result was confirmed by the test for exclusion of the trend and the seasonal dummy variables from the error-correction model. The test proved to be insignificant. Furthermore after visual inspection, we also included and tested for an outlier in the third quarter of 1992. This outlier proved to be significant by the test of exclusion from the error-correction model.

We also report some diagnostic in-sample tests for autocorrelation, heteroscedasticity, normality, functional form and parameter stability for equation (19). The Jarque-Bera test for normality and the tests for serial correlation up to the fourth order, the autoregressive heteroscedasticity (ARCH) up to the fourth order, the test for heteroscedasticity and the Ramsey’s regression-specification error test (RESET) are all insignificant. In other words, the error-correction model indicates normally distributed and serially uncorrelated residuals as well as a homoscedastic variance without an ARCH-property. The insignificant RESET-test indicates that the error-correction model do not apply a functional form mis-specification.

To evaluate the constancy of the parameter estimates we estimated the error-correction model over the sample period by recursive least squares. The coefficient estimates of the log real exchange rate, the difference in real interest rate, the difference in cumulated current account and the error-correction variable are all significantly different from zero and the estimated sign for each significant parameter

\textsuperscript{19} The rejection of the hypothesis of a non-significant error-correction parameter indicates further support to the conclusion that the variables are cointegrated as indicated by table II.

\textsuperscript{20} This is indicated by the significant short-term parameters, $\beta_i$, $\alpha_i$ and $\omega_i$, and the significant long-run parameter, $\lambda_i$, i.e. the error-correction term.
in the model is significant\textsuperscript{21}. The coefficients are, furthermore, inside their 95 percent confidence intervals, which, in turn, are relative narrow indicating a relative narrow random volatility in each variable. This result indicates that the individual coefficients are fairly stable over the sample period. Figure 6 in the figure-appendix displays the 1-step residual of the recursive estimates and its corresponding calculated equation standard errors giving the approximate 95 percent confidence interval. Figure 6 also presents the 1-step Chow test, scaled by its critical value at the 1 percent level, and the N decreasing break-point Chow test. Estimates outside the confidence intervals indicate either a structural break, i.e. a coefficient change in the standard errors, or an outlier. Each point in the break-point Chow test is the value of the Chow F-test for that date against the final period, here the third quarter of 1999, scaled by its 1 percent critical value. The forecast horizon N is decreasing from left to right and, after imposing a dummy variable for the outlier in the third quarter of 1992, it shows that variable constancy cannot be rejected at the 1 percent level for the error-correction model. The 1-step Chow test indicates a possibility for a structural break in the third quarter of 1992 even after imposing a dummy variable. This possibility is though a boarder-line case as the 1-step Chow test is at the 1 percent critical value. To test for this possible structural break in the third quarter of 1992 we used the Chow test, which proved to be insignificant\textsuperscript{22}. Thus, there is no indication of a structural break in 1992:Q3 and the formal tests indicate overall stability of the model.

As a final test of the model stability we performed an out-of-sample forecast of the error-correction model. Figure 7 plots the error-correction model, the fitted value and the forecast with the 95 percent confidence intervals. The forecast period is from the third quarter of 1996 up to the third quarter of 1999, i.e. for a three year period. The test for parameter constancy over the forecast period is yet again insignificant\textsuperscript{23} with the forecast estimates inside the 95 percent confidence intervals.

\textsuperscript{21} The results are not reported here but can be obtained from the author.

\textsuperscript{22} The test statistic for the Chow-test is F(10,49)=0.50 which is insignificant and which indicates parameter stability.

\textsuperscript{23} The test statistic for the Chow-test is F(12,43)=0.44 which is insignificant and which indicates parameter stability over the forecast period.
The formal test for parameter constancy and the forecast estimates and test indicates model stability, relatively low random model volatility and forecast accuracy.

Having provided evidence supporting the adequacy of the estimated error-correction model and its out-of-sample forecast ability, the next issue is whether or not the model provides better out-of-sample forecasts. We reserve the last 18 observations for out-of-sample comparisons. We then sequentially re-estimated the model for every data point from 1995:Q1 onward computing the dynamic forecasts for forecasting horizons of 1, 2 and 4 quarters and the corresponding root-mean-square error (RMSE) and mean absolute error (MAE) of the forecast at each horizon considered.\(^{24}\) As a point of comparison, we also computed the RMSEs and MAEs at the corresponding horizon implied by a random-walk model both with (RWD) and without (RW) a drift included, respectively.

The outcome of the relative forecasting accuracy of the ECM is reported in table V in the figure-appendix. The number in the table is the ratio of the RMSE or MAE of the ECM and the corresponding RMSE or MAE of the RW or RWD, respectively. The lower the ratio, the more the ECM outperforms the comparing RW or RWD. Our dynamic ECM outperforms the forecast ability of the driftless random walk (RW) and the random walk with a drift (RWD) at every forecast horizon. That is, the derived error-correction model produces smaller forecast errors than a random-walk model over multiple forecast horizons both with and without a drift included.

\(^{24}\) The RMSE is defined as follows

\[
RMSE = \left( \frac{\sum_{s=0}^{N_k-1} [F(t+s+k) - A(t+s+k)]^2}{N_k} \right)^{1/2},
\]

and the MAE as

\[
MAE = \frac{\sum_{s=0}^{N_k-1} |F(t+s+k) - A(t+s+k)|}{N_k},
\]

where \(k=1,2\) and 4 denotes the forecast steps and \(\|\) is the absolute value. The \(N_k\) is the total number of forecasts in the projection period for which the actual value \(A(t)\) is known and \(F(t)\) is the forecast value.
To evaluate if the forecast ability of the ECM significantly outperforms the RW and/or RWD at the different forecast horizons we will use the test statistic derived in Diebold and Mariano (1994). Only comparing the values of the mean-square errors does not give any guidance with respect to the significance of the difference in the forecast errors. The test statistic, $S_1$, proposed in Diebold and Mariano (1994) is an explicit test of the null hypothesis of no difference in the accuracy of two competing forecasts. The test statistic is asymptotically standard normally distributed.

Define the loss function as $d_t = (u_{e,t}^2 - u_{w,t}^2)$, where $u_{e,t}$ and $u_{w,t}$ are date-$t$ forecast errors from the ECM and a random-walk model, respectively. The equal accuracy null is equivalent to the null hypothesis that the population mean of the loss differential series is zero. A consistent estimate of the spectral density of the loss differential at frequency zero is obtained using the method of Newey and West (1987). The truncation lag, $S(T)$, for the Bartlett window in Newey and West’s estimator, given by the rule of Andrews (1991), is $S(T) = 1.1447 \sqrt{\hat{\rho}(1) T}$, where

$$
\hat{\rho}(1) = 4 \left( \frac{\hat{\rho}}{(1 - \hat{\rho})(1 + \hat{\rho})} \right)^2
$$

and $\hat{\rho}$ is the estimated first-order autoregressive parameter of the loss differential.

From table V in the figure-appendix we can see that the derived error-correction model is superior to a random-walk model since the test statistic for the loss differential is statistically significant at the 5 percent level. Thus, the null of equal accuracy is rejected at the 5 percent level for different forecasting horizons for both the random-walk model with and without a drift. This finding points out the importance of long-run equilibrium errors, i.e. the error-correction term, in improving the accuracy of the forecast ability and it, furthermore, indicates that the ECM significantly improves the accuracy of the prediction of the real exchange rate.
5. Conclusions.

In this paper the link between the real exchange rate and expected real interest differentials over the period 1980:Q1 to 1999:Q3 for Sweden and the United States were re-examined. The model is a risk-premium adjusted uncovered interest parity model as assumed in the portfolio balance model. The long-run relationship between the real exchange rate and the real interest rate differentials exists in our sample when the difference in the cumulated current accounts relative to GDP is included as a proxy for the expected real exchange rate. The favourable results supports the portfolio balance/risk premium model of exchange rate determination. The estimated error-correction model indicates parameter constancy and no evidence of a structural break during the sample period. As a final test of the stability of the model an out-of-sample forecast of the error-correction model were performed. The test for parameter constancy over the forecast period is yet again insignificant. Thus, the empirical error-correction model indicates model stability, relatively low random volatility and forecast accuracy.

Using the criteria of root-mean-square error or mean absolute error, our ECM dominates a random-walk model, both with and without a drift, in out-of-sample forecasts. The Diebold and Mariano test also support this superiority in out-of-sample prediction.
DATA-APPENDIX.

The data-appendix gives a brief description of the derivation of the data and the sources for the data that will be used in the paper.

Exchange rate: Bilateral nominal exchange rate for SEKUSD.  
Unit: Number of Swedish krona per one U.S. dollar.  
Frequency: Quarterly.  
Source: IFS data tape.

Price index: Consumer price index (CPI).  
Countries: Sweden and USA.  
Unit: Index series.  
Frequency: Quarterly.  
Source: IFS data tape.

Interest rate: Nominal interest rates for 10-year government bond yields.  
Countries: Sweden and USA.  
Unit: Percent.  
Frequency: Quarterly.  
Source: IFS data tape and EcoWin.

