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ANALYSIS OF MINING INVESTMENTS IN ZIMBABWE

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To my parents

Abstract

The papers in this thesis investigate issues related to investment with particular reference to the mining sector in Zimbabwe. Issues analysed are the levels of risk premia that attract investment in minerals in developing countries, whether firms in the sector manage to reduce operation to optimal levels consistent with theoretical predictions, and the extent to which irreversibility has reduced investment expenditures.

Paper I describes the structure of the Zimbabwean mining industry. It is shown that the importance of the mining sector has been declining over time yet it has remained as the most important foreign currency earner for the economy. Depressed mineral prices, foreign exchange shortages and a hostile domestic political climate have impacted negatively on mineral investments. The government is focused on an indigenisation program whose success could depend on a detailed understanding of effects uncertainty in the investment climate, historical mining returns, and the potential of attracting appropriate investment for small-scale operations.

Paper II analyses risk-risk premia on mining investments in Zimbabwe, using the risk adjusted Hotelling model to examine the level of risk-premia required for investment in mining to take place. Empirical results show that a risk-premium higher than 27% is required. For that reason it is highly unlikely that much mining investment will take place in Zimbabwe.

Paper III examines extraction costs for mining firms in Zimbabwe and tests whether the behaviour of firms satisfy optimality conditions derived from inter-temporal profit maximisation using parameter estimates from a dual cost minimisation problem. Results reject the hypothesis that firms optimise inter-temporal profits and show a positive relationship between cumulative extraction and costs that suggests that ore stock depletion matters. For that reason investment in the sector depend on the possibility of raising sufficient funds to enough to cover setting up costs for frequent new operations due to the small-scale nature of deposits.

Paper IV examines the effects of irreversibility on mining investments in Zimbabwe. The path of reversible investment determined by the equality of the marginal-revenue-product of capital to its user-cost is used to predict irreversible investment based on individual-firm uncertainty. It is assumed that the level of capital-stock deviates from its desired level and that the distribution of the deviations can be derived. The distribution is then used to estimate the implied effects of the uncertainty which underly the observed regular investment-pattern. Results show that the individual-firm uncertainty must have deviation values greater than 0.166 contrasted with values less than 0.04 for the observed investment-ratio. The result implies that the irreversible capital stock was less than 68% of the reversible level when there was positive investment.

Keywords: investment, mining, minerals, natural resource, uncertainty, irreversibility, cost function, consumption, return, investment, risk premia, user-cost, capital, Zimbabwe.

JEL Classification: O1 O13

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Göteborg, 13 June 2003
Ronald Chifamba

Contents

Thesis Introduction

Zimbabwe has a diverse mineral potential that could be used to generate foreign exchange. But there is global over-supply of many mineral products because substitutes are being produced so that prices are generally going down. Meanwhile, environmental concerns have impacted negatively on the cost of mining. These factors create an enormous challenge for the Zimbabwean government. They also explain why it is important to investigate and understand the structure of mining costs and the effects of uncertainty and irreversibility on mining investment, including the required risk-premia, in order to determine the potential for increasing investment in minerals.

The broad aim of the thesis is thus to examine the returns to mining investment in Zimbabwe during the period 1969-1995, including the costs of extraction and the effects of uncertainty and irreversibility, and then to determine the prospect of luring further investment. The findings are sufficiently general for them to be applicable to any less-developed country that seeks to increase investment in an exhaustible natural resource. This is important because many of the best opportunities for investment in less-developed countries, particularly in Africa, are still in natural-resource-based projects. The foreword to UNCTAD (1994), for example, states that development of mineral resources may provide one of the few feasible ways of increasing economic growth, even in the case of the least-developed countries that do not currently have a significant mineral-sector activity.

The thesis contains four separate chapters, the first describing the structure of the Zimbabwean mining industry and the remaining three, dealing with a heterogeneous selection of investment problems and using three different broad methodologies applied to aggregate production data for five minerals: chrome, copper, gold, asbestos, and iron.

Projection of future mineral revenues is uncertain, and there is a general decline in the competitiveness. This means that appropriate policies should aim at reducing uncertainty in the controllable aspects of the investment climate so as to increase the benefits derived from mining. The first paper "*The structure of the Zimbabwean* *mining industry*" shows that of mining has been declining in importance yet it remains Zimbabwe's most important foreign-exchange earner. Depressed mineral prices, foreign-exchange shortages and a hostile domestic political climate have impacted negatively on mining investments. The success of the government indigenisation program will depend on a detailed understanding of the nature of mining costs, returns that might attract investment and reducing the uncertainty in the investment climate to enhance the potential of attracting investment appropriate for small-scale operations

Relative to most other industries, mining is characterised by high risk. Hence, private investment is only attracted into mining when mining offers a considerably higher level of return than do other investments. The objective of the paper on "*Risk-premia on mining investments in Zimbabwe*" is to estimate the levels of risk-premia that could attract investment away from alternative investments and into mining in Zimbabwe. The paper uses a modified Hotelling-model that captures risk-premia through the covariance of consumption and returns. The results show that the level of risk premia would have to be very high.

The paper on "*Mineral extraction costs in Zimbabwe*", tests whether Zimbabwean mining firms meet optimality-conditions derived from inter-temporal profit-maximisation. It is generally hypothesized that a price-taking (i.e. competitive) profit-maximizing firm adjusts production so that the difference between price and the marginal cost of production increases at the rate of interest (the Hotelling-rule). Firms in less-developed counties have little if any influence on the price of the commodities produced, and are thus competitive (not monopolistic) but this also means that the existence of resource scarcity cannot be tested. However, with generally falling prices it is expected that marginal costs should also fall for the relationship to hold.

The cost function is specified not only in terms of input prices but also allowing for independent effects of both current and cumulative output and tests are carried out to find out whether marginal costs decrease with extraction, and whether the optimality-conditions are satisfied. The results suggest that mining investments during the study period were not cost effective. New investment should be more suited to the Zimbabwean geology that favours small-scale mining.

The fourth paper, "*Irreversibility and mining investment in Zimbabwe*", addresses the concern that irreversibility of investment expenditure reduces the level of investment. Undertaking an irreversible investment permanently affects cash-flows that can never be recovered. The literature suggests that irreversible investment will be undertaken only when the expected discounted payoff from investment exceeds its user-cost by the opportunity-cost (option-value) of the investment.

The model used in the paper examines the behaviour of firms that seek to maximise the present value of their investments while taking into account the state of the business environment, the price of capital goods, and irreversibility constraints. It also shows the effects of microeconomic irreversibilities on aggregate investment-dynamics in the presence of individual-firm uncertainty. The model was used to explain effects of irreversibility constraints that underlie observed investment-series that have low variation.

Aggregate investment-series do not show the effects of irreversibility on investment, hence are usually ignored by macroeconomic policy-makers. Convex adjustment-costs implied by changes in technology and market structure make it costly to adjust capital stock, and so have been used to explain the observed autocorrelation and low variation in aggregate investment. Bertola and Caballero (1994) have shown that it is possible to explain most of the autocorrelation and low variation using the effect of individual-firm uncertainty on aggregate investment.

The paper uses a model that estimates the path of reversible investment implied by the equality of the marginal-revenue-product of capital to its user cost. It is assumed that the actual level of capital stock deviates from its desired level and that a distribution of the deviations can be derived. The importance of uncertainty suggests that it is important to reduce the volatility of the investment climate. The factors affecting the volatility of the investment climate include prices, interest rates, exchange rates, taxes, tariff structures, and regulatory policies.

