

Institutionen för didaktik och pedagogisk profession

# Making Sense of Negative Numbers

av

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## Abstract

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Numbers are abstract objects that we conceptualize and make sense of through metaphors. When negative numbers appear in school mathematics, some properties of number sense related to natural numbers become contradictory. The metaphors seem to break down, making a transition from intuitive to formal mathematics necessary. The general aim of this research project is to investigate how students make sense of negative numbers, and more specifically what role models and metaphorical reasoning play in that process. The study is based on assumptions about mathematics as both a social and an abstract science and of metaphor as an important link between the social and the cognitive. It is an explorative study, illuminating the complexity of mathematical thinking and the richness of the concept of negative numbers. The empirical data were collected over a period of three years, following one Swedish school class being taught by the same teacher, using recurrent interviews, participant observations and video recordings. Conceptual metaphor theory was used to analyse teaching and learning about negative numbers. In addition to the four grounding metaphors for arithmetic described in the theory, a metaphor of Number as Relation is suggested as essential for the extension of the number domain. Different metaphors give different meanings to statements such as finding the difference between two numbers, and result in incoherent mappings onto mathematical symbols. The analyses show affordances but also many constraints of the metaphors in their role as tools for sense making. Stretching metaphors, from the domain of natural numbers to fit the domain of signed numbers, changes the metaphor, with unfamiliarity, inconsistency and limited applicability as a result. This study highlights the importance of understanding limitations and conditions of use for different metaphors, something that is not explicitly brought up during the lessons or in the textbook in the study. Findings also indicate that students are less apt to make explicit use of metaphorical reasoning than the teacher. Although metaphors initially help students to make sense of negative numbers, extended and inconsistent metaphors can create confusion. This suggests that the goal to give metaphorical meaning to specific tasks with negative numbers can be counteractive to the transition from intuitive to formal mathematics. Comparing and contrasting different metaphors could give more insight to the meaning embodied in mathematical structures than trying to fit the mathematical structure into any particular embodied metaphor. Participants in the study showed quite different learning trajectories concerning their development of number sense. Problems that students had were often related to similar problems in the historical evolution of negative numbers, suggesting that teachers and students could benefit from deeper knowledge of the history of mathematics. Students with a highly developed number sense for positive numbers seemed to incorporate negatives more easily than students with a poorly developed numbers sense, implying that more time should be spent on number sense issues in the earlier years, particularly with respect to subtraction and to the number zero.