Current account: Countries: Sweden and USA.  
Unit: Billions (denominated in the domestic currency).  
Frequency: Quarterly.  
Source: IFS data tape.  
To obtain the cumulated current account we assume, for each country, that the cumulated current account was zero in 1979:Q4 and accumulated the current account thereafter.

Gross domestic product: Countries: Sweden and USA.  
Unit: Billions (denominated in the domestic currency).  
Frequency: Quarterly.  
Source: IFS data tape.
FIGURE-APPENDIX

The figure-appendix displays the figures and tables referred to in the text.

Figure 1: The real exchange rate and the real interest differential for the U.S. and Sweden. To construct the real variables we used the twelve-quarter centered moving average inflation measure. The series are re-scaled in the figure for comparison purposes.

Figure 2: The real and nominal exchange rate for the Swedish krona per one U.S. dollar. The series are re-scaled in the figure for comparison purposes.
Figure 3: The nominal exchange rate and the nominal interest rate differential for the U.S. and Sweden. The series are re-scaled in the figure for comparison purposes.

Figure 4: The real long-term interest differential between the U.S. and Sweden for alternative measures of the inflation rate, i.e. for the 1 quarter (1q), the 4 quarter (4q), and the 12 quarter centered moving average (12q) change. The series are re-scaled in the figure for comparison purposes.
Figure 5: The real exchange rate and the difference in the share of the cumulated current account relative to GDP for the U.S. and Sweden. The series are re-scaled in the figure for comparison purposes.

Figure 6: The 1-step residual (at the top), the 1-step Chow test (in the middle), and the break-point Chow test (at the bottom). The figure indicates model stability over the sample period.
Figure 7: The actual-, fitted-, and forecast values of the error-correction model. The model indicates forecast accuracy as the forecast is within the standard error bands.
**Table I:** Test for order of integration. The variables are log of the real exchange rate (lrealvx), real interest rate differentials (realrd), and the difference in cumulated current accounts relative to GDP (ccdbal). The first difference of the variables is indicated by a d in front of the variable. The t-adf indicates the Augmented Dickey-Fuller t-statistics, where two lags and no deterministic variable and a constant were used, respectively. The ** indicates significance at the 1 percent level. The critical values for the t-adf test at the 1 percent level is –2,595 (No deterministic variable) and –3,522 (Constant) and at the 5 percent level –1,945 (No deterministic variable) and –2,902 (Constant). The significant unit root statistics for the first difference of the series indicates that the series are integrated of order one, i.e. I(1).

<table>
<thead>
<tr>
<th>Variable</th>
<th>t-adf</th>
<th>Lag</th>
</tr>
</thead>
<tbody>
<tr>
<td>lrealvx No constant</td>
<td>0.29</td>
<td>2</td>
</tr>
<tr>
<td>Constant</td>
<td>-1.61</td>
<td>2</td>
</tr>
<tr>
<td>realrd No constant</td>
<td>-1.43</td>
<td>2</td>
</tr>
<tr>
<td>Constant</td>
<td>-2.29</td>
<td>2</td>
</tr>
<tr>
<td>ccdbal No constant</td>
<td>-1.04</td>
<td>2</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.89</td>
<td>2</td>
</tr>
<tr>
<td>drealvx No constant</td>
<td>-3.82**</td>
<td>2</td>
</tr>
<tr>
<td>Constant</td>
<td>-3.77**</td>
<td>2</td>
</tr>
<tr>
<td>drealrd No constant</td>
<td>-4.46**</td>
<td>2</td>
</tr>
<tr>
<td>Constant</td>
<td>-4.43**</td>
<td>2</td>
</tr>
<tr>
<td>dccdbal No constant</td>
<td>-3.10**</td>
<td>2</td>
</tr>
<tr>
<td>Constant</td>
<td>-3.18**</td>
<td>2</td>
</tr>
</tbody>
</table>

**Table II:** The Johansen test for cointegration. The test statistic indicates one cointegrating vector at the 5 percent level (*) by both $\lambda_{\max}$ and $\lambda_{\max}$, where the numbers in parentheses are the 95 percent probability values.

<table>
<thead>
<tr>
<th>Rank</th>
<th>$\lambda_{\max}$</th>
<th>$\lambda_{\max}$</th>
<th>Eigenvalue</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>18.73* (17.9)</td>
<td>40.86* (36.4)</td>
<td>0.226</td>
</tr>
<tr>
<td>1</td>
<td>1.55 (11.4)</td>
<td>1.93 (12.5)</td>
<td>0.021</td>
</tr>
<tr>
<td>2</td>
<td>0.38 (3.8)</td>
<td>0.38 (3.8)</td>
<td>0.005</td>
</tr>
</tbody>
</table>

**Table III:** Granger causality tests on the log real exchange rate (lrealvx), the difference in real interest rates (realrd), and difference in the cumulated current accounts (ccdbal). The $\rightarrow$ indicates “Granger cause”, $\nrightarrow$ indicates “no Granger cause” and * and ** indicates significance at the 5 percent and 1 percent level respectively.

<table>
<thead>
<tr>
<th>Causality</th>
<th>Wald test</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>realrd $\rightarrow lrealvx$</td>
<td>5.02**</td>
<td>0.00</td>
</tr>
<tr>
<td>ccdbal $\rightarrow lrealvx$</td>
<td>3.45*</td>
<td>0.04</td>
</tr>
<tr>
<td>lrealvx $\nrightarrow$ realrd</td>
<td>1.98</td>
<td>0.13</td>
</tr>
<tr>
<td>ccdbal $\nrightarrow$ realrd</td>
<td>2.11</td>
<td>0.11</td>
</tr>
<tr>
<td>lrealvx $\nrightarrow$ ccdbal</td>
<td>5.03**</td>
<td>0.00</td>
</tr>
<tr>
<td>realrd $\nrightarrow$ ccdbal</td>
<td>1.51</td>
<td>0.22</td>
</tr>
</tbody>
</table>
Table IV: The parsimonious error-correction model with specification tests. The parameter estimates are displayed with the standard error within parentheses. The d represents the first difference, q is the log of the real exchange rate, rd is the difference in the real interest rates, cedbal is the difference in the current accounts, ECM is the error-correction parameter and D92 indicates a dummy variable for an outlier in 1992:Q3. The number within parentheses indicates the lag. The ** and * indicates significance at the 1 percent and 5 percent level respectively. The Part R² indicates the squared partial correlation of every regressor with the dependent variable. Not surprisingly is the partial correlation highest for the most significant parameters. RSS(13,68) is the residual sums of squares for 13 variables and 68 observations. R² is the coefficient of determination, i.e. a “goodness-of-fit” estimate. DW indicates the Durbin-Watson statistic for autocorrelation and JB the Jarque-Bera test for normality. F_{ARCH(4)} indicates the diagnostic statistic for residual autocorrelation up to lag four, F indicates the statistic for autoregressive conditional heteroscedasticity up to the fourth lag and  indicates the statistic for a heteroscedastic variance. F_{RESET} indicates the statistic for Ramsey’s regression-specification error test and F_{Chow} indicates the formal Chow test for a structural break where the breakpoint period is set to 1992:Q3. F indicates an F-test with degrees of freedom in parentheses. All the specification tests of the model is insignificant, which indicates normally distributed and no autocorrelated residuals. Furthermore, is the variance homoscedastic with no sign of an ARCH-property. The model proves not to be mis-specified or include a structural break during the sample period by the RESET- and Chow-test. We also tested for a trend and seasonality in the data but with insignificant test statistics.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Part R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>dq(2)</td>
<td>-0.36** (0.113)</td>
<td>0.15</td>
</tr>
<tr>
<td>dq(3)</td>
<td>0.35** (0.106)</td>
<td>0.16</td>
</tr>
<tr>
<td>dq(4)</td>
<td>-0.51** (0.126)</td>
<td>0.23</td>
</tr>
<tr>
<td>dq(5)</td>
<td>-0.22* (0.109)</td>
<td>0.07</td>
</tr>
<tr>
<td>dq(6)</td>
<td>-0.36** (0.113)</td>
<td>0.15</td>
</tr>
<tr>
<td>dq(7)</td>
<td>0.25* (0.104)</td>
<td>0.10</td>
</tr>
<tr>
<td>dq(10)</td>
<td>-0.30** (0.099)</td>
<td>0.14</td>
</tr>
<tr>
<td>drd(3)</td>
<td>0.009* (0.004)</td>
<td>0.07</td>
</tr>
<tr>
<td>drd(7)</td>
<td>0.009* (0.004)</td>
<td>0.09</td>
</tr>
<tr>
<td>drd(9)</td>
<td>0.009* (0.004)</td>
<td>0.09</td>
</tr>
<tr>
<td>cedbal(3)</td>
<td>0.36** (0.084)</td>
<td>0.25</td>
</tr>
<tr>
<td>ECM(1)</td>
<td>-0.009** (0.002)</td>
<td>0.22</td>
</tr>
<tr>
<td>D92</td>
<td>-0.02* (0.009)</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Specification test:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>RSS(13,68)</td>
<td>0.0271</td>
<td>F_{ARCH(4)}(4,47)</td>
<td>1.53</td>
</tr>
<tr>
<td>R²</td>
<td>0.557</td>
<td>F_{Heteroscedasticity} (25,29)</td>
<td>1.29</td>
</tr>
<tr>
<td>DW</td>
<td>1.93</td>
<td>F_{RESET} (1,54)</td>
<td>1.04</td>
</tr>
<tr>
<td>JB</td>
<td>4.08</td>
<td>F_{Chow} (10,49)</td>
<td>0.50</td>
</tr>
<tr>
<td>F_{AR(1)}(4,51)</td>
<td>0.50</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table VII: Evaluation of forecasting accuracy. The columns for RMSE and MAE in the upper part of the table present the ratios of RMSE and MAE of the ECM and the RW or RWD, respectively. MAE and RMSE are the mean absolute error and root-mean-square error. ECM is the error-correction model and RW and RWD are a driftless random walk and the random walk with a drift, respectively. The lower the ratio, the more the ECM outperforms the random walk models. The numbers in the lower part of the table are the Diebold-Mariano statistic, in which the lag truncation parameter is selected based on the Andrews’ AR(1)-rule. The 5 percent critical value is –1.65 since the test is the one-tail test. The significant test statistic implies superiority of the ECM in out-forecasting both the RW and RWD.