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Paper I

The Structure of the Zimbabwean mining industry, 1969-1995

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Abstract: This paper describes the structure of the Zimbabwean mining industry. It shows that mining has been declining in importance yet it remains Zimbabwe's most important foreign-exchange earner. Depressed mineral prices, foreign-exchange shortages and a hostile domestic political climate have impacted negatively on mining investments. The success of the government indigenisation program will depend on a detailed understanding of the nature of mining costs, returns that might attract investment and reducing the uncertainty in the investment climate to enhance the potential of attracting investment appropriate for small-scale operations

Keywords: investment, consumption, mining, Zimbabwe, natural resource.
JEL Classification: O1 O13

1. Introduction

For most countries the share of mining output in total production is low and mining is relatively unimportant, but the mining sector in Zimbabwe generates more than 30% of export earnings, and the government places a lot of emphasis on the possibility of using mineral resources to generate foreign-exchange and increase the growth rate of the economy. Like in many countries, mineral production in Zimbabwe has been achieved mainly by the private sector, although Zimbabwe controls mineral rights and enforces environmental regulations and legislation specifically regarding mineral resources. Nevertheless, on the basis of definitions by Nankani (1985), and Auty (1993), Zimbabwe is not a mineral-based economy. Nankani (1985) and Auty (1993) defined mineral-based economies as those where mining accounts for at least 8-10% of GDP and 40% of export earnings.

Although the foreword to UNCTAD (1994) stated that development of mineral resources may sometimes provide one of the few feasible ways of increasing economic growth, even in less-developed countries (LDCs) that do not have significant mineral-sector activity, Sarmiento (1988, p. 105) argued that economies led by natural-resource production could hardly expect to maintain rates of growth similar to those of the world economy as a whole, while those led by manufacturing may expect to grow at rates well above the average.

Although some economies outside the OECD such as China, the Republic of Korea, and India continue to utilise stockpiling arrangements designed to protect their mineral supplies, most industrialised countries no longer perceive minerals as of strategic value and during the 1990s they started reducing their stockpiles. Production of mineral substitutes has also increased, so that concern over security of their supply is declining. Thus new investment in minerals in less-developed countries, including Zimbabwe, has also been declining. At the same time there has been a growing preoccupation with environmental matters. Thus mining is no longer being vigorously promoted as a vehicle for economic development, and as a result there are few economic studies on which to base consistent policies for mining. In fact, most studies of mining in Zimbabwe, feasibility studies essential for determining extraction possibilities, have focused on geological and engineering aspects of the industry. Also most of Zimbabwean mining companies have run into low-grade ore causing high operating and capital costs.

But since mining is expected to remain quite important for Zimbabwe for the foreseeable future, it is important to analyse how the sector has evolved and directions it could develop. This paper describes the structure of the mining industry in Zimbabwe and relates it to the other sectors of the economy.

The next section outlines the policy regimes and related general economic performance over the past four decades, while Section 3 describes the ownership structure of the mining sector. Section 4 compares mining with the manufacturing and agricultural sectors, and Section 5 discusses the leading mineral products. Section 6 summarises and draws some conclusions.

2. Policy regimes and general economic performance

Prior to the most recent era of rapid global industrialisation, governments' interest in minerals was not questioned. Privately-owned mining enterprises were rare and usually confined to low-value minerals. As economies industrialised, most highly developed countries retained specific prerogatives and authority with regard to mining.

During the colonial era, most African countries provided essential mineral resources to their colonisers, with most mines being operated by companies owned and operated by multinational corporations. As African countries gained independence, they tended to nationalise some of the mining enterprises and thus to make the investment climate rather uncertain for the private sector. Thus there was a decline in private-sector investment in mineral resources and an increase in state control. However, the policies of state control failed, and many African governments have since tried to reverse the situation by adopting policy measures to attract private investment. As in other African countries, the investment climate for mining in Zimbabwe has depended upon the specific policy regimes that the government and the economy has passed through.

Since 1965 Zimbabwe has passed through four distinct policy regimes. During the period 1965-79, international sanctions were imposed on the economy because of the Unilateral Declaration of Independence (UDI) made in 1965. Zimbabwe attained independence in 1980 and adopted socialist policies that remained until 1990. During the period 1991 to 2001 the economy went through the IMF/World Bank type of reform programmes before the government abandoned them.

During the UDI period, exports were severely restricted and companies experienced shortages of foreign-exchange for purchasing imported inputs and spare parts. The government thus adopted interventionist economic policies, including a system of foreign-exchange allocation and import quotas, intended to promote rapid development of an industrial and technological base. There were controls on prices, wages, interest rates, and investment, plus a total ban on repatriation of profits, which forced foreign companies to reinvest in the country, resulting in diversification of production. Most of the basic infrastructure was developed during the UDI period, including a system of paved roads, railway links to all the major mining areas, and an electricity grid covering most of the country. The average growth for the period was 4.9% (Table 1).

After Zimbabwe attained independence in 1980, the government maintained most of the economic controls and restrictions that were in place during the UDI period, on the basis of their perceived potential for achieving growth and equity. However, because companies continued to experience foreign-exchange shortages, the government partially eased foreign-exchange restrictions for meeting verified export orders first through an Export Revolving Fund (ERF) in 1983, and later through an Export Retention Scheme (ERS) in 1989, and an Open General Import Licence (OGIL) in mid 1990.

Repatriation of domestic assets owned by foreign companies and individuals continued to be restricted during the socialist period. Nevertheless foreign finance played an important role in the early 1980s, but was later replaced by domestic purchases of medium and long-term government bonds, often at low or negative real interest. Firms could purchase government bonds with maturity of 12 years and individuals with maturity of and 20 years. In both cases they would incur a significant capital-loss in foreign-exchange terms. Later, in a 1-2 year period around mid-1987, the government allowed repatriation of funds to foreign firms who divested under certain conditions that included a discount on book value of 70% or more. There were strict controls on both new foreign investments and foreign borrowing, enforced through a system of committee approval.

There were some years of impressive growth after Independence at the beginning of the socialist period, and average growth for the period was 5.2%. Nevertheless, economic performance deteriorated; growth had fallen to 1 % by 1990. Economic growth had usually been related to good agricultural seasons, as in the years 1980-1981 and 1984-1985.

Table 1: Zimbabwe average annual GDP growth rates, by period, 1965-2002

UDI (1965-79)	Socialist (1980-90)	Adjustment (1991-99)	2000	2001	2002
4.9	5.2	2.5	-4	-7.3	-12

Source: computations from Zimbabwe Central Statistical Office Reports.

Due to the negative economic effects of the socialist policies, economic reforms were introduced in February 1991 under the economic-structural adjustment programme (ESAP), intended to reduce direct controls and let the economy utilise indirect market based methods of resource-allocation. The measures adopted included deregulation of the economy, trade liberalisation, public-enterprise reform, and fiscal and monetary policy-reforms (Go Z, 1991).

Increased availability of foreign-exchange during the structural-adjustment periods enabled the mining industry to replace some aging and obsolete equipment, but the economic reforms also brought about an escalation of production costs, as wages and costs of raw materials increased. This forced major producers to streamline operations by laying off workers, selling or shutting down unprofitable marginal mines, and reducing production to remain viable.

Despite the launching of the ESAP the economy continued to perform badly at first, with a growth rate of only 1.8% in 1991. However, growth improved to 8.1% in

1996, at the end of the first phase of structural adjustment. The economy then grew by 4% in 1997 and by 2% in 1998, when a year behind schedule, the government launched a second structural-adjustment program, ZIMPREST also targeting an economic growth rate of 5%. But no growth was achieved and in fact the economy continued to decline, though posting 2.5% average annual growth for the period.

During the adjustment-period the government actively promoted development of the mining industry by providing free geological, metallurgical, and advisory engineering services to those mines that did not have their own expertise. Technical support, management services, plant-hire schemes, and advice were also given through the Zimbabwe Mineral Development Corporation (ZMDC), together with some local and international non-governmental organizations that were made responsible for the organized growth of a small-scale mining centre. In addition the government partially funded an Institute of Mining Research that gives advice and assistance to mineral producing companies.

The second structural adjustment program, ZIMPREST, was abandoned in February 2000 when the government announced a Millennium Economic Recovery Programme (MERP). MERP was a domestic version of the structural adjustment advocated by the World Bank and IMF, set out as an 18-month programme encompassing a broad range of short-term measures designed to arrest economic decline and stabilise the economy. It was structured to achieve both short and long-term macro-economic stabilisation and economic recovery.

