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Dynamic Hedge Rations on Currency Futures

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Abstract

In the globalized economy many businesses are exposed to the foreign exchange risk in their daily trading activities. Exchange traded futures contracts are one of the instruments that are designed specifically to hedge such risk. Over the years researchers and practitioners have been interested in designing the optimal hedge ratio as a number of contracts that should be purchased in order to minimize the variance of the hedged portfolio. Early methods of calculating that ratio assumed time invariant variance covariance structure between spot and futures prices resulting in static ratios. However this assumption has been challenged and models that allow for dynamic evolution of variances and covariances gained on popularity. The purpose of this paper is to investigate the performance of the dynamic hedge ratio strategy on EUR/SEK and USD/SEK designed with bivariate error correction GARCH model with diagonal BEKK parameterization. Also we compare this strategy with other most commonly used hedging schemes such as static OLS and naïve hedging. Using daily observations spanning over almost 8 years we found that despite the theoretical feasibility of the bivariate GARCH it fails to outperform static regression based hedges both in and out of sample.

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1. Introduction

Foreign exchange risk is one of the basic risks that economical agents face when dealing with international transactions. Modern risk management techniques provide many different ways of hedging such a risk. One of them is hedging with exchange traded futures contracts. Such contract specifies the price at which a financial asset such as foreign currency can be bought or sold at the specified future time. Trading in futures markets on foreign currencies began in 1972 on the Chicago Mercantile Exchange and since then they have become increasingly popular among investors.

The basic principle of hedging with currency futures is very simple. Assume that Swedish investor will receive a payment in Euro in six months. Since he does not know what the prevailing exchange rate will be in six months he is exposed to the exchange rate risk. He can remove that risk by buying currency futures which will specify the price at which Euro will be sold in the future. In that way the investor is neutralizing his risk. The problem with this strategy is that it only works if the futures contract matures exactly at the date when the investor will receive his payment. Should the payment occur before the maturity of the contract the risk might not necessarily be neutralized. That is due to the fact that both spot and futures prices follow stochastic processes and thus fluctuate substantially prior to maturity. The risk associated with those fluctuations if often referred to as the basis risk. Because of the basis risk this simple strategy of covering the whole position in foreign currency with futures contracts might not be optimal. In his paper, Johnson (1960) developed a hedging model that has proved superior to the naïve hedging described in the example above. He introduced a term of minimum variance hedge ratio which is a number of futures contracts that should be purchased in relation to the spot position held that minimizes the variance of return of the hedged portfolio. Johnson (1960) worked under the assumption that the joint distribution of the spot and futures prices is time invariant which would imply a static hedge ratio. It also implies that we can estimate it using simple techniques such as Ordinary Least Squares (OLS). The time invariance assumption has been however challenged by other researchers (see Bollerslev 1990; Kroner and Sultan 1991) who showed that as new information arrives to the market the shape of the distribution changes. If that is the case then the minimum variance hedge ratio would vary over time as the new information reaches the market. An additional problem with the proposed OLS methodology is that it ignores the theoretical long run relationship (cointegration) between spot and futures prices. According to Brenner and Kroner (1993) this will result in a downward bias on the estimated hedge ratio. In presence of time varying return distribution and cointegration static models such as OLS could yield an inferior hedging performance. Korner and Sultan (1993) address those issues by applying a bivariate error correction Generalized Autoregressive Conditional Heteroskedasticity model (GARCH) thereby allowing the conditional variance covariance matrix to change over time. This model implies that minimum variance hedge ratio is updates as the new information arrives in the marketplace. It is therefore more accurate and has a potential of outperforming both naïve and static hedges.

The purpose of this paper is to evaluate the performance of the time varying minimum variance hedge ratios on futures written on two exchange rates USD/SEK and EUR/SEK. In order to model the conditional variance covariance matrix we will employ the bivariate error correction GARCH methodology with the diagonal BEKK parametrization of Engle and Kroner (1995). To evaluate the performance of the dynamic hedging we will construct different hedge portfolios using four different strategies: unhedged portfolio, naively hedged portfolio, OLS portfolio and the dynamic bivariate GARCH portfolio. We will compare the hedging strategies in terms of variance reduction when compared to the unhedged portfolio. As we are looking at exchange rates that are rarely investigated by researchers we are hoping to contribute to the existing literature by giving an empirical summary of the most common hedging schemes. Additional contribution is the test of the diagonal BEKK specification which is not often used to calculate the minimum variance hedge ratios.

This paper is divided into 8 sections. In section 2 we review the available literature relating to the minimum variance hedging with currency futures. Afterwards we comprehensively discuss the theory of hedging with futures and show derivations of the static and dynamic minimum hedge ratios. Next section gives the description of the methodology used in this study. In section 5 the data and preliminary results are described followed by the empirical results in section 6. Section 7 deals with comparing hedging performance and we conclude with a discussion of results in section 8.

2. Literature Review

Over the years there has been substantial number of research about methods of calculating and the performance of the minimum variance hedge ratio on futures written on a variety of assets including indices, commodities and foreign exchange. Hill and Schneeweis (1982) compute the static OLS hedge ratios on five foreign exchanges: British Pound, Swiss Frank, German Mark, Canadian Dollar and Japanese Yen[1]. They found a substantial performance improvement compared to unhedged portfolios. A year later Grammatikos and Saunders (1983) investigated the same currencies but looked more closely at the stability of the OLS hedge ratios. Authors found that there is considerable time variation in covariances and variances in all currencies except the Canadian Dollar. With the advancements in the field of theoretical econometrics researchers started to look more closely at the dynamic structure of the variance covariance matrix. Particularly Autoregressive conditional Heteroskedasticity model (ARCH) of Engel (1982) and its extension to GARCH by Bollerslev (1986) provided tools necessary to deal with the issue of time varying variance covariance structure.

Bollerslev (1990) successfully showed that major exchange rates can be modeled with a multivariate GARCH model. Kroner and Sultan (1993) investigate again the five currencies that were in center of attention in studies mentioned earlier but this time they employ the bivariate error correction GARCH model for the computation of the minimum variance hedge ratios. The authors show that by applying this model the hedger can reduce the variance of the portfolio compared to the traditional OLS hedge for all currencies with an exception of the British Pound. Gagnon, Lypny and McCrudy (1998) examine the usefulness of the multivariate GARCH models to hedge two currency portfolios one of German Marks and Swiss Francs and second of German Marks and Japanesee Yen. By applying a trivariate GARCH they conclude that there is a substantial gain in hedgers' utility compared with traditional hedging methods. Harris, Shen and Stoja (2007) in their paper examine various hedging schemes on USD/EUR, USD/GBP and USD/JPY. Their results are show that dynamic hedging outperforms marginally unconditional OLS hedges for Euro and Pound while OLS hedge seem to be superior for the Japanese Yen.

[1] All foreign exchange rates were in relation to USD

The methodology of calculating the dynamic minimum variance hedge ratios has been obviously widely applied on other financial assets. Baillie and Myers (1991) examine the performance of bivariate GARCH on six different commodities. The results seem to wave in favor of using the dynamic hedging schemes although the as with currencies the scope of reduction varies from commodity to commodity. Brooks, Henry and Persand (2002) examine the performance of the dynamic hedgers on the FTSE 100 futures and again find a significant improvement over traditional schemes.