<table>
<thead>
<tr>
<th>Forecast horizon</th>
<th>ECM/RW</th>
<th>ECM/RWD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RMSE</td>
<td>MAE</td>
</tr>
<tr>
<td>1</td>
<td>0.262</td>
<td>0.379</td>
</tr>
<tr>
<td>2</td>
<td>0.229</td>
<td>0.295</td>
</tr>
<tr>
<td>4</td>
<td>0.33</td>
<td>0.417</td>
</tr>
</tbody>
</table>

Diebold-Mariano statistic:

<table>
<thead>
<tr>
<th>Forecast horizon</th>
<th>ECM/RW</th>
<th>ECM/RWD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-3.99</td>
<td>-3.37</td>
</tr>
<tr>
<td>2</td>
<td>-1.75</td>
<td>-1.66</td>
</tr>
<tr>
<td>4</td>
<td>-2.10</td>
<td>-1.78</td>
</tr>
</tbody>
</table>
References.


Long memory effects and credibility in the Swedish monetary policy.

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School of Economics and Commercial Law at Göteborg University,
Göteborg, Sweden.

ABSTRACT

The issue of central bank credibility for the Swedish Central Bank is analysed by the use of the long memory concept. The concept is used to analyse the credibility for the Swedish Central Bank before and after the adoption of a strict inflation target for its monetary policy objective combined with a relative independent central bank. The proposed hypothesis is that if the central bank is very highly to fully credible, then should the memory structure in the inflation rate be insignificant, i.e. there should be no significant feedback effect from past realisations of the inflation rate into today’s inflation rate. This then implies that the market participants interpret the policy objective of the Central Bank of Sweden as very highly to fully credible in the sense that they believe that the central bank are “living up to its words”. The long memory parameter is significant for the pre-independent period but not so for the post-independent period using the GPH-estimator. Furthermore, the estimates are significantly different from each other using the Chow-test. Thus, there is evidence of that the pre-independent period do contain a long memory dependence structure but not so for the post-independent period. The long memory parameter is not significant for either the pre-, or the post-period for Germany, which indicates that the credibility for the Bundesbank has not changed significantly during the period. This strengthens the indication that the credibility for the monetary policy objective of the Swedish Central Bank has increased from the pre- to the post-independent period.

KEYWORDS: Credibility, GPH-estimator, Long memory, Sweden.
1. Introduction.

During the last decade, the concept of credibility has become a central concern in not only the scholarly literature on monetary policy but also in practical central-banking circles. This heightened interest in the credibility of monetary-policy pronouncements is in part tied to the increased awareness of the importance of the link between expectations and the affect that a proposed economic policy will have on the economy. The credibility issue of monetary policy focuses also on the resolution of the so-called inconsistency problem\(^1\) in the conduct of monetary policy, as identified by Kydland \textit{et al} (1977) and Barro \textit{et al} (1983a, 1983b). Suggestions for removal of the inflation bias implied by the inconsistency problem include among other things (1) the building up of an anti-inflationary reputation by the public sector\(^2\), and (2) an institutional reform aimed at establishing an independent and anti-inflationary central bank(er)\(^3\). These suggestions were aimed at strengthening the credibility of the policy objective of central banks and their independence.

Although the concept of credibility has become of central concern, there appears to be no generally agreed-upon definition of the term in either central bank circles or the academic literature. This lack of a precise definition of the concept allows each market participant to attach his/her own preferred meaning to the term. The most common definitions of the concept in central bank circles and in the academic literature is analysed in a paper by Blinder (2000). The most common definition in central bank circles that was found in the paper is of a purely pragmatic nature such as “A central bank is credible if people believe that it will do what it says”\(^4\). However, in the academic literature credibility is often identified with one of three things: (1) strong aversion to inflation, i.e. more inflation-averse central banks are in general

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\(^{1}\) The “traditional” inconsistency problem arises when a policymaker is tempted to raise output and employment above its “natural rate” level by creating unanticipated monetary shocks. See Välijä (1996) for a further discussion.

\(^{2}\) See Barro \textit{et al} (1983b) for a further discussion.

\(^{3}\) Rogoff (1985) and Lohmann (1992) have shown that separating the objective function of the central banker from the objective function of the policymaker is possible, costless and credible in equilibrium.

\(^{4}\) Given that many countries recently adopted an inflation target as their monetary policy regime, the central bankers often take the degree of dedication to price stability as synonymous with credibility. See Blinder (2000) for a further discussion.
more credible; (2) incentive compatibility, i.e. one way to induce the central bank to carry out its pledge to fight inflation is for the government to write an incentive-compatible contract for its central bank, which might penalise the central banker if inflation is “too high”; or (3) precommitment towards a relative low and stable inflation rate, i.e. a central bank is not credible unless it is bound by a rule or other “commitment technology” to live up to its word of a relative low inflation rate despite the temptation to “cheat” by delivering a relative higher inflation rate after the market has formed its expectations about future inflation⁵.

There is furthermore, in the literature, other aspects of the credibility-concept in the sense that there is a distinction between the credibility for the policy objective/target of the central bank, i.e. the credibility of its “conservativeness”, and the credibility of its independence from the political sphere. It has also been recognised that there may be a difference between the de facto status (political) of the central bank and its de jure status (legal), which also might affect belief in the independence of the central bank. The influence of the political sphere on the independence of the central bank can also be attributed to the objectives of or instruments for the central bank policy, i.e. a central bank might have “instrument independence” but not “objective independence”⁶. Yet another way for the political sphere to obtrude upon the independence of a central bank is through the appointment of its board members⁷.

⁵ See Blinder (2000) for references and a more formal discussion about the definition of credibility in the case of a central bank.

⁶ The Reserve Bank of New Zealand Act from 1989 implies “instrument independence” but not “objective independence” because the Reserve Bank of New Zealand is free to choose by what means the established objective is to be achieved. See Välijä (1996) and Walsh (1995). In a similar fashion, the government sets the goal for the Bank of England but the Monetary Policy Committee sets the instrument (see King, 1998). A similar separation of goal and instrument is to be found in Sweden, where the Swedish Central Bank has formalised the government-established goal to an inflation rate of 2% +/- 1% over a two-year period. The instrument is set by the Swedish Central Bank. Similar arrangements are to be found in other countries that have the same framework for the monetary policy.

In the paper by Blinder (2000)\(^8\), two main issues about the central bank credibility were addressed: first, Why is credibility so important to central bankers? and; second, How can a central bank create or enhance credibility? For the first issue, Blinder (2000) proposed seven reasons why credibility is important\(^9\) and the central bankers generally favour four of them, namely: Greater credibility (1) makes disinflation less costly; (2) helps hold down inflation once it is low; (3) makes it easier to defend the currency when necessary; and (4) helps garner public support for central bank independence. Most economists agree on the first two of the proposed reasons why credibility is important, i.e. (1) reducing the costs of disinflation, and (2) keeping inflation low\(^{10}\).

For the second issue in the paper by Blinder (2000), i.e. methods of building or creating credibility\(^11\), the views of the central bankers and the economists are in general closely aligned. Establishing a history of living up to its word is ranked as the most important method followed by central bank independence. Two of the most strongly emphasised methods in the scholarly literature\(^{12}\) were rated as least important by both groups, i.e. precommitment and incentive-compatible contracts. It would appear then that the respondents of the questionnaire by Blinder (2000) feel that central bankers earn credibility with the market participants more by building a track record for honesty and inflation aversion than by limiting their discretion via commitment technologies or by entering into incentive-compatible contracts.

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\(^8\) The paper builds on a survey (a questionnaire) mailed to the heads of 127 central banks (84 responded implying a response rate of 66 percent) and to a similar sized sample of academic economists who specialise in monetary economics or macroeconomics.