MERP was intended to specifically attract external investment into mining and increase investment and to mineral-based manufacturing; to re-capitalise the Mining Industry Loan Fund to meet increased demand for credit from small and medium-size mines; to speed up the privatisation of mining parastatals; to give greater financial support to research and development institutions and other services supporting investment in the mining sector; and to enforce environmental laws and encourage environmental best practices.

The World Bank and IMF-type reform programs were abandoned in October 2001 when the President declared them dead and stated that Zimbabwe would re-introduce price controls and take over any businesses that would close due to the controls. An agro-based recovery programme was adopted, but it failed to revive the economy, with GDP growth rates declining to a low of –12% in 2002.

Nevertheless, economic reforms had improved access to new technology in Zimbabwe's mining sector. But threats of mine invasions, depressed world mineral-prices, high borrowing costs, and foreign-exchange shortages had increased the uncertainty and deterred private investment.

The growth rate of the economy during the 1980-1990 decade was above that of South Africa and Sub-Saharan Africa (Table 2). Later on the growth of the economy has not been impressive and did not meet the structural-adjustment targets of the 1990s and was below that of South Africa and Sub-Saharan Africa.

Table 2: Average annual percentage growth of real GDP, Zimbabwe, South Africa, and Sub-Saharan Africa

	1980–90	1990–2001
Zimbabwe	3.6	1.8
South Africa	1.0	2.1
Sub-Saharan Africa	1.6	2.6

Source: World Development indicators (World Bank).

3. Ownership structure

The mining sector is oligopolistic, heavily dominated by foreign owners and the government intends to reverse the situation, and to indigenise the economy in general. There are also some very small producers such as gold panners and chrome mining co-operatives. Through companies such as ZMDC and the Zimbabwe Iron and Steel Company (ZISCO), the government is actively involved in the extraction and processing of minerals. In some cases ZMDC has taken over collapsing companies. Through the Minerals Marketing Corporation of Zimbabwe (MMCZ) the government is involved in the external marketing of most minerals except gold.

The industry is thus highly concentrated overall. A few major domestic mining companies and some large multinational companies undertaking relatively large-scale mining operations produce most of the mineral output of Zimbabwe. For example a single company produces 30% of Zimbabwe's gold output. Refractory ores containing gold are treated at a company whose ownership was recently transferred from the government to a subsidiary company of the Reserve Bank of Zimbabwe.

The chromium industry of Zimbabwe is composed of large companies vertically integrated, from chromite mines to ferro-chromium production. Only two major companies produce ferro-chrome, and they are also involved in the extraction process. There are a number of small independent chromite mines, operated by co-operatives, and others operated independently but on behalf of the large vertically-integrated companies.

Three major companies operate in the copper industry, partially owned by the government through ZMDC. The government also partially owns ZISCO, which operates some iron mines and processes the mineral. A single company produces asbestos.

As part of restructuring, the government intends to partially privatise some of its mining interests. The government also encourages the indigenisation of mining companies through the purchase of interests in mining operations, mainly through ZMDC.

In February 1999 the government launched an indigenisation policy-framework promoting increased investment in the economy by black nationals (GoZ, 1999). The government set up a National Investment Trust to gradually acquire and warehouse investment portfolios in privatised enterprises for sale to indigenous people as unit trusts. In addition, the government required a minimum of 10% of the shares of privatising enterprises to be reserved for the previously disadvantaged Zimbabweans. Whether or not the ambitious government programme will succeed depends on the profitability of the mining operations. Historical performance of the sector can be used to assess the possibilities that lie ahead for the sector.

4. A comparison of the mining, manufacturing, and agricultural sectors

Mining has been declining in importance in Zimbabwe. In 1999 it employed 4.5% of the labour force and produced 4% of the country's GDP, while earning 30% of the country's foreign-exchange. Two decades earlier, in 1979, mining employed 7.1% of the labour force and produced over 10% of Zimbabwe's GDP. The sector has been adversely affected by a hostile domestic political environment, declining worldwide commodity prices, shortages of imported materials and spare parts for use in mineral extraction, and rising energy costs.

Mining output went up absolutely during the period 1964-76, fell during 1977 –83, and started to go up again slowly thereafter. But manufacturing output has been rising steadily with downturns only during 1975-1979 and 1985. Thus the percentage contribution of mining to GDP, at factor cost, declined from an average of 9% during the UDI period to 6% during the structural-adjustment period (Table 3). By contrast manufacturing has held almost constant at 23-24%. Agriculture has also declined slightly although its contribution fluctuated heavily year-to-year in response to weather.

Table 3: Percentage contribution to GDP at factor cost by sector and period 1965-99

	manufacturing	mining	agriculture
UDI (1965-79)	23	9	15
Socialist (1980-90)	24	8	14
Adjustment (1991-99)	23	6	13

Source: computations from Zimbabwe Central Statistical Office Reports.

The percentage of manufacturing investment in total gross fixed capital formation rapidly increased from 18% during the UDI period to 30% during the adjustment period (Table 4). By contrast, mining rapidly declined from 13% during the UDI period to 5% during the adjustment period. The continued decline suggests that mining will not be a source of high future growth for the economy, as manufacturing might, although mining might still be able to provide the foreign-exchange necessary for the growing manufacturing sector.

Table 4: Percentage of investment in total gross fixed capital formation

	manufacturing	mining	agriculture
UDI (1965-79)	18	13	10
Socialist (1980-90)	21	8	10
Adjustment (1991-99)	30	5	9

Source: computations from Zimbabwe Central Statistical Office Reports.

In addition to the direct contribution to GDP mining provides essential inputs for some manufacturing industries. In fact, Zimbabwean manufacturing industries based on metallic and non-metallic mineral inputs were created during the UDI period when international sanctions were imposed on the economy. These industries have produced about a quarter of manufacturing output consistently over the last four decades. The extent to which these industries would be adversely affected by a decline in mining output is limited since current exports could be diverted for use by domestic industries.

During the adjustment period mining employed on the average 4.3% of the labour force (Table 5), down form 6.2% during the UDI period. The drop can be explained partly by the decline of the lower contribution of mining to GDP, and also perhaps to some extent by an increase in the capital-intensity of operations. Manufacturing's percentage of the labour force increased correspondingly, while agriculture dropped from 35.3% to almost 26% compensated for by gains in services (not shown).

Table 5: Employment percentage of labour force, by sector and period, 1965-99

	manufacturing	mining	agriculture
UDI (1965-79)	13.7	6.2	35.3
Socialist (1980-90)	16.5	5.5	26.2
Adjustment (1991-99)	15.5	4.3	25.9

Source: computations from Zimbabwe Central Statistical Office Reports.

The average real wage-rate for the mining sector increased from Z$5 thousand under UDI to Z$7.3 thousand during the socialist period, but declined to Z$6.2 thousand during the adjustment period (Table 6). The real wage-rate in mining has consistently been lower than in the manufacturing, but considerably higher in the agriculture. Not only the average annual wage but also the proportion of the value of net output that is paid to workers is generally higher for manufacturing sector than for mining.

Table 6: Annual average real wage-rate by sector and period, 1965-99

	manufacturing	mining	agriculture	total
UDI (1965-79)	7.8	5.0	1.4	5.4
Socialist (1980-90)	9.5	7.3	2.3	7.0
Adjustment (1991-99)	7.4	6.2	1.3	5.1

Source: computations from Zimbabwe Central Statistical Office Reports.

5. Leading mineral products

Zimbabwe has diverse mineral deposits and currently produces about 50 different minerals some of them used as essential raw materials for manufacturing industries. Only a few minerals produce most of the export revenue. The highest percentages of total mineral value come from gold, nickel, asbestos, coal, copper, iron ore, chromite, granite, diamond, limestone, and phosphate though in all cases the level of production is relatively small by international standards[1].