3. Futures prices and Minimum Variance Hedge Ratio

A future contract is an agreement to buy or sell underlying asset at the specified price and specified time in the future. Futures contracts are highly standardized and traded on exchanges. Let F_0 and S_o denote the natural logarithm of futures and spot price of currency at time 0 respectively. Then the relationship between futures and spot price is usually written as

$$F_0 = S_0 e^{(r-r_f)T} \tag{1}$$

Where r and r_f are the risk free interest rate home and abroad respectively and T is time to maturity of the contract. This relationship is derived from a non arbitrage condition and is subject to certain assumptions such as no transaction costs, constant tax rate and possibility of borrowing and lending at the same risk free interest rate. If the above expression didn't hold a market participant could lock in an arbitrage profit.

Futures on currencies are widely used to manage the exchange risk exposure. If the hedged instrument matches the underlying of the contract and hedger wants to close his position at the maturity date of the contract then a simple hedge strategy is to buy contracts covering the entire position in foreign currency (naive hedge). In practice however it is rarely the case that the hedger can close his position when contract matures. If a hedge has to be closed prior to maturity of the contract the hedger is exposed to a so-called basis risk. Basis (b_t) at time t is defined as

$$b_t = S_t - F_t \tag{2}$$

When closing the position prior to maturity the hedger is not sure whether he will get the contracted price and the basis represents his payoff at that time. In the presence of basis risk the simple hedging strategy covering whole exposure might not be optimal. Instead the hedger who is only interested in reducing his risk would like to make sure that the basis risk is as small as possible. In other words the desired position will have as small variance as possible.

Following Brooks, Henry, Persand (2002) we define $\Delta S_t = S_t - S_{t-1}$ and $\Delta F_t = F_t - F_{t-1}$. Then at time t-1 the expected return (basis) at time t can be rewritten as

$$E_{t-1}(R_t) = E_{t-1}(\Delta S_t) - \beta_{t-1} E_{t-1}(\Delta F_t) \tag{3}$$

The $\beta_{\tau-1}$ is referred to as the hedge ratio and in naive hedging this ratio equals to 1. The variance of this expected return is

$$h_{R,t}^2 = h_{S,t}^2 + \beta_{t-1}^2 h_{F,t}^2 - 2\beta_{t-1} h_{SF,t} \tag{4}$$

Similarly to Brooks, Henry, Persand (2002) we will assume that hedger has two moment utility function expressed as

$$U(E_{t-1}(R_t), h_{R,t}^2) = E_{t-1}(R_t) - \varphi h_{R,t}^2 \tag{5}$$

In this utility function φ is the risk aversion coefficient. Having expressions for both variance of return and hedgers utility we can specify the maximization problem as

$$\max U(E_{t-1}(R_t), h_{R,t}^2) = E_{t-1}(\Delta S_t) - \beta_{t-1} E_{t-1}(\Delta F_t) - \varphi(h_{S,t}^2 + \beta_{t-1}^2 h_{F,t}^2 - 2\beta_{t-1} h_{SF,t}) \tag{6}$$

The hedger wishes to maximize his utility which is solely derived from the return on the hedged position and variance of that return. We solve the maximization problem with respect to β_{t-1}. In order to achieve it however we need an assumption that futures prices are martingales i.e. we assume that $E_{t-1}(\Delta F_t) = 0$. The assumption about futures prices being martingales is consistent with a random walk theory which states that the best prediction of tomorrow's price is the price today. The hedge ratio that maximizes hedger's utility is

$$\beta_{t-1}^* = -\frac{h_{SF,t}}{h_{F,t}^2} \tag{7}$$

The minimum variance hedge ratio is a covariance of spot and futures prices divided by the variance of futures prices. The payoff from the position at time *t* can be calculated as follows

$$R_t = \Delta S_t - \beta^*{}_t \Delta F_t \tag{8}$$

4. Methodology

In this section we will describe the methodology employed in this study of minimum variance hedge ratios. We will compare four different hedging strategies; no hedge, the naïve hedge ratio, the OLS hedge ratio and the bivariate GARCH hedge ratio. In so doing we divide the sample into two parts. In-sample analysis will be used for hedge ratio estimation and 85% of the data set will be used to achieve that. Final 15% of the data will be saved in order to perform out-of-sample analysis which will evaluate the performance of the hedge. The software package used in this thesis is EViews 6.0.

4.1 No hedge and naïve hedging

Unhedged and naively hedged portfolios are straightforward to compute. For the no hedge scenario we simply assume zero hedge ratio in equation 8. The naïve hedge corresponds to the hedge ratio equal to unity in the equation 8.

4.2 OLS

Following Johnson (1960) we set up a framework for calculating static hedge. The simplest way of estimating the minimum variance hedge ratio is by using OLS regression. By doing so we are implicitly assuming that variances and covariances are time invariant. This means that we can drop the time index in equation 7. An additional shortcoming of that model is that we are ignoring the possible long run relationship between spot and futures prices. The model we are going to estimate is

$$\Delta S_t = \alpha + \beta \Delta F_t + \varepsilon_t \tag{9}$$

Coefficient β is the estimated minimum variance hedge ratio. Since the ratio is static in nature the same β will be used for in-sample and out-of-sample analysis.

4.3 Multivariate GARCH

To capture the dynamics of the variance-covariance matrix in the estimation of hedge ratio we employ bivariate GARCH model. Since the spot and futures prices seem to be cointegrated there exists a long term relationship between those prices. To capture that fact the mean equation in the bivariate GARCH setting will be modeled with Vector Error Correction Model (VECM) according to the specification below

$$\Delta Y_t = \mu + \sum_{i=1}^{l} \Gamma_i \Delta Y_{t-i} + \Pi v_{t-i} + \varepsilon_t \tag{10}$$

$$\Delta Y_t = \begin{bmatrix} F_t \\ S_t \end{bmatrix} \; ; \; \mu = \begin{bmatrix} \mu_F \\ \mu_S \end{bmatrix} ; \; \Gamma_i = \begin{bmatrix} \Gamma_{i,F}^{(F)} & \Gamma_{i,S}^{(F)} \\ \Gamma_{i,F}^{(S)} & \Gamma_{i,S}^{(S)} \end{bmatrix} \; ; \; \Pi = \begin{bmatrix} \pi_{F,t} \\ \pi_{S,t} \end{bmatrix} \; ; \; \varepsilon_t = \begin{bmatrix} \varepsilon_{F,t} \\ \varepsilon_{S,t} \end{bmatrix}$$

In this model a vector of futures and spot returns ΔY is regressed upon a constant μ, previous lags and ν_{t-1}, which is the error correction term. The residuals from the VECM specification will be saved and used for the modeling of conditional variance covariance matrix.

The variance equation can be written as a bivariate GARCH model with the following specification

$$vech(H_t) = \begin{bmatrix} h_{C,t} \\ h_{CF,t} \\ h_{F,t} \end{bmatrix} = C_0 + A_1 vech\left(\varepsilon_{t-1}\varepsilon'_{t-1}\right) + B_1 vech H_{t-1} \tag{11}$$

$$\varepsilon_t | \psi_{t-1} \sim N(0, H_t)$$

$$C_0 = \begin{bmatrix} c_{11} \\ c_{12} \\ c_{22} \end{bmatrix} \; ; \; A_1 = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \; ; \; B_1 = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

This specification was first developed by Bollerslev, Engle and Wooldridge (1988). Using this model we can describe the evolution of variance of spot and futures prices. In this model H_t is the 2 x 2 conditional variance covariance matrix at time t which is a function of a 3 x 1 constant vector C_0, a 2 x 1 error term vector ε_{t-1} and a 2 x 2 conditional variance covariance matrix at time t-1. The multivariate GARCH models are traditionally estimated using the maximum likelihood method. The necessary assumption for this model is that the error term ε_t given the information set ψ_{t-1} is approximately normally distributed with mean 0 and variance covariance matrix H_t. According to Brooks (2008) one of the biggest shortcomings of that model is the number of the parameters that need to be estimated. In this bivariate setting we would need to obtain estimates of 21 parameters in total. Moreover the conditional variance covariance matrix should be positive semi definite which according to Brooks (2008) might not be the case if a non linear optimization procedure as in multivariate GARCH is used.