\(^9\) The seven reasons set forth by Blinder (2000) for importance of credibility are (1) Reducing the costs of disinflation (credibility hypothesis); (2) Helping to keep low inflation (a version of the credibility hypothesis); (3) Flexibility to change tactics; (4) Serving as a lender of last resort; (5) Defending the exchange rate; (6) A duty to be open and truthful; and (7) Public support for central bank independence.

\(^10\) Beyond that the economists have markedly different rankings than the central bankers. See Blinder (2000) for a more detailed discussion.

\(^11\) The methods of building or creating credibility put forth by Blinder (2000), were: (1) A history of living up to its word; (2) Central bank independence; (3) A history of fighting inflation; (4) Openness and transparency; (5) Fiscal discipline by the government; (6) Precommitment; and (7) Incentive-compatible contracts. The methods appear in the order of ranking by the two groups.

\(^12\) See Barro et al (1983a) and Blinder (2000) for a further discussion.
Several attempts to formalise, measure, and estimate the degree of central bank independence and credibility has been considered in the literature during the last decade\textsuperscript{13} as many countries have adopted an inflation target as their monetary policy regime and a relative independent central bank\textsuperscript{14}. The highest ranked methods to build or create credibility in the paper by Blinder (2000) was (1) the establishment of a history of living up to its word, followed by (2) central bank independence. Furthermore, the central bankers in those countries often take the degree of dedication to price stability as synonymous with credibility with the market participants\textsuperscript{15}. The hypothesis that will be proposed and tested in this paper is that a perfectly credible central bank will have no feedback effect or memory from previously realised outcomes in the monetary policy making process, i.e. in its targeted inflation process. In other words, there will be no significant serial autocorrelation or memory in the inflation process in the sense that past realised monetary policy and inflation rates will significantly affect expectations about and the outcome of the current monetary target and associated policy. I will, in this paper, address the credibility issue of the central bank and the creation of credibility with the market participants by using the concept of a long memory process\textsuperscript{16}. That is, the concept whether or not realised outcomes of the monetary policy objective at relative long lags are significant or not for the outcome of the monetary policy and, therefore, in the end for the credibility of the central bank as the authority that is implementing the monetary policy.

The paper is organised as follows. Section two outlines the theoretical model and section three introduces the fractionally differenced time series models and the Geweke and Porter-Hudak estimator for long memory. Section four presents the empirical results and section five concludes.

\textsuperscript{13} See Cukierman (1992) and Blinder (2000) among others.

\textsuperscript{14} Some of the countries that have adopted this regime in recent years are New Zealand (1990), Canada (1991), the United Kingdom (1992), Sweden (1993), Finland (1993), Australia (1994), and Spain (1994).

\textsuperscript{15} See Blinder (2000) for a further discussion.

\textsuperscript{16} A covariance stationary time series exhibits a long memory property if \( \sum_{j=n}^{\infty} |\rho_j| \rightarrow \infty \) as \( n \rightarrow \infty \) where \( \rho_j \) is the autocorrelation at lag \( j \). This infinite-sum condition suggests that correlations at long lags are not negligible. For a further discussion on long memory models and application to inflation series see Maneschiöld (2000).
2. Central bank independence and credibility.

The interaction between the output-inflation trade-off and credibility can be considered by a one time-period\textsuperscript{17} characterisation of a closed economy

\[ y = \bar{y} + \beta(\pi - \pi^e) + \varepsilon \quad \beta \geq 0 \]  \hspace{1cm} (1)

where \( y \) denotes the log of aggregate real output, \( \bar{y} \) denotes the log of aggregate real output achieved with the natural rate of unemployment and \( \pi \) and \( \pi^e \) are the actual and expected rate of inflation, respectively. The \( \beta \)-parameter measures the non-negative effect of surprise inflation on real output and \( \varepsilon \) denotes a stochastic real shock, which is realised from \( N(0, \sigma^2_\varepsilon) \). The output is assumed to be able to deviate from its natural level as a consequence of surprise inflation or real shocks.

The public sector of the economy consists of two independent authorities, the Fiscal Authority (FA) and the Monetary Authority/Central bank (CB). The FA is assumed to control the tax rate and to possess the power to either delegate the conduct of monetary policy to the CB, whose characteristics are assumed to be exogenous, or to make the CB comply with the fiscal policy\textsuperscript{18}. The CB is assumed to control the money growth and thereby inflation. Public expenditures (\( g \)) are financed by tax revenues (\( \tau \))\textsuperscript{19} and money creation only. Thus, public expenditures are residually determined by taxes and money creation or seignorage such that the FA is restricted by a binding budget constraint of the form

\[ g \equiv \tau + \pi \]  \hspace{1cm} (2)

\textsuperscript{17} It can, without loss of generality, be extended to a multi-period model. For a further discussion see Kydland \textit{et al} (1977) and Barro \textit{et al} (1983a) among others.

\textsuperscript{18} Any institutional reform, such as granting the CB independence, is assumed to be operationalised as changes in the legal framework of concern for the central bank, which then also indicates the independence of the CB. The real shock is assumed to occur after expectations formation and so is also the choice of the FA of the actual policy of delegation. This policy of delegation might or might not deviate from the announced policy, i.e. the FA has an informational advantage (last-mover advantage) in the game.

\textsuperscript{19} For simplicity, we ignore dynamic considerations associated with public debt.
Equation (2) equates the public deficit with seignorage income, which, in turn, is assumed to be equal to the inflation rate \( \pi \). The (intermediate) economic policy goal is assumed to include both monetary stability and stability of aggregate output \( y \) around a target level \( y^* \), which is assumed to exceed the natural rate level \( \bar{y} \).

The two authorities may differ in the importance they place on the two policy goals. Consequently, the periodic loss function \( L \) and the objectives of the CB and the FA can be written as follows:

\[
\min_{\pi} L_{\text{CB}} = \mu_1 \pi^2 + \mu_2 (y - y^*)^2 \quad \mu_{1,2} > 0, \quad y^* = k \bar{y} > \bar{y} \tag{3}
\]

20. The underlying assumption behind equation (2) is that inflation is determined by money growth, which in turn is assumed to be controlled by the CB. *Ex post* this is the case in a natural rate world where the quantity theory of money applies. *Ex ante*, fiscal policy would affect prices though, but for simplicity, we retain the assumption implied by equation (2). See Jensen (1992) for a further discussion.

21. The nominal public spending relative to nominal output is denoted by \( g \) and \( \tau \) denotes nominal tax receipts relative to nominal output with seignorage in the same relative terms. The seignorage is assumed to be equal to the domestic rate of inflation, which is denoted by \( \pi \). It is, furthermore, assumed that the only source of change in the price level is generated out of changes in the money stock, which, in turn, is assumed to be the available policy instrument to the CB. Remark that taxes are assumed to have no demand effects, as any induced change in public expenditures is exactly offset by the equivalent change in private expenditures. For a more in debt discussion see Alesina *et al* (1987), De Kock *et al* (1992), Jensen (1992) and Välilä (1996).

22. Given the reaction functions of the authorities, the first order condition of the two optimisation problems in equation (3) and (4) are respectively

\[
\pi_{\text{CB}}^* = \frac{\mu_2 \beta}{\mu_1 + \mu_2 \beta^2} \left[ \bar{y}(k - 1) + \beta \pi^e - \varepsilon \right] \Rightarrow
\]

\[
\pi_{\text{RE}}^{\text{CB}} = \pi_{\text{CB}}^* \bigg| \pi^* = E[\pi_{\text{tn}}] = \frac{\mu_2 \beta}{\mu_1} \left[ \bar{y}(k - 1) + \beta \left( \frac{\mu_1}{\mu_1 + \mu_2 \beta^2} \right) \right] \tag{5}
\]

and

\[
\pi_{\text{FA}}^* = \frac{\lambda_2 \beta}{\lambda_1 + \lambda_2 \beta^2} \left[ \bar{y}(k - 1) + \beta \pi^e - \varepsilon \right] \Rightarrow
\]

\[
\pi_{\text{RE}}^{\text{FA}} = \pi_{\text{FA}}^* \bigg| \pi^* = E[\pi_{\text{tn}}] = \frac{\lambda_2 \beta}{\lambda_1} \left[ \bar{y}(k - 1) + \beta \left( \frac{\lambda_1}{\lambda_1 + \lambda_2 \beta^2} \right) \right] \tag{6}
\]

where \( k \) is an arbitrary constant such that \( k \bar{y} > y \). The second order conditions for minima for equation (3) and (4) is met. See Välilä (1996) for proof and a further discussion.
and

\[
\min_{\pi} L_{FA} = \lambda_1 \pi^2 + \lambda_2 (y - y^*)^2 \quad \lambda_{1,2} > 0
\] (4)

where \( \mu_1 > 0 \) and \( \lambda_1 > 0 \) indicates the degree of inflation aversion, and \( \mu_2 < 1 \) and \( \lambda_2 < 1 \) indicates the temptation to “cheat” by raising aggregate output above the natural rate level and creating unexpected inflation, i.e. the source of inflation bias.\(^{23}\)

The FA announces, to the private sector at the beginning of the time period, the policy of delegation that they intend to employ, i.e. whether the FA itself (indirectly) or the CB will determine the inflation rate. This announcement is assumed to be conducted by the FA by the designation of the legal status of the CB. The credibility of this signal is assessed by the private sector, which then sets its expectations about inflation rationally by minimizing, with respect to \( \pi^e \), the absolute difference between the actual and expected inflation rates\(^{24}\), i.e.

\[
E|\pi - \pi^e|
\] (8)

\(^{23}\) Assume that real shocks cannot raise output to the target level \((y^* - \bar{y} > \varepsilon)\) and that

\[
(\mu_1 > \lambda_1; \mu_2 < \lambda_2) \Rightarrow \frac{\mu_2}{\mu_1} < \frac{\lambda_2}{\lambda_1}
\] (7)

This then implies that the CB is relative more conservative concerning inflation than the FA, i.e. \( \pi_{CB}^* < \pi_{FA}^* \). Furthermore, it implies that the FA accommodates real shocks to a relative larger extent than the CB. For a further discussion see Barro et al (1983a), Blinder (2000) and Kydland et al (1977).