During the period 1965-96 all minerals together consistently contributed about a third of Zimbabwe's total exports, with gold consistently contributing a large and increasing fraction of mineral exports (Table 7). Its contribution had reached 42.4% during the adjustment period. The percentage contribution by all minerals started to go down in 1989, and had reached 30.5% by the end of 1996. The reasons for the decline include diversification of manufacturing into exports as a direct result of structural adjustment.

Table 7: Mineral export as percentage of total exports and gold exports as percentage of mineral exports, by period, 1965-96

	UDI (1965-79)	Socialist (1980-90)	Adjustment (1991-96)
Mineral exports as percentage of total exports	37.9	38.7	30.5
Mineral exports as percentage of total exports (excluding gold)	32.1	29.3	20.2
Gold export as percentage of total mineral exports	22.4	34.2	42.4

Source: computations from Zimbabwe Central Statistical Office Reports. Data for 97-99 are not available.

[1] The scale of operations is defined by the estimated reserves and the production rate that vary with the type of mineral (see Alpan, 1986).

There has generally been a decline in the percentages of world mineral exports accounted for by Zimbabwe (Table 8).

Table 8: Percentage of average of world export

	chrome	copper	gold	asbestos	iron	nickel
UDI (1965-79)	8.64	0.50	0.94	5.44	0.19	1.64
Socialist (1980-90)	5.27	0.25	0.94	3.80	0.21	1.55
Adjustment (1991-97)	5.37	0.13	0.93	4.76	0.27	1.76

Source: calculated from the United Nations Commodity Trade Statistics (COMTRADE) and Industrial statistics yearbook (volume ii).

Demand for gold is generally by speculative activity related to inflationary expectations, the level of real interest rates, and exchange rates. Gold prices are also influenced not only by production but also by IMF auctions and by selling from central-bank stocks. Bullion prices fell when the IMF, the Bank of England, and the Swiss Central Bank reduced gold reserves in 1999–2000. Other countries were also planning to offload bullion stocks, in a process that could decouple major currencies from gold. In more recent years, producer forward-selling and options-trading have dominated in setting prices. Trade in gold derivatives has to some extent replaced the use of gold itself as a political and economic hedge (Weston, 1983); hedging and speculative demand in turn affect market performance (Slade et al. 1993).

The Reserve Bank of Zimbabwe (RBZ) purchases all gold produced in the country at an official gold-support price. Before the introduction of market reforms, the RBZ guaranteed this price to producers irrespective of the world price. When the world price rose beyond the guaranteed RBZ-price, gold-mining firms paid the difference to the RBZ. However, gold and other mining operations in Zimbabwe have recently been adversely affected by the international prices and by the fixed exchange-rate policy adopted by Zimbabwe. The government fixed the currency despite the magnitude of inflation, and thus destroyed the viability of many mines.

Some mineral commodities such as non-ferrous metals are sold under two-tier pricing systems whereby major firms in the industry set their own prices and commodity exchanges set different prices. All buyers of metals sold under the producer-pricing system are direct users of the metals. In contrast, hedgers and speculators can also purchase metals on commodity exchanges. When producer-prices dominate, the major producers set prices for delivery, based to large extent on production costs that are moderately stable (Crowson, 1998). Most copper is sold through annual supply-contracts, but producer-prices tend to operate in protected markets such as Japan. Even when producer-pricing operates, the prices are usually linked to prices at the London Metal Exchange (LME) and to a lesser extent to Commodities Exchange (COMEX).

Most chromium materials are not openly traded but are sold via long-term contracts. Zimbabwe does not export much iron ore since most of it is used to meet domestic demand. World demand for iron ore is relatively high by historical standards but supply-capacity is nevertheless said to be excessive (Papp, 1997). World demand for asbestos is expected to decline because of health and environmental concerns. However, production of asbestos in Zimbabwe has mainly been of the long fibre chrysotile type whose use has not been banned. Worldwide, there is a tendency towards producer-pricing (by the large producers) in fixed contracts, with discounting of the prices depending on quality. Zimbabwean producers are relatively small compared with those in other parts of the world, however. Zimbabwe thus does not have much influence on the prices and might only raise profit margins by increasing productivity and cutting costs

During the period 1965-99, the intensity of use of capital for extraction of minerals was inversely related to the levels of output, suggesting a negative impact of recessions on productivity and possible rigidities in capital use. A possible explanation of the rigidity of capital is its specific nature for particular mining activities, with, high-capacity machines used to produce little output, failing to achieve economies of scale, and contributing to costly levels of energy. A World Bank (1987) study showed that energy use in some industrial sub-sectors in Zimbabwe was 40-80% above that in more highly developed economies. Thus investments in minerals are essentially sunk costs, not easily reversible. Enforcement of hiring and firing regulations has severely restricted labour layoffs and has thus contributed to increased costs of production. Any new investments in mining will have to be appropriate for small-scale deposits in order to attain low operating costs.

6. Summary and conclusions

Mining has been declining in importance, though it has remained the most important foreign-exchange earner for Zimbabwe, as it will likely continue to be for the foreseeable future. Mining investment has been declining compared with other sectors, and it has not been able to maintain similar growth-rates. Creating an enabling environment to attract investment into mining thus presents a great challenge for the government.

Generally the success of investment in mineral extraction will depend on the possibility of reducing extraction-costs using least-cost production processes since Zimbabwe has little influence on the prices of commodities. New technologies offer these possibilities but they usually require new investment.

Since the government of Zimbabwe continues to rely on mining for foreign-exchange and intends to indigenise the sector, it is important to analyse the historical returns that have characterised mining, and find out whether the firms have been able to achieve optimal levels of output, and the effects of uncertainty on investment.

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Paper II

Risk-premia on mining investments in Zimbabwe, 1969-1995

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Abstract: This paper analyses risk-premia on mining investments in Zimbabwe, using the risk adjusted Hotelling model to examine the level of risk-premia required for investment in mining to take place. Empirical results show that a risk-premium higher than 27% was required. For that reason it is highly unlikely that much mining investment will take place in Zimbabwe.

Keywords: risk-premium, investment, consumption, Hotelling, mining, Zimbabwe, natural resource, generalised method of moments.
JEL Classification: O1 O13

1. Introduction

Zimbabwe, like many other less-developed countries, relies to a great extent on mineral exports to generate foreign exchange. The mining sector generates more than 30% of export earnings. In addition, the government places great emphasis on the possibility of using mineral resources to increase the rate of economic growth. Government also created institutions to avert collapse of major mining companies, and promote indigenous and local small-scale mining investments.

The mining sector in Zimbabwe is export-oriented and exposed to world market fluctuations. Humphreys (1995) states that most industrialised countries no longer perceive minerals as strategic, and have reduced concern over the security of their supply. This perception has important implications for some of the factors that have been driving investment in minerals in less-developed countries, including Zimbabwe.

Relative to most other industries, the mining industry is characterised by high risk. Hence, private investment is only attracted into mining when it offers a higher level of return than other investments. In particular, most mineral investments in less-developed countries have been undertaken by multinational corporations. Competition for their investment requires especially attractive returns, including risk-premia for risky ventures. But even local investors, choosing between investing in alternative assets, require risk-premia to invest in risky ventures. Thus, it can be useful to estimate the premia that mineral investments must offer above returns on other investments in order to attract funding.

The purpose of this paper is to analyse risk-premia on mining investments in Zimbabwe. The analysis is based on the assumption that private investment depends on both potential returns and the relative riskiness of assets. We will examine the level of risk-premia derived from the covariance between marginal consumption, and then returns on investment and compare it with that derived by explicitly discounting the value of minerals stocks. Levels of risk-premia have been estimated using the empirical risk-adjusted Hotelling model of efficient resource extraction.