One of the solutions to the problems described above is using so called BEKK parameterization developed by Engle and Kroner (1995). The variance equation in the BEKK model has the following form

$$H_t = C_0'^* C_0^* + A_{11}'^* \varepsilon_{t-1} \varepsilon_{t-1}' A_{11}^* + B_{11}'^* H_{t-1} B_{11}^*$$

$$C_0^* = \begin{bmatrix} c_{11}^* & c_{12}^* \\ 0 & c_{22}^* \end{bmatrix} \quad A_{11}^* = \begin{bmatrix} \alpha_{11}^* & \alpha_{12}^* \\ \alpha_{21}^* & \alpha_{22}^* \end{bmatrix} \quad B_{11}^* = \begin{bmatrix} \beta_{11}^* & \beta_{12}^* \\ \beta_{21}^* & \beta_{22}^* \end{bmatrix} \qquad (12)$$

This model requires estimation of only 11 parameters and ensures that the variance covariance matrix is always positive definite. The semi positives of variance covariance matrix will ensure that the numbers on the leading diagonal (variances) will be positive and that the matrix will be symmetrical. In this paper we will use the diagonal BEKK specification which is supplied in the software package. Restricting matrices A and B to be diagonal further reduces number of parameters to 7. From this model we will extract the conditional variance covariance matrix. Particularly we will be interested in covariance between spot and future prices and the variance of future prices. The hedge ratio will be computed according to equation 7.

4.4 Evaluating Hedge Performance

In order to evaluate the performance of the hedge we will compare our four different hedging strategies. The benchmark scenario in which the spot position is left unhedged corresponds to the 0 hedge ratio. Second, we will compute a naïve hedge strategy which comprises of equal position in spot and futures markets (1 hedge ratio). Third strategy presented will be a static hedge ratio calculated using OLS methodology. Finally we will evaluate the performance of dynamic hedge ratio computed using the bivariate GARCH. The evaluation will be done in-sample and out-of-sample. For the most practical purposes however the out-of-sample analysis is much more important since it tests the model in a real market situation. Out-of-sample estimation will be done by using last 15% of the data set spanning from 1/01/2007 to 17/03/2008. The payoff of the position will be calculated on the daily basis according to the equation 8

Evaluating OLS results is straightforward since we calculate static hedge ratio once and use it on the rest of the sample. In order to evaluate the performance of the dynamic hedge ratio we must make a conditional variance covariance matrix forecast from the diagonal BEKK model. We used software supplied modeling tool in order to get a forecast of BEKK residuals first. This was done using the Bootstrap methodology provided by the software package which generates innovations by randomly drawing residuals from the sample period. Once we have residuals generated we can forecast the movements of variances and covariances in the last 15% of the data set. This is done by solving equation 12 using the estimated parameters of the diagonal BEKK model and residuals obtained from the mean equation.

Next the variance of this return will be calculated and finally the reduction in variance compared to the unhedged position. The reduction in variance can be expressed as

$$reduction = \frac{h_{R,unhedged} - h_{R,hedged}}{h_{R,unhedged}} \tag{14}$$

5. Data and preliminary results

In this paper we use 2033 daily observations of spot and future prices of USD/SEK and EUR/SEK exchange rate. The time period covered spans from June 1 2000 to March 17 2008. Futures contracts on the USD/SEK and EUR/SEK employed in this study trade on the

Intercontinental Exchange in New York (former New York Board of Trade). The maturity dates are March, June, September and December. Each contract starts trading one year prior to maturity. In this study we use the continuous series of futures prices computed by the DataStream Advance. Figure 1 presents the evolution of daily spot and futures prices over the study period.

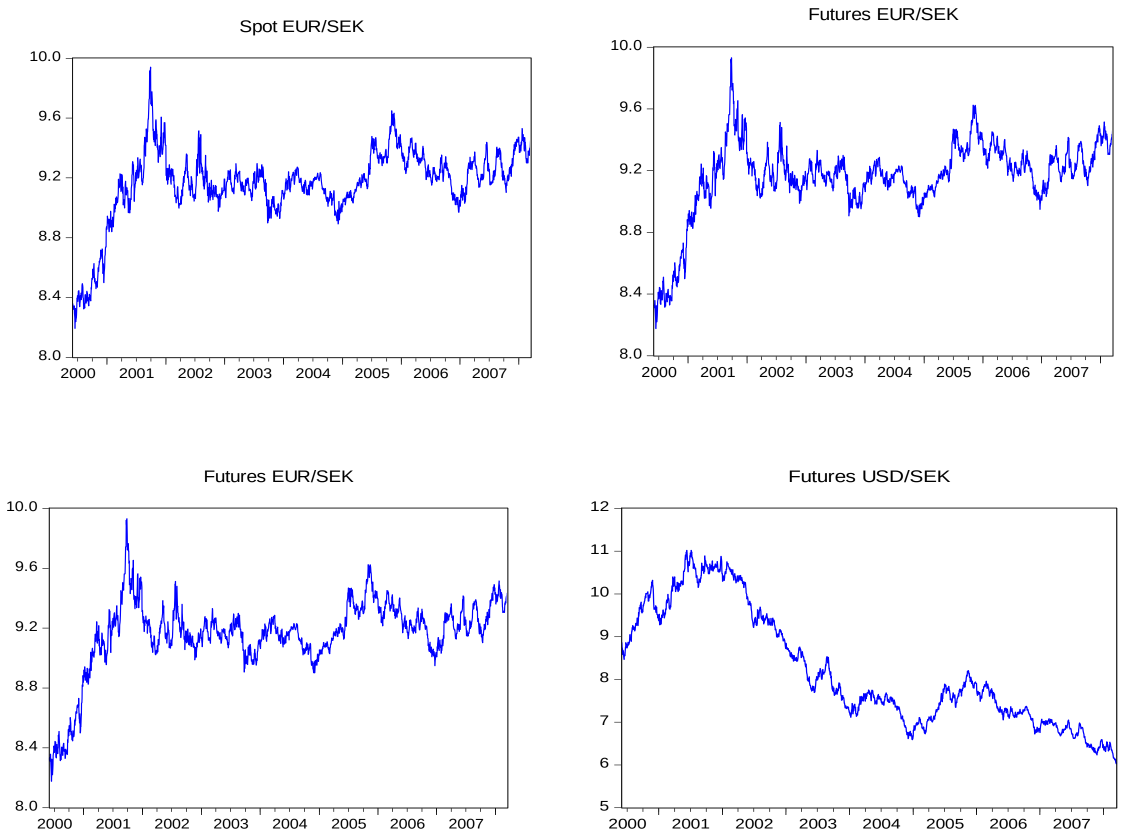

Figure 1. Evolution of Spot and Futures prices on EUR/SEK and USD/SEK

Both spot and futures prices of EUR/SEK exchange rate experienced a sharp increase from 2000 up until middle of 2001. In the rest of the sample the price of the currency stabilized; it shows no apparent trend and its evolution resembles a mean reverting process. On the other hand, spot and futures prices of USD/SEK clearly follow a downward deterministic trend through the whole sample. The summary statistics for the natural logarithms of spot and futures prices on both exchange rates are presented in Table 1.