\(^{24}\) For analytical tractability, equation (8) is minimised instead of the expected squared deviation between actual and expected inflation rates. The optimisation of equation (8) will always result in a “pure strategy” in the sense that the private sector expects either equation (5) or (6) to occur, whichever it deems more likely, and never a (linear) combination of them. If the private sector assigns both outcomes equal probabilities, we assume that it expects equation (5) to occur. For a further discussion see Välilä (1996).
The postulated generation of the output in equation (1) can, by following Lucas (1973) and Välilä (1996), be re-written as:

\[ y = \bar{y} + \beta (\bar{\pi} - \pi^e) + \varepsilon = \bar{y} + \beta \rho (\bar{\pi} - \pi^e) + \varepsilon = \bar{y} + \beta_0 (\bar{\pi} - \pi^e) + \varepsilon \quad (11) \]

It then follows that

\[ \beta = \beta_0 \left( \frac{\sigma_u^2}{\sigma_u^2 + \sigma_{\pi}^2} \right)^{-1} \quad (12) \]

The output-inflation trade-off, \( \beta \), is thus a function of the variance of the prior inflation rate. There is, furthermore in the present framework, a straightforward relationship between the independence of the CB and the prior inflation rate, i.e. a change in the expected independence of the CB implies, by assumption, a change in

\[ u = \pi - \pi^e \sim N \left( 0, \sigma_u^2 \right) \Rightarrow \sigma_u^2 = \text{var}(\pi - \pi^e) \quad (10) \]

Assume, furthermore, that \( \sigma_u^2 \) is more sensitive to changes in expectations than \( \sigma_{\pi}^2 \) in the sense that inflation expectations \( \pi^e \) are more sensitive than the actual outcome of the inflation process \( \pi \) to new information. This then implies that \( \sigma_u^2 \) is more volatile to changes in expectations about future inflation as a consequence to a change in the credibility of the CB concerning its degree of inflation aversion relative to the change in \( \sigma_{\pi}^2 \).
the prior rate of inflation. The $\beta$ is, furthermore, determined by the variance of $u$, which is, according to equation (10), equal to the variance of $(\pi - \pi^c)$. Consequently

$$
\beta = \beta_0 \left( \frac{\sigma_u^2}{\sigma_u^2 + \sigma^2_{\pi}} \right)^{-1} = f \left( \beta_0, (1 - \theta), \lambda_1, \lambda_2, \mu_1, \mu_2, k, e \right)
$$

(15)

where the sign indicates the partial derivatives. The output-inflation trade-off, $\beta$, is thus a function of the independence of the CB, which, in turn, will affect the magnitude of the effect of the surprise inflation on the aggregate output in equation (1). This functional relationship is, furthermore, state-dependent, i.e. it depends on the state of nature as ultimately determined by the parameters characterising the two authorities and the real shock.\(^{27}\)

\(^{26}\) The variance of $(\pi - \pi^c)$ can be expressed by considering

$$
\text{E}[\pi - \pi^c] = \left( 1 - \theta \right) \left[ \pi^*_{\text{CB}}|_{\pi^c = E(\pi^c_{\text{FA}})} - E(\pi^c_{\text{FA}}) \right] = (1 - \theta)B_1; \quad \theta \geq \frac{1}{2}
$$

$$
\theta \left[ \pi^*_{\text{FA}} |_{\pi^c = E(\pi^c_{\text{CB}})} - E(\pi^c_{\text{CB}}) \right] = \theta B_2; \quad \theta < \frac{1}{2}
$$

(13)

where $\theta = \Pr(\pi = \pi^c_{\text{FA}})$ as (unbiasedly) assessed by the private sector. By the use of equation (13), the variance can then be characterised by

$$
\text{var}(\pi - \pi^c) = \text{E}\left[ (\pi - \pi^c)^2 \right] - \text{E}[(\pi - \pi^c)^2]
$$

$$
\approx \tilde{p}(1 - \theta)^2 B_1^2 + (1 - \tilde{p})\theta^2 B_2^2 - [\tilde{p}(1 - \theta)B_1 + (1 - \tilde{p})\theta B_2]^2
$$

(14)

where $\tilde{p} = \Pr(\theta \geq \frac{1}{2}) = \Pr[\pi^c = E(\pi^c_{\text{FA}})]$, i.e. the probability of the private sector to expect institutional cheating by the FA or the probability of the private sector not believing in the independence of the CB and therefore not believing in the credibility in the policy goal of the CB – be there an initial announcement by the FA of the CB’s independence or not. In the event that the FA initially announces the CB’s independence and the announcement is regarded as credible (non-credible) by the private sector, then is $\tilde{p}$ likely to assume a small (large) value. For a further discussion see Välilä (1996).

\(^{27}\) $\beta$ is assumed to increase/decrease with credibility under the assumption that the volatility in $\pi^c$, i.e. $\sigma_{\pi^c}^2$, changes to a larger degree than the volatility in $\pi$, i.e. $\sigma_{\pi}^2$, as credibility increases/decreases. The credibility, $\beta$, is assumed to increase with initial credibility, $\beta_0$, for the CB. For $\beta$ to increase with $(1 - \theta)$ it can be shown that $\tilde{p} = \Pr(\theta \geq \frac{1}{2}) = \Pr[\pi^c = E(\pi^c_{\text{FA}})] < \frac{1}{2}$, i.e. prior to expectations formation, and $\left( B_1 + B_2 \right) \geq 0 \wedge \tilde{p} \leq \frac{1}{2}$, i.e. after the realisation of the shock $e$ and hence after expectations formation. The intuition behind this is that, if the private sector is more likely than not to expect the independence of the CB ($\tilde{p}$ not being greater than $\frac{1}{2}$), then will a “sufficiently large”
2.1 The long memory approach and credibility.

One of the fundamental parts of time series analysis is the relationship between observations made at different points in time, i.e. the memory in the time series. The memory is often indicated by a significant autocorrelation between observations at time $t$ and $t-j$ and it can, as will be done in this paper, be defined as a short or long memory process.

**Definition 1.** A stationary process is defined by its moments:

$$E(\tilde{X}_t) = \mu \quad \text{for } t = 0, 1, 2, \ldots, \infty,$$

$$E[(\tilde{X}_t - \mu)^2] = \sigma_{\tilde{X}_t}^2 = \gamma_0 < \infty \quad \text{and}$$

$$E[(\tilde{X}_t - \mu)(\tilde{X}_{t-j} - \mu)] = \sigma_{\tilde{X}_t,\tilde{X}_{t-j}} = \gamma_j < \infty \quad j = 1, 2, \ldots, \tag{16}$$

where $t =$ time, i.e. the moments are finite and independent of time.

Define a linear process over time of the credibility-process of the market participants for the monetary target of the central bank, $\beta$, as

$$X_t = \sum_{i=-\infty}^{\infty} a_i \frac{1}{\beta_i} \nu_{t-i} \tag{17}$$

negative output shock increase the variance of $\left(\pi - \pi^\varepsilon\right)$ by increasing the probability of institutional cheating ($\theta$), which in turn decreases $\beta$. For a further discussion see Välilä (1996) among others. If the inflation aversion by the FA ($\lambda_1$) and/or by the CB ($\mu_1$) increases then it is assumed that the credibility will increase and it is, furthermore, assumed that it will decrease if FA’s ($\lambda_2$) and/or CB’s ($\mu_2$) temptation to cheat increases. Furthermore, the credibility is assumed to decrease with an increase in $k$, i.e. by an increase in aggregate output over the natural rate level ($k \bar{y} > y$) and, therefore, an increase in the volatility of $\pi^\varepsilon$. It is also assumed that it will decrease with $\varepsilon$ as a negative real shock, e.g. a supply-side shock such as an increase in energy prices, is assumed to increase the volatility of $\pi^\varepsilon$ and therefore $\sigma_{\pi^\varepsilon}^2$ relative larger than the volatility in $\pi$, i.e. in $\sigma_\pi^2$. For a further discussion see Välilä (1996) among others.
where \( \nu_i \sim \text{NID}(0, \sigma_\nu^2) \) and \( 0 \leq a_i \leq 1 \) is the weight realised by the market participants to the credibility of the CB in period \( i \) with the assumption that \( a_i \geq a_{i-j} \), i.e. periods further back in time do not have as large weight as periods closer to the present time period upon forming expectations about the credibility of the CB and the announced policy of delegation and (intermediate) economic policy goal.\(^{28} \) As the variance of the process in (17) is finite, it then follows that

\[
\sum_{i=-\infty}^{\infty} \left( a_i \frac{1}{\beta_i} \right)^2 < \infty
\]  \hspace{1cm} (18)

which makes the linear process in (17) stationary.