The Hotelling model assumes that, holding other things constant, and in the absence of extraction costs the rate of return for holding a resource-stock should equal the rate of return on other assets. Empirically, either the price of the resource, or the profit derived from its sale is used as a proxy for the rate of return, which is not measurable since the resource has not yet been extracted. A formal test of the rule requires a model that controls for uncertainty, improvement of technology over time, exploration for new deposits, and other factors (Krautkraemer, 1998).

Among others, Young and Ryan (1996) used the consumption-capital asset-pricing model (CCAPM) to capture uncertainty or risk. Their empirical test showed an improvement in the performance of the Hotelling model, due to the inclusion of parameters that capture uncertainty. In this paper the validity of the rule has not been tested, but rather the model has been used as the theoretical basis for assessing the implied risk-premia that could attract investment into the mining sector in Zimbabwe.

Empirical research has failed to support the hypothesis that, in the absence of risk, equilibrium rates-of-return on non-renewable resource assets and on alternative assets will be equalised[1]. Therefore the basic Hotelling model has been modified to allow for risk. In order to include risk, Gaudet and Howitt (1989) and Gaudet and Khadr (1991) showed that the Hotelling-type arbitrage-rule takes the form of the inter-temporal capital asset-pricing rule of portfolio theory. In the risk-adjusted model competitive firms expect the rate of return on an exhaustible resource to differ from the return on alternative assets by a risk-premium associated with the resource asset. Gaudet and Howitt (1989) developed a two-period discrete-time version of the risk-adjusted Hotelling rule in the context of a discrete time CCAPM[2]. Gaudet and Khadr (1991) developed the rule for the continuous-time case. Young and Ryan (1996) applied a two-period discrete-time model to Canadian data and found significant risk-premia for some mineral commodities.

In this paper the Young and Ryan model has been modified by discounting the value of mineral stocks. It will be shown that the level of risk-premium required when there

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[1] Gaudett and Howitt (1989) emphasise that the Hotelling rule is just one equilibrium condition in a complete model where rates of return are determined by technological conditions.
[2] The CCAPM has been studied extensively in the finance literature; see Fama (1991) for a survey.

is explicit stock-discounting is higher than that derived using the basic CCAPM. Mineral stock-discounting was included since depletion of mineral stocks increases extraction costs. Further, the investor must be compensated for uncertainty in deposit size and the possibility of exhaustion. Hence, risk-averse investors are expected to discount the value of mineral stocks. Cumulative extraction is normally used to capture these effects in models of resource extraction.

The model used for estimation had a non-linear rational-expectations form, capturing rates-of-return on assets and consumer preferences. Early attempts to estimate the model used a linear transformation of the non-linear version. The linearisation is based on strong restrictions on preferences and underlying sources of uncertainty in the economy. Hansen and Singleton (1982, 1983) showed how the non-linear model could be estimated directly. The non-linear model was thus used here in order to avoid the restrictions of the linear transformations. The model yields higher levels of risk-premia than those derived from the consumption-beta capita asset-pricing model (CCAPM) when stock-discounting is not negligible. This is likely to be more relevant for less-developed countries such as Zimbabwe, and would yield more appropriate levels of risk-premia.

This paper shows that the levels of risk-premia required to attract investment in mineral extraction in Zimbabwe, and in most African countries, is in general high. Through the discounting of mineral stocks, the modified model used captures the higher levels of risk-premia that tend to characterise investments in less-developed countries. The policy implication is that, at the levels of risk-premia characterising current investment in minerals, it is unrealistic to expect more investment to be forthcoming if uncertainty is high.

The next section reviews the mining sector in Zimbabwe. Section 3 outlines the model and Section 4 presents the data. Section 5 describes the estimation technique and Section 6 presents the results. Section 7 summarises and draws conclusions.

2. The mining sector in Zimbabwe

Mining in Zimbabwe has been declining in importance. By 1999 the sector produced only 4% of the country's gross domestic product (GDP) and employed only 4.5% of the formal-sector labour force. Two decades earlier, in 1979, mining produced over 10% of the country's GDP and employed 7.1% of the formal-sector labour force. The sector has been adversely affected by a hostile domestic political environment, declining commodity prices, shortages of spares parts and other imported materials for use in mineral extraction, and rising energy costs.

During the study period the average rate of return for minerals was declining, a trend not affected by three distinct policy-regimes that the economy passed through. During the period 1965-79 international sanctions were imposed on the economy; the country then attained independence in 1980 and adopted socialist policies through 1990; due to the negative economic effects of the socialist policies, economic reforms were introduced in 1991. These changes did not affect the downward trend of returns. During the sanctions period South Africa was the conduit for mineral trade, so sanctions had little effect. The main feature of the mining sector after independence was direct investment by the government. The major regime shifts have thus merely changed the ownership structure of mineral investments, rather than the returns on these investments. The reason is probably that the market for the minerals has not changed and that, each time, the new owners have maintained supply.

In spite of the decline, Zimbabwe continues to rely heavily on the export revenues generated from minerals and mineral-related goods. Moreover, available geological and engineering studies for Zimbabwe confirm the availability of ores that could be economically exploited using current technology; see for example the geological surveys of Zimbabwe, GoZ (2000).

Strongman (1994) argued that Zimbabwe has an attractive mineral potential warranting increased exploration expenditures by the private sector. He also noted that investors require competitive terms and conditions, including assurances that the investment environment will be stable.

Current mining laws in Zimbabwe allow easy acquisition of title and security of tenure. However, mining claims that confer on their holders the exclusive right to mine are subject to annual inspection, with risk of the licence being revoked if the mines are not being worked. Mining claims that are worked continuously do not have expiry dates, but companies that fail to keep their operations in production run the risk of having their licences revoked. Consequently, it is sometimes argued that mining firms can be shut down and nationalised[3].

Even now, the government actively participates in production of some minerals and in supervision of sales. In 1982 the government formed the Zimbabwe Mining Development Corporation (ZMDC) so it could participate directly in the mining sector and save companies that were threatening to close. The government also encourages indigenisation of mining companies through the purchase of shares of the ZMDC. In February 1999 the government launched an indigenisation policy-framework promoting increased investment in the economy by black nationals (GoZ 1999). The government set up a National Investment Trust to gradually acquire and warehouse investment portfolios in privatised enterprises for sale to indigenous people as unit trusts. In addition the government required a minimum of 10% of the shares of privatising enterprises to be reserved for the previously disadvantaged Zimbabweans.

The drive to increase local ownership was partly due to government mistrust of existing largely foreign-owned firms, but it also had political and economic reasons. On the political front, the government wanted to please the electorate. On the economic front, the government and indigenous investment organisations blamed multinational corporations for extracting huge profits from mining investments without reinvesting.

Except for gold, external marketing of minerals is done through the Minerals Marketing Corporation of Zimbabwe (MMCZ). Gold is sold through the Zimbabwe

[3] Following forced acquisitions of white owned farms in the country the President of Zimbabwe, Robert Mugabe, threatened nationalize several industries, including foreign-owned mines. Formally, all mining activities are conducted under the Mines and Minerals Act [Chapter 165], which ensures security of tenure from exploration through to mining and production.

Reserve Bank. In 2001 the Reserve Bank launched the Gold Mining and Minerals Development Trust in order to, among other things, improve the production of gold and other minerals that have the potential to contribute significantly to economic growth and employment creation in Zimbabwe. The trust is intended to mobilise financial resources for lending to gold miners and producers of other minerals.

Government efforts, and indeed those of indigenous organisations to increase investment in mining, will not work if key issues of concern to investors are not addressed. It is important to analyse how investment in mining evolved, and to examine the risk-premia that might be able to lure investment into the sector. The next section outlines the model used here for doing so.

3. The model

3.1 The Young-and-Ryan model

The basic Hotelling model can be expressed as $\frac{\Delta\lambda}{\lambda} = \rho$ where λ is the return on a resource asset and ρ is the rate of return on an alternative asset. The model states that the rate of return on the resource should be equal to the rate of return on the alternative asset. Young and Ryan (1996) included a risk-premium such that $\frac{\Delta\lambda}{\lambda} = \rho + risk\ premium$. Because the risk-premium is taken into account when making investment decisions, the equilibrium rates-of-return may not be equalised with the difference based on the perception relative risk. It is assumed that the market is efficient.