Both spot and future prices of EUR/SEK are leptokurtotic which means that they have a positive kurtosis. Kurtosis measures peekness of the distribution so a distrubition with a positive kurtosis will put more probability around the mean than a normal distribution would. Moreover, the EUR/SEK exchange rate in both spot and futures prices has a negative skew. Negative skew implies that the mass of the distribution is shifted to the right compared with normal distribution.

Variable	Mean	Variance	Skewness	Kurtosis
EUR/SEK Spot (S)	2,214238	0,000752	-1,490389	6,458388
EUR/SEK Futures (F)	2,214160	0,000756	-1,570975	6,682682
USD/SEK Spot (S)	2,092099	0,024872	0,384610	1,888522
USD/SEKFutures (F)	2,091353	0,025313	0,386229	1,886641

Table 1. Summary Statistics

In that case more probability is put on values greater than the mean compared with the normal distribution. Spot and futures prices of USD/SEK are also leptokurtotic but have a positive skew.

We are also interested in whether the spot and futures prices on both exchange rates are stationary or not. The notion of stationarity is an important one in time series econometrics particularly if we want to work with OLS regression models. According to Brooks (2008) using non-stationary series in regression analysis might lead to spurious results, which means that the model might find a strong relationship between variables when there actually is none. The effects of the news (error term) is also different for stationary and non-stationary processes. If a series is stationary the effect of the shock gradually dies out while if series is non-stationary the effect of the same shock is permanent. Finally, the statistical inference for non-stationary series might not be valid. In Table 2 we report the results of Augmented Dickey Fuller (ADF) test which is a standard test for series stationarity.

Under the null hypothesis the series contains unit root and is therefore non-stationary. The number of lags used in the test is determined by the Akaike Information Criterion. To test EUR/SEK we use ADF test without a deterministic trend since except the period between 2000 and mid 2001 the series did not seem to follow any apparent trend (see Figure 1).

USD/SEK on the other hand clearly follows a descending trend and therefore we use ADF test with trend.

	Test Statistic	P-value	Lags
EUR/SEK Spot (S)	0,90037	0,90190	7
EUR/SEK Futures (F)	0,89209	0,90060	3
USD/SEK Spot (S)	-2,51130	0,32260	2
USD/SEK Futures (F)	-2,54067	0,30830	0

Table 2.Augmented Dickey Fuller Test

The results imply that both spot and futures prices of EUR/SEK and UDS/SEK are non-stationary. According to Brooks, Henry, Persand (2002) this result is to be expected and it is consistent with weak form efficiency of the spot and futures market. For econometric analysis non-stationarity implies that we will work on the first differences (returns) rather on level data when calculating the minimum variance hedge ratios.

EUR/SEK				
No. of CE(s)	Eigenvalue	Trace Statistic	Critical Value (0.05)	Prob
None	0,07582	160,72240	12,32090	0,00010
At most 1	0,00041	0,82372	4,12991	0,41990
USD/SEK				
No. of CE(s)	Eigenvalue	Trace Statistic	Critical Value (0.05)	Prob
None	0,04601	95,52529	15,49471	0,00000
At most 1	4,1E-08	0,00008	3,84147	0,99360

Table 3 .Johansen’s Cointegration Test

We also check whether there exists a long-run cointegrating relationship between spot and futures prices of currencies. It is of particular interest because as we mentioned in the introductory part the existence of such relationship might undermine the validity of the OLS approach. Table 3 present results of the most commonly used test for cointergration, Johansen cointegration test.

The null hypothesis in the Johansen's test is that there are *r* cointegrating equations. If the null hypothesis is rejected the null is modified and we test whether there are *r+1* cointegrating equations. The procedure is repeated until the right number of cointegrating equations is found. According to the test spot and futures prices of both EUR/SEK and USD/SEK have a one cointergrating equation. This means that there exists a long-run relationship between those two prices and we should take this into account when calculating the minimum variance hedge ratio.

6. Empirical Results

In this section we will present the empirical estimates of minimum variance hedge ratios and evaluate their performance. We start with discussing in sample estimates of both OLS and bivariate GARCH hedge ratios. The section will conclude with evaluating out of sample model performance and comparing it to benchmark cases of unhedged and naively hedged portfolios.

6.1 OLS

We estimate the minimum variance hedge ratio using OLS according to the equation 8 and report the estimates in table 4.

Currency	Hedge Ratio	Standard Error	p-value
EUR/SEK	0,334832	0,023633	0,0000
USD/SEK	0,281163	0,023213	0,0000

Table 4 .Minimum Variance Hedge Ratios OLS Estimates

The minimum variance hedge ratios estimated using the OLS are static. This means that once estimated the hedger uses this ratio of futures to spot during the entire hedging period. As we can see from regression outputs the OLS hedge ratio for EUR/SEK is about 0,334832 while for USD/SEK it is 0,281163. Clearly the OLS implies much smaller position in the futures market compared with naïve hedging. The scatter plots of the spot and futures prices for the OLS regressions can be seen in Apendix A.

6.2 Multivariate GARCH

To obtain the estimates for dynamic minimum variance hedge ratio we start with modeling the mean equation using VECM model as specified by 9. According to BIC information criterion the optimal lag length is eight in both cases. The estimation output is presented in the appendix A. The residuals after VECM are then saved and multivariate GARCH specification (equation 12) is estimated. The coefficient estimates are presented in the Appendix B. In order to get an estimate for the dynamic minimum variance hedge ratio we need to extract conditional covariance and variance of futures prices. This is done by solving equation 12 using residuals from the VECM model estimated earlier and coefficients of the diagonal BEKK. Figure 2 presents the dynamic evolution of covariance between spot and futures prices and variance of futures. Clearly both covariance and variance futures prices varied substantially during the study period[2].