Using (16) and (17), the definition of the variance will be defined as

\[
\gamma_0 = \sigma_X^2 = \sigma_\nu^2 \left( \sum_{i=-\infty}^{\infty} \left( a_i \frac{1}{\beta_i} \right)^2 \right)
\]  \hspace{1cm} (19)

which then explains the condition for stationarity (18) for the linear process (17). The autocovariance function of the linear process (17), assuming the definition of the white noise process, will then be defined, similar to (19), as

\[
\gamma_j = \mathbb{E}(X_t X_{t-j}) = \sigma_\nu^2 \left( \sum_{i=-\infty}^{\infty} \left( a_i \frac{1}{\beta_i} \right) \left( a_{i-j} \frac{1}{\beta_{i-j}} \right) \right)
\]  \hspace{1cm} (20)

\(^{28} \) It is assumed that \( a_i \frac{1}{\beta_i} \geq a_{i-j} \frac{1}{\beta_{i-j}} \), by the assumption that \( a_i \geq a_{i-j} \) and \( \beta_i > 0 \), but still that the credibility can be larger or equal in period \( i-j \) to the credibility in period \( i \) and vice versa, i.e. that \( \frac{1}{\beta_i} \geq \frac{1}{\beta_{i-j}} \) or \( \frac{1}{\beta_i} \leq \frac{1}{\beta_{i-j}} \).
By the assumption that $a_i \frac{1}{\beta_i} \geq a_{i-j} \frac{1}{\beta_{i-j}}$, for $j > 0$, and the definition of equation (18), then will the autocovariance (20) always be smaller or equal to the variance (19). The autocorrelation is then obtained by dividing (20) by (19), i.e.

$$
\rho_j = \frac{\gamma_j}{\gamma_0} = \frac{\sum_{i=-\infty}^{\infty} \left( a_i \frac{1}{\beta_i} \right) \left( a_{i-j} \frac{1}{\beta_{i-j}} \right)}{\sum_{i=-\infty}^{\infty} \left( a_i \frac{1}{\beta_i} \right)^2}
$$

which then implies that $|\rho_j| \leq 1$.

**Definition 2.** Short and long memory processes: It can be shown that the $j$:th autocorrelation of a stochastic process can be defined by\(^{29}\)

$$
\rho_j = A_j^{2d-1}
$$

where $A$ is a suitable constant and $d$ is the memory parameter.

A covariance stationary time series is defined as a short memory process when the decay of the autocorrelation function is exponential or more rapid and if the autocorrelation function is absolute summable, i.e.

$$
\sum_{j=-\infty}^{\infty} |\rho_j| < \infty
$$

The autocorrelation function is, for the stationary long memory process, bounded by a hyperbolic decaying function. Furthermore, it is characterised by an autocorrelation function that is not absolute summable, i.e.\(^{30}\)

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\(^{29}\) See Brockwell *et al* (1991), Lyhagen (1997) and Maneschiöld (2000) for proof and a further discussion.

\(^{30}\)
\[ \sum_{j=-\infty}^{\infty} |\rho_j| = \infty \quad \text{(24)} \]

The infinite sum condition for the autocorrelation of a long memory process suggests that correlations at long lags are not negligible for a stochastic process, like the credibility-process in (17), with this characteristic.

A credible central bank will then have an insignificant autocorrelation structure in the sense that there will be no significant feedback effect from previous realised outcomes of the inflation process, i.e. the central bank is credible in the way that the market participants believe that the central bank will “live up to its word” and deliver the pre-announced inflation level. Thus, the memory process of previous realised outcomes of the inflation process will then be insignificant and the credibility-process will belong to the short memory processes as defined by equation (23). On the other hand, a central bank that is not credible by the market participants will have a significant feedback effect from previous realised outcomes in the inflation process upon forming expectations about the credibility of the CB and the announced policy of delegation and (intermediate) economic policy goal. This will be indicated by a significant memory parameter. The less credible the central bank is the larger will the significant feedback effect be in to the realised outcomes in the inflation process in the sense of a relative larger memory in the credibility-process (17) and a significantly larger estimate of the memory parameter.

Note that the definition of a long memory process does not tell us anything about the magnitude of the autocorrelation only that the decay is slow enough to be classified as a long memory. The stationary short memory process is characterised by (23) and a significant parameter estimate \(-0.5<d<0\) in (22). The stationary long memory process is characterised by (24) and a significant parameter estimate \(0<d<0.5\) in (22). See Brockwell et al (1991) for a further discussion.
3. Fractionally differenced time series models.

Granger et al (1980) and Hosking (1981) introduced a model, the ARFIMA-model (the method of fractional differencing), which incorporates long memory dynamics into time series models. Whether a data series displays a structure of long-term dependency or not is in this model conditional on a fractional differencing parameter, which can be estimated.

A general class of long memory processes can be described by

\[
\phi(L)(1-L)^d y_t = \theta(L)\epsilon_t
\]  

(25)

where \( \phi(L) \) and \( \theta(L) \) are polynomials in the lag operator \( L \) with \( p \) and \( q \) lags, respectively. The \( d \)-parameter is the difference parameter, which is allowed to take fractional values indicating the dynamics of the memory process in the series. All roots of \( \phi(L) \) and \( \theta(L) \) are stable and \( \epsilon_t \) is a white noise disturbance term. The fractional parameter, \( d \), can assume any real value. This fractional model includes the usual autoregressive moving average model when \( d=0 \) as a special case, i.e. the ARMA\((p,q)\)-model. Upon using a return series as a dependent variable\(^{31} \), a value of \( d \) greater than zero indicates a mean diverting property while a value less than zero indicates a mean reverting property in the original series. The extension to allow for non-integer values of \( d \) raises the flexibility in modelling long-term dynamics. This extension allows for a more rich class of spectral behaviour at low frequencies.

Geweke et al (1983) proposed a frequency domain estimator for the long memory parameter \( d \) (hereafter abbreviated as the GPH-estimator). The GPH-estimator is a semi non-parametric test for fractional processes that does not require any specification of the short memory process, i.e. the ARMA-part. This is so since only a fraction of the first \( T^{\alpha} \) frequencies are used. The fractional differencing test serves to uncover a fractal structure in a time series based on spectral analysis of its low frequency dynamics. The spectral regression is specified as

\(^{31} \)As will be done in this paper as the log CPI-series for both periods contain a unit root but not the first difference of the log CPI-series of the two periods, i.e. for the inflation rate (see table I).
\[
\ln[I(w_m)] = \phi_0 - \phi_1 \ln \left[ 4 \sin^2 \left( \frac{w_m}{2} \right) \right] + \varepsilon_i, \quad m = 1, \ldots, n
\]  

(26)

where \( \ln[I(w_m)] \) is the periodogram at the frequency \( w_m = 2\pi m/T \). The asymptotic variance of the error term \( \varepsilon_i \) is \( \pi^2/6 \), which is imposed in the estimation to raise the efficiency of the estimator. Geweke et al. (1983) also show that the ordinary least square estimate of \( \phi_i \) provides a consistent estimate of the long memory parameter, \( d \), and hypothesis testing concerning the value of the long memory parameter can be based on the usual \( t \)-statistics\(^{32}\). There is evidence of a long memory process in the data if the ordinary least square estimate of the long memory parameter is significantly larger than zero\(^{33}\).

With a proper choice of \( n \) in equation (26) then will the asymptotic distribution of the long memory parameter, \( d \), neither depend on the order of the ARMA-components nor on the distribution of the error term of the ARFIMA-process. Cheung (1993) and Cheung et al. (1995) suggest to set \( n = T^\alpha \) with \( \alpha \)-values of 0,50 and 0,55 as a fair compromise between low and high frequencies\(^{34}\). The GPH-estimator can also be

\(^{32}\) The estimation of the long memory parameter, using the GPH-estimator, does not make any assumptions about the shape of the underlying distribution, e.g. that it must be Gaussian, more than that the underlying process is independent. Furthermore, the estimation of the long memory parameter is asymptotically standard normally distributed, which makes the hypothesis tests that are based upon the normal distribution applicable. Using synthetic data, Geweke et al. (1983) shows that the asymptotic theory proves to be reliable in samples of 50 observations or more. For a more detailed derivation and discussion of the GPH-estimator see Geweke et al. (1983) and Maneschiöld (2000) among others.