The Young-and-Ryan model assumes competitive equilibrium in the production and consumption of two goods, one a non-renewable resource, and the other a composite commodity. Production and consumption take place over a two-period horizon. Producers take the prices of the goods as given in making their extraction or production decisions. Efficient extraction and production conditions generate rates of return on the assets used in production. Consumers are assumed to have common information, as well as identical preferences measured by their utility functions.

Taking returns on assets as given the consumers optimally invest in the different assets depending on their preferences over portfolio allocation.

In the first period the composite commodity is either consumed or accumulated in the form of risk-free bonds or of physical capital. In the second period the remaining assets are consumed. It is assumed that the return on the risk-free bond is certain, but that the returns on the capital and resource assets are uncertain due to technological shocks that affect the second-period production-function.

A representative consumer chooses investment in risk-free and risky assets, and the rate of extraction of the non-renewable resource, in order to maximise inter-temporal utility associated with an inter-temporal feasible-consumption stream. In equilibrium the competitive economy must satisfy the first-order condition for the optimisation problem

$$\max_{b,k,x} V(C_1, C_2) = \left[C_1^{\gamma} + \beta \left(E_1 \left[C_2^{\delta} \right] \right)^{\gamma/\delta} \right]^{(1/\gamma)}$$

such that

$$\begin{aligned} & C_1 = G(K - zx, x) - k + \rho B - b \\ & C_2 = F(K + k - z(X - x), (X - x), \varepsilon) + K + k + (1 + \rho)(B + b) \qquad (1) \\ & 0 \neq \delta < 1 \qquad 0 \neq \gamma < 1 \qquad \beta < 1 \\ & 0 \leq x \leq X \qquad 0 \leq k \leq K \end{aligned}$$

where V is inter-temporal utility; C_i is consumption in period i (i is 1, 2); β is a discount factor that is used to calculate the rate of time preference as $\beta^{-1} - 1$; γ captures elasticity of substitution that is defined as $1/(1 - \gamma)$;[4] δ is the risk-aversion parameter where $(1 - \delta)$ is relative risk aversion.[5] This specification implies risk-

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[4] Elasticity of substitution is defined by $\sigma_i = \dfrac{d \ln\left(\frac{C_{t+1}}{C_t}\right)}{d \ln\left(\frac{V_{C(t+1)}}{V_{ct}}\right)}$.

[5] The $(1 - \delta)$-mean (or constant relative-risk-aversion expected-utility) specification for U is given by $U(C) = [EC^{\alpha}]^{1/\alpha}$, $0 \neq \alpha < 1$. When $(1 - \delta) = 1$, the natural logarithm is used, and division by δ is necessary when

preference if $(1-\delta)<0$; risk-neutrality if $(1-\delta)=0$; and risk-aversion if $(1-\delta)>0$. E is the expectations operator, G is the first-period production-function, F is the second-period production-function, X is a non-renewable resource stock, x is the extracted resource, z is unit extraction-cost, B is the number of risk-free bonds, b is the rate of accumulation of bonds, ρ is the rate of return on the risk free bond, K is capital, k is the rate of investment in capital, and ε is a random technological shock in the second-period production-function.

Intertemporal utility $V(C_1, C_2)$ is defined using the infinite-horizon model developed by Epstein and Zin (1989, 1991) as an extension of Kreps' and Porteus' (1978) model of consumption[6]. The model incorporates time preference and risk aversion in order to capture uncertainty in the second-period consumption, C_2. Risk aversion and substitution are represented by different parameters. The function is homogeneous of degree one. It is concave when $\gamma \leq 1$ and convex when $\gamma \geq 1$.

The first-order conditions for the optimisation problem are given by the Euler equations

$$C_1^{\gamma-1} - \beta^* E_1\left(C_2^{\delta-1}\right)(1+\rho) = 0 \tag{2.1}$$

$$C_1^{\gamma-1} - \beta^* E_1\left(C_2^{\delta-1}\right)(1+r) = 0 \tag{2.2}$$

$$C_1^{\gamma-1}.\lambda_1 - \beta^* E_1\left(C_2^{\delta-1}\right)\lambda_2 = 0 \tag{2.3}$$

$$\beta^* = \beta\left(E_1\left(C_2^{\delta}\right)\right)^{(\gamma/\delta)-1} \tag{2.4}$$

where,

$r = F'$

$\lambda_1 = G' - z\,G'$

$\lambda_2 = F' - z\,F'$

$\delta < 0$.

[6] Epstein and Zin (1991, p. 265) show the aggregator-function that generalizes the recursive structure for lifetime utility. Intertemporal aggregation is encoded in the aggregator-function. The expected-utility formulation has an explicit infinite time-horizon when $\alpha = \rho$; thus $U_t = \left[(1-\beta)E_t \sum_{j=0}^{\infty} \beta^j \tilde{c}_{t+j}^{\alpha}\right]^{\frac{1}{\alpha}}$ when β is appropriately redefined (page 266).

r = return on the capital asset,

λ_i = return on the resource in period i (i = 1, 2).

Thus in a competitive economy, the market rate-of-interest is set such that the return on the capital asset r, and the first and second-period returns on the resource, λ_1 and λ_2, will be equal, respectively, to their marginal products F', $G' - z\,G'$, and $F' - zF'$.

The equations can be manipulated to give

$$E\left(\frac{\Delta\lambda}{\lambda}\right) = E(r) - \frac{\operatorname{cov}\left(C_2^{\delta-1}, \frac{\Delta\lambda}{\lambda}\right) - \operatorname{cov}\left(C_2^{\delta-1}, r\right)}{E\left(C_2^{\delta-1}\right)}$$

$$= E(r) - \frac{\operatorname{cov}\left(C_2^{\delta-1}, \frac{\Delta\lambda}{\lambda} - r\right)}{E\left(C_2^{\delta-1}\right)} \qquad (3.1)$$

$$E\left(\frac{\Delta\lambda}{\lambda}\right) = \rho - \frac{\operatorname{cov}\left(C_2^{\delta-1}, \frac{\Delta\lambda}{\lambda}\right)}{E\left(C_2^{\delta-1}\right)} \qquad (3.2)[7]$$

$$\frac{\Delta\lambda}{\lambda} = (\lambda_2 - \lambda_1)/\lambda_1 \qquad (3.3)$$

where E is again the expectations-operator, and $\frac{\Delta\lambda}{\lambda}$ is the rate-of-return on the resource. The expected rate-of-return on the resource asset, $E\left(\frac{\Delta\lambda}{\lambda}\right)$, will differ from the (expected) rates-of-return on the alternative asset when there is uncertainty. The differences are the risk-premia associated with the resource asset given by $-\frac{\operatorname{cov}\left(C_2^{\delta-1}, \frac{\Delta\lambda}{\lambda} - r\right)}{E\left(C_2^{\delta-1}\right)}$ and $-\frac{\operatorname{cov}\left(C_2^{\delta-1}, \frac{\Delta\lambda}{\lambda}\right)}{E\left(C_2^{\delta-1}\right)}$, corresponding to the risky and the risk-free alternative assets, respectively.

Whether the expected return on the resource asset exceeds or falls short of the expected return on the capital assets depends on the relative riskiness of the assets. A

[7] Equation (3.2) does not have a second covariance-term because ρ is non-stochastic and $\operatorname{cov}\left(C_2^{\delta-1}, \rho\right) = 0$. Cov($a$, $b - c$) = cov(ab) – cov(a, c).

negative correlation implies that, in periods of low consumption, the return from the resource asset will be low. In that case the resource asset must offer the investor a risk-premium of at least $-\frac{\operatorname{cov}\left(C_2^{\delta-1}, \frac{\Delta\lambda}{\lambda}\right)}{E\left(C_2^{\delta-1}\right)}$ above the certain return from the risk-free asset. If there is zero covariance between the level of consumption and the rates-of-return, then there is a level of return that make investors indifferent between investing in mining and in alternative assets. In that case the Hotelling hypothesis of returns being equalised holds.