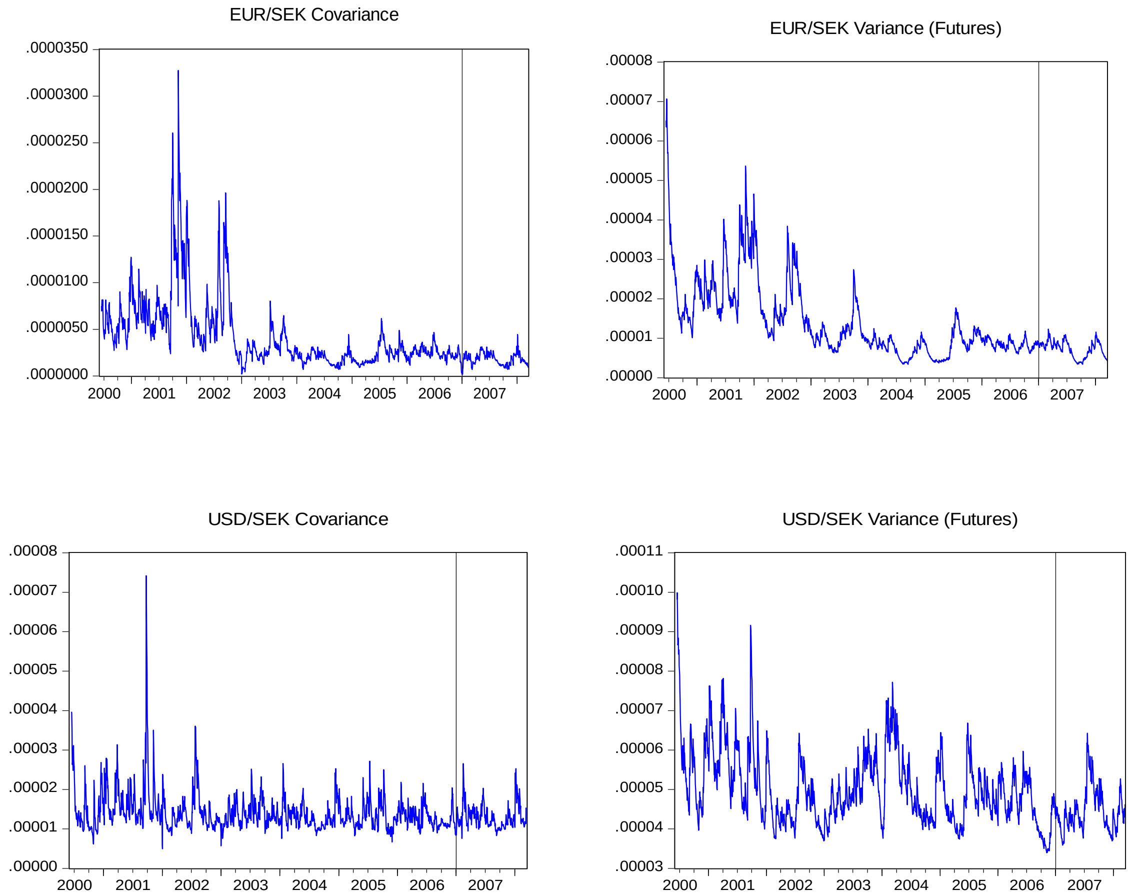

Figure 2. Evolution of covariance between spot and futures prices and variance of futures prices.

[2] Variance of the spot prices also exhibit a time varying patter however it is not presented here since it is not a part of the hedge ratio equation.

In order to examine the effectives of the bivariate GARCH hedging strategy we need a forecast of conditional variance covariance matrix which is obtained as described in section 4. Appendix C presents the innovations generated with bootstrapping. Having the predicted residuals we solve equation 12 for the conditional variance covariance matrix using coefficients that were estimated on the first 85% of the data set. In Figure 2 the series behind the black vertical line are the forecasted values of covariance and futures variance.

Having the conditional variance covariance matrix extracted from the model we can now compute the dynamic hedge ratio according to the equation 7. Figure 3 presents the dynamics of the minimum variance hedge ratio computed with the bivariate GARCH. For comparision we have also included the static OLS hedge ratio in the figure.

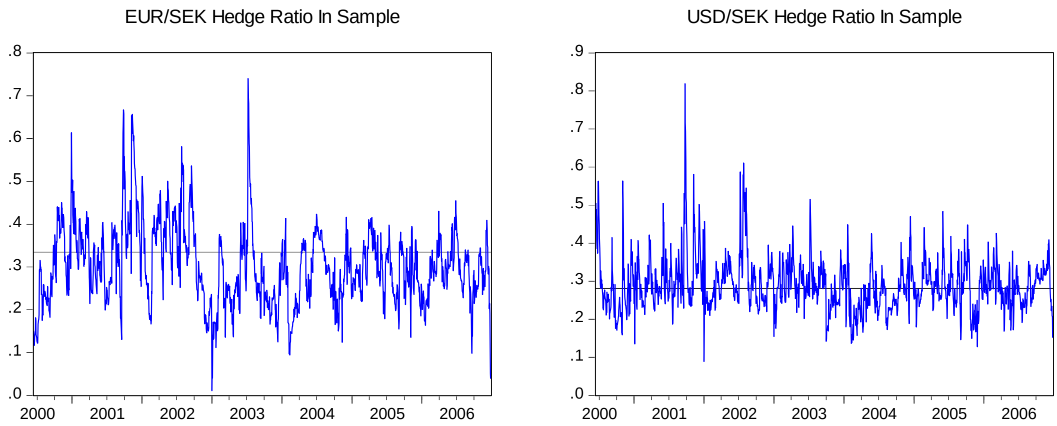

Figure 3. In sample dynamics of the M-GARCH Minimum Variance Hedge Ratio. Horizontal black line represents OLS hedge ratio.

From the inspection of Figure 3 we can clearly see that the bivariate GARCH hedge ratio on both currencies varies substantially across the sample. For EUR/SEK the dynamic hedge ratio ranges from 0,01 to 0,74 while for the USD/SEK the ratio takes values between 0,09 and 0,81. This implies that the hedger would sometimes have a portfolio close to the unhedged position and sometimes close to the naively hedged portfolio. This variability of the hedge ratio was to be expected as we have already seen in Figure 2 that both covariances and variances changed substantially during the whole sample period.

To compute the out of sample forecasted hedge ratios we use the covariance and variance predicted by the diagonal BEKK model. Again the ratio is computed according to the equation 7.In figure 4 we present the forecasted dynamic hedge ratios for both exchange rates.

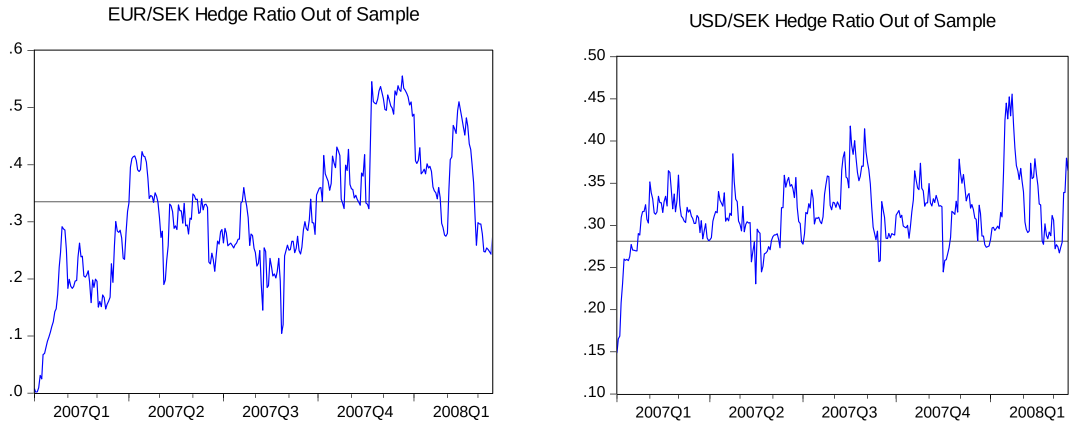

Figure 4. Out of Sample dynamics of the M-GARCH Minimum Variance Hedge Ratio. Horizontal black line represents OLS hedge ratio.

The out-of-sample bivariate GARCH ratio also varies across prediction period. The EUR/SEK ratio takes values between 0,00 and 0,55 while USD/SEK ratio implies values between 0,14 and 0,46.

7. Hedging Performance

The performance of the hedge is evaluated both in sample and out of sample. Table 5 reports the results for in sample analysis covering period from June 1 2000 to January 1 2007. All mean returns and variances are in values per annum. The benchmark unhedged EUR/SEK portfolio yields an average mean return of 1,736% with a variance of 0,000256. Constructing the simplest naïve hedge position reduces the return to -0, 088% and increases the variance of the portfolio by 23,23 %. Static OLS hedging performs better with a reduction in variance of 11,86% but the mean return is also reduced to 1,125%. The dynamic hedge strategy gives the hedger a reduction in variance of 10,43% and mean return of 1,429%. Based solely on this results it would seem that the hedger who whishes the smallest variance possible in his portfolio should choose the static hedging scheme.

In case of USD/SEK the benchmark portfolio yields a negative mean return of 5,946% with a variance of 0, 00088. Similar to the Euro results the naïve hedging actually increases the portfolio variance by 30,24% but return is improved to -0,792%. Static hedging reduces the variance by 8,48% and yields a mean return of -4,497%. The dynamic scheme performs similarly to the static one with variance reduction of -8,49% and mean return of -4,135%. Hence it would seem that there is an improvement in variance reduction by using the dynamic hedging. However, the improvement is marginal.