\(^{33}\) The \( j^{th} \) autocorrelation of a stochastic process can be shown to be defined by \( \rho_j = A^{2d-1} \) where \( A \) is a suitable constant and \( d \) is the memory parameter. A stationary short memory process is characterised by \( \sum_{j=-\infty}^{\infty} |\rho_j| < \infty \) and a significant parameter estimate within the bounds \(-0.50<d<0\). A stationary long memory process is characterised by \( \sum_{j=-\infty}^{\infty} |\rho_j| = \infty \) and a significant parameter estimate within the bounds \( 0<d<0.50 \). When \( 0.50<d\leq1 \), then does the process possess a long memory but it is non-stationary and mean-reverting and for \( d>1 \) it is non-stationary but mean-diverting. For a further discussion see Brockwell et al. (1991), Hosking (1981) and Maneschiöld (2000) among others.

\(^{34}\) The \( \alpha \)-value represents the proportion of the observations included in the GPH-estimator. The number of low frequency ordinates, \( T^\alpha \), used in the spectral regression is a choice variable in the GPH-estimator. A too large number of \( T^\alpha \) will cause contamination of the estimate of the long memory parameter, \( d \), due to that medium- or high frequency components and thus more short-term influences are included in the analysis. A too small value of \( T^\alpha \) will lead to imprecise estimates due to limited degrees of freedom in the estimation. For a further discussion see Cheung et al. (1995) and Maneschiöld (2000) among others.
used as a unit root test by determining whether the long memory parameter from the
first-differenced data is significantly different from zero\textsuperscript{35}.

4. Empirical results.

I will in this section examine the memory structure in the inflation series for
Sweden and Germany as a test for the credibility of the central bank for its monetary
policy objective. The consumer price index (CPI)\textsuperscript{36} series for Sweden was obtained
from EcoWin and from IFS for Germany. The series range from January 1980 to
March 2000 for Sweden and from January 1980 to September 1999 for Germany and
are at the monthly frequency.

The inflation series for Sweden is divided up into two sub-series, which are
graphically presented in the appendix. The first period, January 1980 to November
1992, was characterised by a policy objective for the Central Bank of Sweden that
was not explicitly targeted at price stability but also at the exchange rate relative to a
trade weighted currency basket. Furthermore, the central bank was not independent
from the political sphere to the same degree as is the case for the period that follows.
The period January 1980 to November 1992 will be referred to as the pre-independent
period.

The other sub-period runs from January 1995 to March 2000 and will be referred
to as the post-independent period. This period is characterised by an adoption of an
inflation target, a floating exchange rate regime, and a relative independent central
bank with a relative larger degree of openness and transparency\textsuperscript{37}. The period

\textsuperscript{35} This has a particular interest in the examination of the dynamics of the consumer price index (CPI) as
standard unit root tests, which have a stationary ARMA-process as their alternative hypothesis (e.g. the
ADF-test), usually cannot reject the hypothesis of a unit root in the CPI-data.

\textsuperscript{36} The inflation series was created by taking the log and first difference of the corresponding CPI-
series.

\textsuperscript{37} The greater degree of openness and transparency hinges on e.g. the quarterly published inflation
report and the announcement of the minutes from the monetary policy meeting of the Board of
December 1992 to December 1994 will be referred to as the accumulation-of-credibility period. This period will not be included in the empirical estimates of the long memory parameter as it is assumed that the Swedish Central Bank (Riksbanken) is building up a track record of credibility during this period. Thus by the words of Blinder (2000), the central bank is building up “A history of living up to its words” during this period in the sense that it is creating credibility by the public and the market participants for its monetary policy and target.

Under the condition that the creation of credibility during the accumulation-of-credibility period has been successful by the market participants then should the autocorrelation structure in the target variable for the central bank be significantly reduced in the post-relative to the pre-independent period. If the central bank is very highly to fully credible, then should the memory structure in the inflation rate be insignificant, i.e. there should be no significant feedback effect from past realisations of the inflation rate into today’s inflation rate. This then implies that the market participants interpret the policy objective of the Central Bank of Sweden as very highly to fully credible in the sense that they believe that the central bank is “living up to its words”.

The empirical estimates of the long memory parameter are displayed in table II. The long memory parameter $d$ is significant for the pre-independent period at both the OLS $t$- and asymptotic $t$-statistic calculated at the 0,50 frequency value. However, the memory parameter for the post-independent period is not significantly different from zero. It is insignificant by both the OLS $t$- and asymptotic $t$-statistic estimated at the 0,50 frequency value. Furthermore, the estimate for the pre-independent period is not significantly different from 0,50 indicated by the hypothesis test displayed in table II. This then indicates that both the pre- and post-independent estimate belongs to the domain of stationary memory processes\(^{38}\), i.e. a significant $-0,50<d<0,50$. The estimates of the pre- and post-independent period are furthermore significantly

\(^{38}\) This is also supported by the, in table I, presented unit root tests where the inflation series do not contain a unit root, i.e. they are stationary.
Different from each other using the Chow-test\textsuperscript{39}. This then indicates, using the terminology in this paper, that the credibility for the monetary policy objective of the Swedish Central Bank has increased from the pre- to the post-independent period through the accumulation-of-credibility period by the new policy regime and independent structure of the central bank.

One point of contact between policy makers and academics is that many central bankers take the degree of dedication to price stability as synonymous with credibility. Thus the Bundesbank was long considered to be one of the world’s most credible central banks even though it missed its professed money-growth target more than 50 percent of the time.\textsuperscript{40} When people declared that the Bundesbank had high credibility, they meant that no one questioned its determination to fight inflation. This consideration by the public and market participants that the Bundesbank was a highly credible inflation fighter has in general been present during the period considered for the Swedish Central Bank. Thus, we can use Germany as a comparison to examining the concept of credibility using the long memory concept. The CPI-series for Germany was obtained from IFS and range from January 1980 to September 1999 and are at the monthly frequency. The corresponding pre-independent period runs from January 1980 to June 1991, the corresponding post-independent period runs from January 1993 to September 1999, and the period July 1991 to December 1992 correspond to an accumulation-of-credibility period. The empirical estimates of the long memory parameter are displayed in table II. The long memory parameter $d$ is not significant for either the pre-, post-, or total period at both the OLS $t$- and asymptotic $t$-statistic, i.e. there is no indication of a long memory process in the German inflation process. Furthermore, the estimates of the pre- and post-independent period are not significantly different from each other using the Chow-test\textsuperscript{41}. This then indicates that the credibility for the monetary policy objective of the Bundesbank in the pre- and the post-period has not changed significantly. Thus, the insignificant long memory estimate indicates that the Bundesbank has by the public and market participants

\begin{itemize}
  \item \textsuperscript{39} The Chow-test is $F(2,214) = 52.03$. The critical value at the 5 percent level is 3.04 and 4.71 at the 1 percent level.
  
  \item \textsuperscript{40} See Blinder (2000).
  
  \item \textsuperscript{41} The Chow-test is $F(2,225) = 2.21$. The critical value at the 5 percent level is 3.04 and 4.71 at the 1 percent level.
\end{itemize}
remained significantly equally and highly credible during this period concerning its determination to fight inflation.

A series with an indication of a significant long memory process might be the result of two possible structures. The first possibility is that there is actually a long memory process in the series, which then should be encountered upon modelling the series. The second possibility is that the series actually has an independent structure of random variables but that the observations do scale according to a value of the long memory parameter that is significantly different from zero. More precisely, an estimate of the long memory parameter that is significantly different from zero has these two possible explanations:

1. There is a long memory component in the time series, which implies that each observation is correlated to some degree with the observation that follows.

2. The analysis itself is flawed and an anomalous value of the long memory parameter does not mean that there is a long memory effect at work. Perhaps do we not have enough data for a valid test as the guidelines in the literature to the correct amount is somewhat fuzzy. Still, the series being studied is an independent series of random variables, which happens to (1) scale according to a value significantly different from zero, or (2) to be an independent process with fat tails, as suggested by Cootner (1964).

We can test the validity of the results by randomly re-arrange the order of the data of the original time series into a scrambled data set and recalculate the long memory parameter. To confirm the results of the long memory estimates found in the Swedish pre-independent period I will perform this informal test\(^\text{42}\) of the long memory parameter in the inflation series. The frequency distribution of the observations, upon which the GPH-estimator hinges, will remain unchanged. The reason is that all observations are still incorporated in the re-arranged series. The estimates of the parameter of the original series and the corresponding re-arranged series could then be

\(^{42}\) See Peters (1991) and Peters (1994) for a discussion about the so-called scramble test.
used to distinguish between the two possible explanations for a significant long memory parameter.

The order of the data is important if there is a long memory effect in the series. The scrambling of the data should then destroy this structure and the recalculated long memory parameter should then be insignificantly different from zero but significantly different from the estimated long memory parameter of the original series. If the series instead is truly an independent series then should the recalculated long memory parameter be virtually unchanged. The reason is that there did not exist a long memory effect, or correlation, between the observations in the original series. Therefore, scrambling the data would have no effect on the qualitative aspect of the data.