Equation (3.3) was used to calculate individual rates of return on minerals from the price of each mineral. Because the rate of return is not observable, Young and Ryan (1996) and others have suggested that one can use either the rate of change of the price of the natural resource, or the rate of change of profits, as a proxy. Using profits rather than prices gives a better proxy, since the marginal extraction-cost is not zero. Thus $\lambda = p - C_q$ where p is the price of the resource and C_q is the marginal extraction-cost. It is clear that if the marginal extraction cost were zero then $\lambda = p$ and $\frac{\Delta\lambda}{\lambda} = \frac{\Delta p}{p}$ so that the rate of change in prices could be used to estimate the shadow price, p. Since, the extraction cost is sometimes unobservable it is assumed to be zero. Assuming zero marginal cost would have implications on the value of the unit-cost of extraction, z, used in the utility function; would imply that unit extraction-costs fall when production increases. However, both profit and price variables were used in our estimation in order to check the consistency of results.

One would expect an increase in production to reduce unit costs and to increase consumption and utility. When δ is large, individuals dislike risk and want consumption in different states to be roughly similar. One would expect the estimates of risk-aversion, δ, to have lower values and the discount rate, β, to have higher values, when prices are used rather than profits to measure the rate of return on the resource.

3.2 The modified model

Young and Ryan (1996) used the discount-factor, β, that households apply to the utility derived from future consumption, for all the estimated equations. Increasing β leads consumers to save more. The interaction between the discount-factor used by consumers and the effect of mineral stock-depletion is not specified in the Young and Ryan (1996) model.

The effect of stock-depletion can be incorporated by adjusting the estimate of β in the equations. Thus for mineral investments we expect the estimated value of β to be affected by the perceived levels of extractable mineral resources. There is usually a gradual reduction of ore grades and a rise in extraction costs, due to depletion of mineral stocks, which may become exhausted, and the investor must be compensated for the uncertainty. This means that risk-averse investors discount stocks so as to remain on the safe side.

In cost-functions cumulative extraction is normally used to capture these effects, which can also be captured directly by applying a discount rate on the stock of mineral resources. Thus we can distinguish these rates by assuming that the value of the remaining resource-stock is discounted at a rate ψ with an absolute value lying between 0 and 1. A value of ψ close to zero reduces second period consumption by a large margin. We can adjust the second-period production-function such that the stock-variable $X - x$ is discounted by the factor ψ. As a result the second-period consumption-level is defined by the modified equation

$$C_2 = F\,(K + k - z\psi\,(X - x),\ \psi(X - x)\,,\ \varepsilon\,) + K + k + (1 + \rho)(B + b\,)$$

The first-order derivatives with respect to b and k remain unchanged. However, the first-order derivative with respect to x becomes

$$C_1^{\gamma-1}\lambda_1 - \beta\left(E_1\left(C_2^{\delta}\right)\right)^{(\gamma/\delta)-1} E_1\left(C_2^{\delta-1}\right)\lambda_2\psi = 0$$

By setting $\psi = 1 + \frac{\Delta\beta}{\beta}$ and manipulating the first-order conditions as before, we obtain

$$E\left(\tfrac{\Delta\lambda}{\lambda}\right) = E(r) - \frac{\Delta\beta}{\Delta\beta + \beta}(1 + E(r)) - \frac{\beta}{\Delta\beta + \beta}\frac{\operatorname{cov}\left\{C_2^{\delta-1}, \left(\frac{\Delta\lambda}{\lambda} + \frac{\Delta\beta}{\beta}\frac{\Delta\lambda}{\lambda} - r\right)\right\}}{E\left(C_2^{\delta-1}\right)} \tag{4.1}$$

$$E\left(\tfrac{\Delta\lambda}{\lambda}\right) = \rho - \frac{\Delta\beta}{\Delta\beta + \beta}(1 + \rho) - \frac{\operatorname{cov}\left\{C_2^{\delta-1}, \frac{\Delta\lambda}{\lambda}\right\}}{E\left(C_2^{\delta-1}\right)} \tag{4.2}$$

The above equations replace equations (3.1) and (3.2),[8] and indicate that the rate-of-return on the resource asset is greater by $\frac{\Delta\beta}{\Delta\beta + \beta}(1 + E(\rho))$ if $\Delta\beta < 0$ and $\Delta\beta < \beta$, without taking into account the covariance term. This is the effect of stock-discounting rather than expected-utility discounting.

The overall effect is a reduction in the estimated value of β, implying less investment in the mining. The specification allows us to estimate different parameters for discount-rates for financial and mineral assets. Hence, in addition to the risk-premium derived from the covariance of marginal consumption, we have another component introduced through discounting of mineral stocks. The equations indicate that higher levels of risk-premium are required when $\Delta\beta < 0$. The above have been used to estimate the modified model using the generalised method of moments (GMM) that was developed by Hansen (1982) and by Hansen and Singleton (1982).

4. Data

The model was estimated using data on the net price of the resource, λ; the rate of return on the alternative asset, ρ; the rate of return on the capital asset, r; and

[8] In the case where the alternative asset is risk-free then there is no correlation between the alternative asset and the marginal utility of consumption, so that $\operatorname{cov}\left\{C_2^{\delta-1}, \left(\frac{\Delta\lambda}{\lambda} + \frac{\Delta\beta}{\beta}\frac{\Delta\lambda}{\lambda} - \rho\right)\right\} = \operatorname{cov}\left\{C_2^{\delta-1}, \left(\frac{\Delta\lambda}{\lambda} + \frac{\Delta\beta}{\beta}\frac{\Delta\lambda}{\lambda}\right)\right\}$ $= \operatorname{cov}\left\{C_2^{\delta-1}, \frac{\Delta\lambda}{\lambda}\right\}$ since $\operatorname{cov}\left(C_2^{\delta-1}, \rho\right) = 0$.

consumption per capita, *C*. In addition data was needed to form an instrumental variable-set in order to estimate the model using GMM. The instrumental variables constitute the representative consumer's information set. The consumer forms preferences based on the variables in the information-set.

The choice of the number of instrumental variables is limited by degrees-of-freedom considerations. Hence only those that other researchers have considered to be most important were included: real GDP per capita, the consumer price-index, second-period lags of all returns, and consumption per capita.

Proxies were constructed for the return on the resource, capital, alternative assets, and the remaining variables, using data obtained from Government Central Statistical Office (CSO), the Reserve Bank Of Zimbabwe Quarterly, and International Financial Statistics (IFS). The data is believed to be reliable and reasonably accurate for making meaningful inferences. It would have been appropriate to use firm-level data but that was not available.

Annual aggregate data was used for chrome, copper, gold, asbestos, and iron for the period 1969 to 1995; after 1995 the available data is more aggregated and the various minerals are not distinct. In 1995 the five minerals accounted for over 80% of mining output.

The model requires data on the rate of return on the risk-free alternative asset, ρ. The asset must give a certain return, and should not be affected by technological shocks that affect second-period consumption. Government bonds and treasury bills are normally considered to be relatively risk-free and give a certain nominal rate of return based on a fixed cash payout, including interest payments and repayment of the principal at maturity.

Hence, returns on investments in government bonds and treasury bills were used as proxies for ρ. Data was available for various periods of maturity. For government bonds we use a three-year maturity period with a yield curve different from that of the stock market. We use the two types of assets to check consistency or results. In the case of returns on the alternative assets we calculate real rates of return by subtracting the inflation rates from the nominal.

Another variable used in the model is the return on the capital asset, *r* which is set equal to the marginal product of capital in the second-period production-function, $F^{/}$. The production function is affected by technology shocks and uncertainty. The return is therefore uncertain and more risky than that on government bonds or treasury bills. Thus, *r* is the return on the risky alternative asset. Stock bought on the stock market is assumed to be having a more uncertain return than the risk-free alternative assets.