	Dynamic	Static	Naive	Unhedged
		EUR/SEK		
Mean	0,01233	0,00971	-0,00076	0,01498
Variance	0,00022	0,00021	0,00031	0,00024
Reduction	-10,43%	-11,86%	23,23%	
		USD/SEK		
Mean	-0,03569	-0,03881	-0,00683	-0,05131
Variance	0,00075	0,00075	0,00117	0,00081
Reduction	-8,49%	-8,48%	30,24%	

Table 5 .Hedging Performance In-Sample

	Dynamic	Static	Naive	Unhedged
		EUR/SEK		
Mean	0,03860	0,03126	-0,00291	0,04847
Variance	0,00018	0,00018	0,00028	0,00019
Reduction	-0,85%	-1,84%	33,43%	
		USD/SEK		
Mean	-0,09557	-0,09896	-0,01114	-0,13332
Variance	0,00056	0,00055	0,00087	0,00059
Reduction	-4,43%	-6,23%	32,18%	

Table 6 .Hedging Performance Out-of-Sample

In Table 6 we report the results of out of sample analysis performed on dataset spanning from 1/01/2007 to 17/03/2008. The unhedged EUR/SEK portfolio yields the mean return of 5,616% and variance of 0,0002. If the hedger chooses the naïve strategy he will again increase the variance of his portfolio by 33,43% and reduce the mean return to -0,337%. Static hedging performs better and reduces the variance by -1,84% with a mean return of 3,623%. As in the in sample case the dynamic hedging scheme seem to underperform the static strategy yielding the reduction in variance by -0,85% and mean return of 4,472%

Unhedged USD/SEK portfolio gives the hedger a mean return of -15,448% with a variance of 0,000633. Should he choose to hedge naively he will increase return to -1,291% but also increase the variance by 32,18%. Static OLS scheme provides some improvement by reducing the variance by 6,23% but mean return is reduced to -11,467%. Finally the dynamic strategy reduces the variance by 4,43% and yields mean return of -11,073%. Clearly in terms of variance reduction the out of sample OLS seem to provide superior results.

8. Conclusions and Discussion

Based on our findings we conclude that dynamic hedging scheme presented in this paper for EUR/SEK provides improvement in variance reduction compared with unhedged and naively hedged portfolios both in and out of sample. However, it is strictly outperformed by the static OLS hedge also both in- and out-of-sample. For the second exchange rate USD/SEK in sample dynamic hedging strictly outperforms unhedged position and naively hedged portfolio. It also marginally outperforms the static OLS hedge. However the out-of-sample results the put dynamic ratios to a disadvantage, and OLS seem to be a superior strategy.

The results obtained in this paper are very interesting from the theoretical point of view, since it would seem that OLS which has a number of major methodological drawbacks performs better. Recall that OLS assumes time invariant variance covariance matrix, however previous research and the evidence presented earlier in the paper evidently undermines this assumption. Simple OLS also ignores the possibility of a cointegrating relationship between spot and futures prices. The bivariate GARCH procedure on the other hand provides solutions to all these problems. By construction, it models the time development of the variance covariance matrix and by allowing the mean equation to be modeled with a VECM specification it accounts for the long-term relationship between spot and futures prices. However despite all these merits the model failed to outperform OLS. One of the reasons for this result could be the bivariate GARCH specification used in this paper. Most of the studies quoted in the review used full BEKK specification according to the equation 12. In our setting we used a simplified diagonal version specified in equation 13 which could result in loss of important information in the variance covariance matrix. Inclusion of more parameters into the model makes it however more difficult to estimate.

Poor out-of-sample performance could be a result of keeping the BEKK coefficients constant throughout whole prediction period and misspecification of the forecasting algorithm. Harris, Shen and Stoja (2007) argue that forecasts generated by the multivariate GARCH models are systematically biased in a sense that forecasts they generate are on average incorrect. On the other hand the in-sample performance of dynamic hedging also failed to outperform other strategies.

Another explanation in more asset specific. As noted by Kroner and Sultan (1993) their dynamic hedging scheme on British Pound underperformed static strategies. Also, Harris, Shen and Stoja (2007) could not improve hedging outcomes on Japanese Yen with help of bivariate models. Other researchers (e.g. Baillie and Myers (1991)) also found that the extend of dynamic hedging efficiency varies from asset to asset.

For all the reasons stated above a further extension of the thesis could be to examine the performance of the full BEKK specification. It could also be interesting to pull together those exchange rates for which bivariate GARCH models proved inferior and compare them for common features in order to examine why certain assets fit the model better than others.

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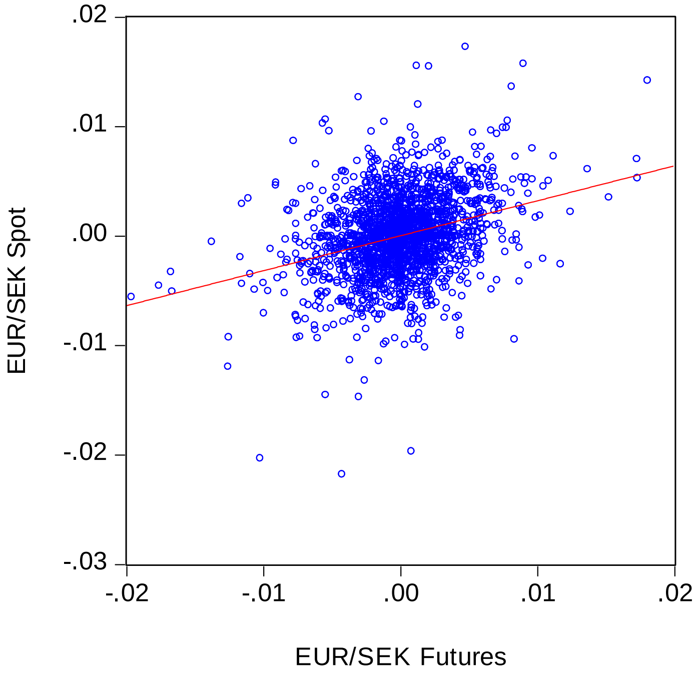
APPENDIX A - Scatterplots for OLS Hedge Ratio Estimates



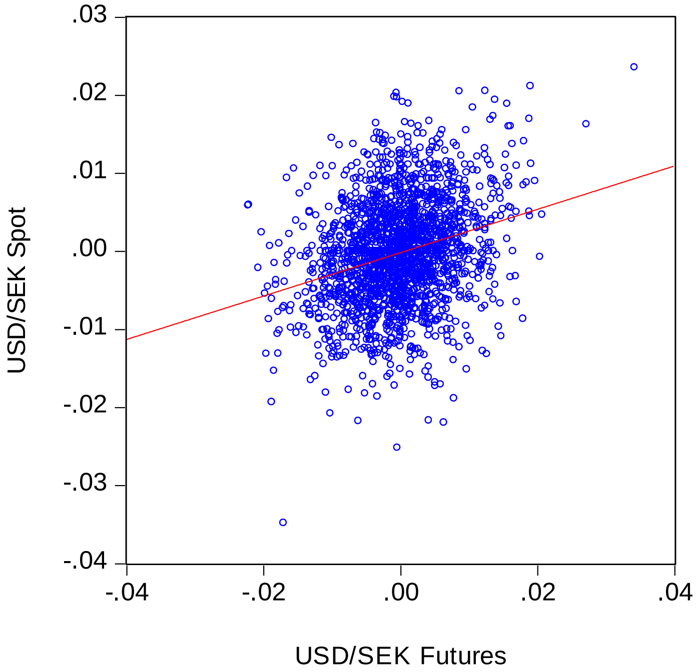


Exhibit A1. Scatterplots for OLS hedge ratio estimation

APPENDIX B – VECM Estimation Results and Residual Histograms

EUR/SEK MEAN EQUATION

$$\mu = \begin{bmatrix} 1.51\text{E-}06\,(5.5\text{E-}05) \\ 9.60\text{E-}06\,(8.9\text{E-}05) \end{bmatrix}$$