The result of the scramble test for the inflation series of Sweden is reported in table III together with the corresponding long memory estimates of the original series estimated by the GPH-estimator at the 0,50 frequency value. The estimates of the long memory parameter for the scrambled series are not significantly different from zero using the OLS \( t \)-statistics. Furthermore by the use of the \( t \)-test, the long memory estimates of the scrambled series of the post-independent period is not significantly different from the long memory estimates of the original series (see the \( t \)-test in table III). The long memory estimate of the scrambled series is for the pre-independent period however significantly different from the corresponding estimate of the original series.

Using the GPH-estimator and the corresponding hypothesis tests, there is evidence of that the pre-independent period do contain a long memory dependence structure but not so for the post-independent period. This strengthen the indication, by the use of the terminology in this paper, that the credibility for the monetary policy objective of the Swedish Central Bank has increased from the pre- to the post-independent period through the accumulation-of-credibility period. The objective by which this increase in credibility has occurred is through the new policy regime of an explicit inflation-target and the implementation of an independent structure of the central bank. The independent structure is furnished by a much greater degree of openness and transparency, e.g. through the quarterly published inflation report and the
announcement of the minutes from the monetary policy meeting of the Board of Governors (protokoll från direktionens sammanträde) although that this announcement is published with a time lag. A further strengthen of the credibility for the central bank has occurred through the adoption of a framework for fiscal discipline for the government during the post-independent period.

5. Conclusions.

The issue of credibility for the policy objective of the Swedish Central Bank has been analysed by the use of the long memory concept. If the central bank is very highly to fully credible, then should the memory structure in the inflation rate be insignificant, i.e. there should be no significant feedback effect from past realisations of the inflation rate into today’s inflation rate. This then implies that the market participants interpret the policy objective of the Central Bank of Sweden as very highly to fully credible in the sense that they believe that the central bank are “living up to its words”.

The long memory parameter for Sweden is significant for the pre-independent period at both the OLS $t$- and asymptotic $t$-statistic calculated at the 0,50 frequency value using the GPH-estimator. However, the memory parameter for the post-independent period is not significantly different from zero. It is insignificant by both the OLS $t$- and asymptotic $t$-statistic estimated at the 0,50 frequency value. Furthermore, the estimates of the pre- and post-independent period are significantly different from each other using the Chow-test. The estimate of the randomly rearranged series are insignificant for both periods, where the long memory estimate of the pre-independent period is significantly different from the corresponding estimate of the original series. The long memory parameter is not significant for either the pre-, or post-period at both the OLS $t$- and asymptotic $t$-statistic for Germany, i.e.

43 Altogether, this newly established framework, in which the Central Bank of Sweden now operates, does include five, by the central bankers and economists, most important methods of building or creating credibility (see Blinder (2000) and footnote 12 above).
there is no indication of a long memory process in the German inflation process. Furthermore, the estimates of the pre- and post-independent period are not significantly different from each other using the Chow-test. This then indicates that the credibility for the monetary policy objective of the Bundesbank in the pre- and the post-period has not changed significantly as is the case for its monetary policy objective and relative independence.

There is, therefore, evidence of that the pre-independent period in Sweden do contain a long memory dependence structure but not so for the post-independent period. This strengthens the indication, by the use of the terminology in this paper, that the credibility for the monetary policy objective of the Swedish Central Bank has increased from the pre- to the post-independent period through the accumulation-of-credibility period. The objective by which this increase in credibility has occurred is through the new policy regime of an explicit inflation-target and the implementation of an independent structure of the central bank. The independent structure is furnished by a much greater degree of openness and transparency, e.g. through the quarterly published inflation report and the announcement of the minutes from the monetary policy meeting of the Board of Governors (protokoll från direktionens sammanträde) although with a time lag. A further strengthen of the credibility for the Central Bank of Sweden has occurred through the adoption of a framework for fiscal discipline for the government during the post-independent period.
Appendix.

Table I: Test for order of integration, using the ADF-test with an intercept included, of the log CPI series (indicated by lnCPI) and the first difference of the log CPI series, i.e. for the inflation series (indicated by infl). The optimal lag length of the ADF-regression is chosen by adding lags until the Ljung-Box test rejects the residual serial correlation at the 5 percent level. The ADF-test is insignificant for the log CPI but significant for the inflation series for both periods, i.e. the log CPI series contains a unit root but the inflation series are stationary.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Period</th>
<th>ADF; H0: I(1)</th>
<th>Critical value</th>
<th>Lag</th>
</tr>
</thead>
<tbody>
<tr>
<td>lnCPI</td>
<td>Pre-independent</td>
<td>-2,71</td>
<td>-2,88</td>
<td>0</td>
</tr>
<tr>
<td>lnCPI</td>
<td>Post-independent</td>
<td>-2.82</td>
<td>-2.91</td>
<td>0</td>
</tr>
<tr>
<td>infl</td>
<td>Pre-independent</td>
<td>-11.56</td>
<td>-2.88</td>
<td>0</td>
</tr>
<tr>
<td>infl</td>
<td>Post-independent</td>
<td>-6.01</td>
<td>-2.91</td>
<td>1</td>
</tr>
</tbody>
</table>

Table II: GPH-estimates, indicated by \( d \), for the inflation series of the pre-independent (1980:01-1992:11), post-independent (1995:01-2000:03), and total period (1980:01-1992:11; 1995:01-2000:03) for Sweden. The corresponding periods for Germany is for the pre-independent period (1980:1-1991:6), the post-independent period (1993:1-1999:9), and the total period (1980:1-1999:9). OLS \( t \) and asymptotic \( t \) indicates the \( t \)-values for \( H_0: d=0 \) at the OLS standard error and the asymptotic standard error respectively. \( H_0: d=0,50 \) indicates, using the OLS standard error, the absolute value of the \( t \)-test for testing if the GPH-estimate of the series belongs to the domain of stationary memory processes, i.e. a significant \( d<0,50 \). The * indicates significance at the 5 percent level and RSS indicates the sum of squared residuals.

<table>
<thead>
<tr>
<th>Period</th>
<th>Frequency value</th>
<th>( d )</th>
<th>OLS ( t )</th>
<th>Asymptotic ( t )</th>
<th>RSS</th>
<th>( H_0: d=0,50 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-independent (Sweden)</td>
<td>0,50</td>
<td>0,65*</td>
<td>2,41 (0,27)</td>
<td>2,50 (0,26)</td>
<td>19,53</td>
<td>0,55</td>
</tr>
<tr>
<td>Post-independent (Sweden)</td>
<td>0,50</td>
<td>0,20</td>
<td>1,43 (0,14)</td>
<td>0,63 (0,32)</td>
<td>1,87</td>
<td>2,14*</td>
</tr>
<tr>
<td>Total period (Sweden)</td>
<td>0,55</td>
<td>0,51*</td>
<td>2,55 (0,20)</td>
<td>2,68 (0,19)</td>
<td>31,81</td>
<td>0,05</td>
</tr>
<tr>
<td>Pre-independent (Germany)</td>
<td>0,55</td>
<td>0,17</td>
<td>0,68 (0,25)</td>
<td>0,74 (0,23)</td>
<td>22,72</td>
<td>1,32</td>
</tr>
<tr>
<td>Post-independent (Germany)</td>
<td>0,50</td>
<td>0,15</td>
<td>0,75 (0,20)</td>
<td>0,43 (0,35)</td>
<td>3,27</td>
<td>1,75</td>
</tr>
<tr>
<td>Total period (Germany)</td>
<td>0,55</td>
<td>0,09</td>
<td>0,53 (0,17)</td>
<td>0,50 (0,18)</td>
<td>26,50</td>
<td>2,41*</td>
</tr>
</tbody>
</table>
Table III: The long memory estimates, which are indicated by \( d \), of the unscrambled, original, series and the scrambled, randomly rearranged, series with the corresponding OLS \( t \)-test in parenthesis estimated at the frequency value of 0,50. The * indicates significance at the 5 percent level. The scrambled \( d \)-estimates are not significantly different from zero. The \( t \)-test is the significance test that the long memory estimate of the unscrambled, original, series is not significantly different from the long memory estimate of the scrambled, randomly rearranged, series, i.e. 
\[ t = \frac{d_{\text{unscramble}} - d_{\text{scramble}}}{\text{se}_{\text{unscramble}}} \]. A \( t \)-test value above the critical value implies rejection of the hypothesis that the unscrambled and scrambled long memory estimates are statistically equal. The critical value, at the 5 percent level, for the \( t \)-test is 1,96.

<table>
<thead>
<tr>
<th>Period</th>
<th>Unscrambled ( d )-parameter</th>
<th>Scrambled ( d )-parameter</th>
<th>( t )-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-independent</td>
<td>0.65 (2.41)*</td>
<td>0.10 (1.03)</td>
<td>2.04*</td>
</tr>
<tr>
<td>Post-independent</td>
<td>0.20 (1.43)</td>
<td>0.07 (0.96)</td>
<td>0.93</td>
</tr>
</tbody>
</table>
Figure 1: Graph of the Swedish monthly inflation series for the pre-independent period, i.e. 1980:01–1992:11.

Figure 2: Graph of the Swedish monthly inflation series for the post-independent period, i.e. 1995:01–2000:03.
References.