$$\Gamma_1 = \begin{bmatrix} -0.772398(0.35979) & -3.434743(0.22139) \\ -0.140610(0.34280) & 2.599829(0.21094) \end{bmatrix}$$

$$\Gamma_2 = \begin{bmatrix} -0.680023(0.33392) & -2.666725(0.20547) \\ -0.218923(0.30766) & 1.894948(0.18932) \end{bmatrix}$$

$$\Gamma_3 = \begin{bmatrix} -0.563115(0.29648) & -1.968535(0.18243) \\ -0.208576(0.26458) & 1.364598(0.16281) \end{bmatrix}$$

$$\Gamma_4 = \begin{bmatrix} -0.437375(0.25106) & -1.411890(0.15449) \\ -0.242452(0.21723) & 0.921389(0.13367) \end{bmatrix}$$

$$\Gamma_5 = \begin{bmatrix} -0.306173(0.20150) & -0.970590(0.12399) \\ -0.320518(0.16648) & 0.516947(0.10244) \end{bmatrix}$$

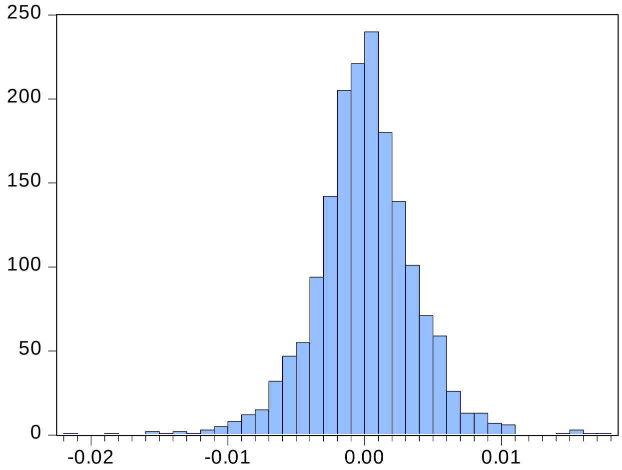
$$\Gamma_6 = \begin{bmatrix} -0.120126(0.14911) & -0.543152(0.09176) \\ -0.341307(0.11512) & 0.167831(0.07084) \end{bmatrix}$$

$$\Gamma_7 = \begin{bmatrix} -0.014050(0.09645) & -0.217316(0.05935) \\ -0.248913(0.06738) & 0.010119(0.04146) \end{bmatrix}$$

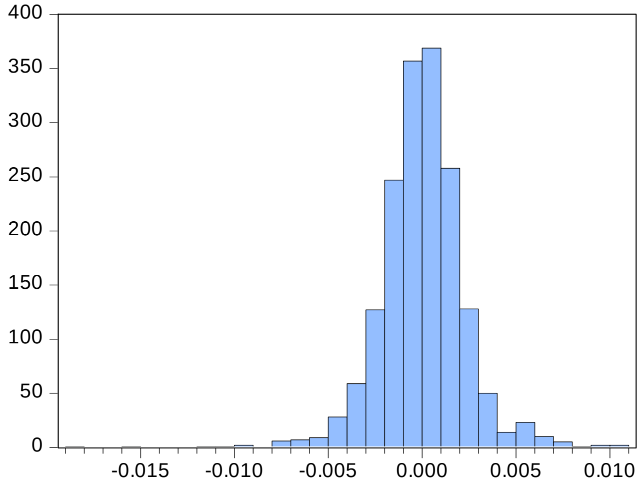
$$\Gamma_8 = \begin{bmatrix} 0.013505(0.04770) & -0.044101(0.02935) \\ -0.142942(0.02786) & -0.029785(0.01714) \end{bmatrix}$$

$$\Pi = \begin{bmatrix} 0.103187\,(0.36507) \\ -4.386744\,(0.22464) \end{bmatrix}$$

Exhibit B1 EUR/SEK mean equation output



Residuals EUR/SEK Futures after VECM	
Sample 6/01/2000 1/01/2007	
Observations 1708	
Mean	-1.15e-19
Median	4.26e-05
Maximum	0.017709
Minimum	-0.021154
Std. Dev.	0.003680
Skewness	-0.155233
Kurtosis	5.696679
Jarque-Bera	524.3893
Probability	0.000000



Residuals EUR/SEK Spot after VECM	
Sample 6/01/2000 3/17/2008	
Observations 1708	
Mean	-5.54e-20
Median	3.89e-05
Maximum	0.010970
Minimum	-0.018200
Std. Dev.	0.002265
Skewness	-0.379957
Kurtosis	8.837779
Jarque-Bera	2466.432
Probability	0.000000

Exhibit B2 EUR/SEK mean equation residuals

USD/SEK MEAN EQUATION

$$\mu = \begin{bmatrix} 3.63\text{E}-07\,(0.00010) \\ 1.46\text{E}-05\,(0.00017) \end{bmatrix}$$

$$\Gamma_1 = \begin{bmatrix} -0.407309(0.34942) & -3.137237(0.20547) \\ -0.464411(0.33638) & 2.383833(0.19780) \end{bmatrix}$$

$$\Gamma_2 = \begin{bmatrix} -0.373410(0.32324) & -2.445583(0.19008) \\ -0.405013(0.30177) & 1.757720(0.17745) \end{bmatrix}$$

$$\Gamma_3 = \begin{bmatrix} -0.334841(0.28617) & -1.832582(0.16828) \\ -0.337553(0.25917) & 1.261800(0.15240) \end{bmatrix}$$

$$\Gamma_4 = \begin{bmatrix} -0.317975(0.24186) & -1.338378(0.14222) \\ -0.240663(0.21207) & 0.830971(0.12470) \end{bmatrix}$$

$$\Gamma_5 = \begin{bmatrix} -0.257523(0.19356) & -0.892189(0.11382) \\ -0.210470(0.16264) & 0.479826(0.09564) \end{bmatrix}$$

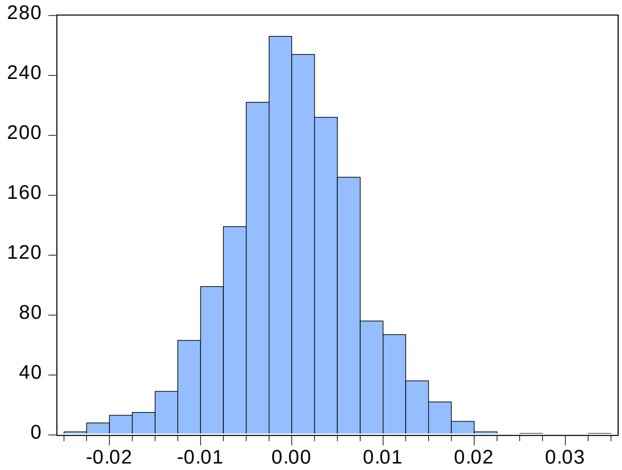
$$\Gamma_6 = \begin{bmatrix} -0.185398(0.14376) & -0.523188(0.08454) \\ -0.147722(0.11288) & 0.226555(0.06638) \end{bmatrix}$$

$$\Gamma_7 = \begin{bmatrix} -0.127875(0.09383) & -0.243076(0.05517) \\ -0.087603(0.06657) & 0.060242(0.03915) \end{bmatrix}$$

$$\Gamma_8 = \begin{bmatrix} -0.040359(0.04728) & -0.075011(0.02781) \\ -0.050178(0.02793) & -0.009508(0.01642) \end{bmatrix}$$

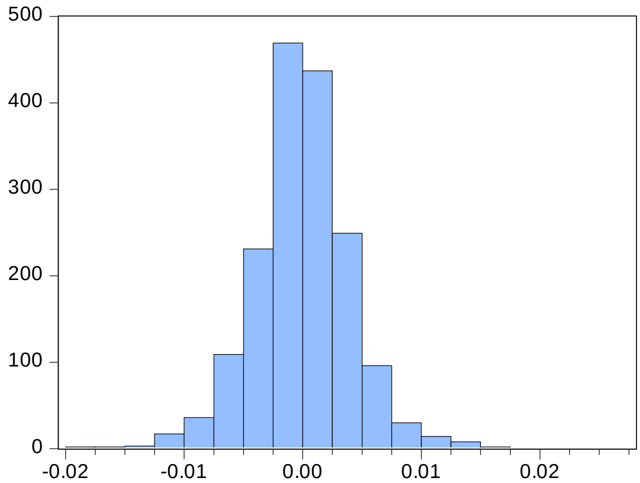
$$\Pi = \begin{bmatrix} 0.493610\,(0.35867) \\ -4.085871\,(0.21091) \end{bmatrix}$$

Exhibit B3 USD/SEK mean equation output



Residuals USD/SEK Futures after VECM
Sample 6/01/2000 3/17/2008
Observations 1708

Mean	-1.47e-20
Median	-2.78e-05
Maximum	0.034612
Minimum	-0.023221
Std. Dev.	0.007062
Skewness	0.009952
Kurtosis	3.631486
Jarque-Bera	28.40769
Probability	0.000001



Residuals USD/SEK Spot after VECM
Sample 6/01/2000 3/17/2008
Observations 1708

Mean	1.80e-19
Median	-4.87e-05
Maximum	0.025730
Minimum	-0.018253
Std. Dev.	0.004153
Skewness	0.201146
Kurtosis	5.689400
Jarque-Bera	526.2571
Probability	0.000000

Exhibit B4 USD/SEK mean equation residuals

APPENDIX C – Bivariate GARCH variance equation coefficient estimates

Variance Equation:

$$H_t = C_0'^* C_0^* + A_{11}'^* \varepsilon_{t-1} \varepsilon_{t-1}' A_{11}^* + B_{11}'^* H_{t-1} B_{11}^*$$

Coefficients Estimates:

EUR/SEK

$$C_0^* = \begin{bmatrix} 7.63\mathrm{E}-08(1.69\mathrm{E}-08) & 5.28\mathrm{E}-08(1.41\mathrm{E}-08) \\ 0 & 1.03\mathrm{E}-07(1.03\mathrm{E}-07) \end{bmatrix}$$

$$A_{11}^* = \begin{bmatrix} 0.296250(0.014618) & 0 \\ 0 & 0.226255(0.014617) \end{bmatrix}$$

$$B_{11}^* = \begin{bmatrix} 0.948193(0.004892) & 0 \\ 0 & 0.970747(0.003419) \end{bmatrix}$$

USD/SEK

$$C_0^* = \begin{bmatrix} 2.91\mathrm{E}-06(5.63\mathrm{E}-07) & 1.73\mathrm{E}-06(3.46\mathrm{E}-07) \\ 0 & 2.07\mathrm{E}-06(7.88\mathrm{E}-07) \end{bmatrix}$$

$$A_{11}^* = \begin{bmatrix} 0.322770(0.025682) & 0 \\ 0 & 0.175143(0.021417) \end{bmatrix}$$

$$B_{11}^* = \begin{bmatrix} 0.851118(0.026125) & 0 \\ 0 & 0.963340(0.010524) \end{bmatrix}$$

Exhibit C1 Variance equation estimation output

APPENDIX D – Actual and predicted (via Bootstrap) BEKK residuals for out of sample analysis

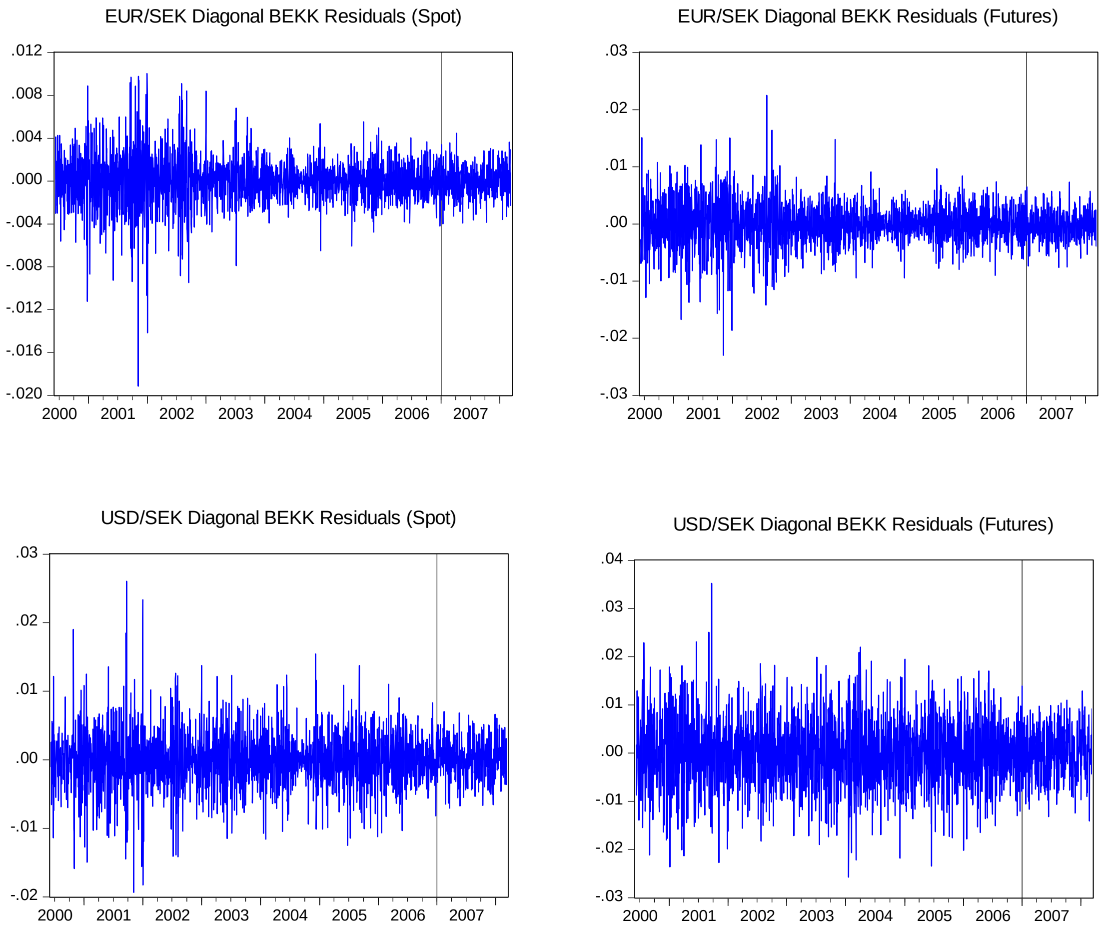


Exhibit D1 Actual and predicted mean equation residuals