Data from the Zimbabwe Stock Exchange was used as a proxy for *r*, on the assumption that dealings on the Zimbabwe Stock Exchange rather than on international exchanges would more appropriately reflect the expected returns on investments in Zimbabwe, since international participation is limited by government policies on foreign investment.

Zimbabwe's stock market is relatively small with 65 companies listed and trading is quite thin, and most small companies are closely held. In September 1996 the government opened both the stock market and the money market to limited foreign-portfolio investment. Because these limited foreign-portfolio investments include Zimbabwe’s largest companies, they account for 40-50% of industrial output.

Earnings yields and dividend yields on stock bought on the Zimbabwe stock exchange were thus used to represent the return on the risky alternative asset. Earnings per share are defined as the last reported profit or loss divided by the total number of ordinary shares held by shareholders. Earnings yield is then calculated as earnings per share for the most recent twelve months divided by the current price per share. It is the reciprocal of the price-earnings ratio that gives the amount of earnings purchased for every dollar’s worth of stock.

Dividends are the part of a company’s net profit that is distributed to shareholders as a cash reward for investing in the company. Dividend yield is calculated as the ratio of the dividend to the last sales price for the shares.

Usually the per-capita level of consumption of durable goods in the economy is used for the consumption variable in CCAPM-estimation. The data available aggregated consumption of both durable and non-durable goods, thus overstating the level of consumption appropriate for estimation. However, it assumed that this would have negligible effect on the parameter estimates.

To measure risk-premia the model uses covariances between the risk-free rates of returns and the rates of return to various mineral investments with the growth-rate of real per-capita consumption. The average rate of per-capita consumption-growth measured by $\log(C_t) - \log(C_{t-1})$ was 0.009 over the sample period.[9]

During the study period, the covariances of per-capita consumption-growth with real stock-market returns was 0.0031 and 0.0043, using the dividend-yield and earnings-yield, respectively (Table1). These are both greater than the covariance of per-capita consumption-growth with the real return on Treasury bills (0.0007) and Government bonds (0.0023).

Table 1: Covariance of real per capita consumption growth and rates of returns to assets, 1969-95

	chrome	copper	gold	asbestos	iron
real profit	0.0080	0.0007	0.0060	-0.0005	0.0113
price	0.0017	0.0054	0.0041	-0.0040	-0.0033

	Treasury bills	Government bonds	Dividend yield	Earnings yield
real return	0.0007	0.0023	0.0031	0.0043
nominal return	-0.0027	-0.0010	-0.0003	0.0009

Source: computations from Zimbabwe Central Statistical Office Reports.

Since stock-market stocks returns had a higher covariance with consumption-growth investors would be expected to have seen them as a poorer hedge against consumption-risk, and thus stock-market shares would be expected to have earned a higher average return.[10] However, this was not the case in Zimbabwe during the study

[9] According to Benartzi and Thaler (1995) the link between stock-returns and consumption is quite tenuous, because most people hold no stocks outside their pension wealth. Most pensions are of a defined-benefit variety such that a fall in stock prices is inconsequential to the beneficiaries. Most of the stock market is owned by three groups of investors: pension funds, endowments, and very wealthy individuals.

[10] Although the risk-free assets have a higher yield than stock that doesn't provide any guaranteed return, the

period, since the real rate-of-return on market stocks was lower than that on bills and bonds.

Although the average nominal rates-of-return were positive for shares bought on the stock exchange, as well as for investment in treasury bills or government bonds the real rates-of-return were all negative, due to high rates of inflation in Zimbabwe. The average rates of return were -0.02 on treasury bills, -0.01 on government bonds, and -0.04 on shares from the stock market (Table 2). Though negative, the average returns on treasury bills and government bonds were thus higher than those on market stocks.[11]

Table 2**:** Average rates of return, 1969-95

	chrome	copper	gold	asbestos	iron
real profit rate	0.10	0.02	0.02	0.02	0.08
nominal profit rate	0.18	0.11	0.14	0.13	0.16
price rate less inflation	0.02	0.07	0.11	0.04	0.24
price rate	0.15	0.20	0.23	0.16	0.37

	Treasury bills	Govt. bonds	Dividend yield	Earnings yield
average real rate of return	-0.02	-0.01	-0.04	-0.05
average nominal rate of return	0.10	0.12	0.08	0.07

Source: computations from Zimbabwe Central Statistical Office Reports.

With the exception of copper and asbestos, the covariances between real per-capita consumption growth and the rates to mineral investments using real profits were greater than those of the alternative assets (Table 1). The greater covariances imply that investors would have treated investments in chrome, gold, and iron as poorer hedges against consumption-risk. Returns on these investments would normally be expected to be higher.

When rates of return on the mineral investments are measured the using the rate of change of prices, a different picture emerges. In this case it is copper (covariance

results change when growth is taken into account. The value of stock generally increases over time, and will likely have an income stream at a later date that is above that of the risk-free assets.

[11] Negative returns are unusual on international markets, where returns to market stocks tend to be positive. Foreign participation on the bond market is restricted to the primary market and only 35% of invested capital may be invested in them.

0.0054), gold (covariance 0.0041), and asbestos (covariance –0.004), that provide a poorer hedge against consumption-risk. However, since prices do not deduct production costs, they are less informative about returns and hence give a less reliable picture.

Average rates of return on chrome, copper, gold, asbestos, and iron were all positive (Table 2), and all greater than the average real returns on the alternative assets. In addition, except for chrome, the average rate of change of price (real or nominal) was greater than the average rate of change of profits (real or nominal). The result is expected, since marginal extraction-costs are positive.

The major reason for the difference in covariances results from prices and profits variables is that the latter depend on efficiency in the use of capital, labour, and raw materials. The capital equipment in use in most mines is obsolete, and hence world-efficiency levels cannot be achieved but instead firms take advantage of the low wages that characterise the economy. Although Zimbabwe is an important world producer of chrome, copper, gold asbestos, iron, as well as lithium, emeralds, coal, granite, and nickel, mining investments are generally small-scale.

Because returns on mineral investments were higher than those on alternative assets, we would expect more mining investment, unless there is higher risk. Low investment could be due to low risk-premia, not high enough to cover risk and uncertainty. Investment in minerals is risky because of the difficulty of knowing the exact quantity of reserves. In addition, huge initial capital outlays are required and more caution is necessary. Unlike investment in other sectors of the economy such as manufacturing investment in minerals depends on availability of the resource. Uncertainty and risk are captured by $\Delta\beta$. Hence, we would expect some risk-premium to be required for mining investment to take place. Even gold, whose output is traded by the Reserve Bank, has as average rate of return considerably higher than that on treasury bills. Instead of reinvesting in minerals some firms diversify and invest in other sectors in order to protect themselves against high mineral investment risk.

5. Estimation technique

As with most studies that estimate a first-order condition from a dynamic-optimisation problem, the generalised method of moments (GMM) was used to estimate elasticity of substitution (γ), risk aversion (δ), utility discounting factor (β), and the differential in the utility discounting factor due to interaction with mineral resource stock discounting ($\Delta\beta$).

Jagannathan et al. (2002) give an overview of some recent applications of GMM, and Hansen and West (2002) survey its use. The method is explained in appendix 1. GMM estimates unknown parameters by matching population moments or theoretical moments to appropriate sample-moments. There is no need to specify a parametric model for conditional heteroskedasticity or to include any distributional assumptions. The equations need only satisfy the moment conditions. The unknown parameters are estimated by setting the sample-averages of these moment-functions as close to zero as possible. It is not necessary to linearise the model.

Unlike other methods GMM does not require the rates of return or consumption to be drawn from any particular family of distributions, but rather is usually used in a situation where only the Euler equations are available. Other methods such as maximum-likelihood (ML) require the correct distribution of errors to be specified, otherwise the estimators will be biased[12]. In this case the distribution of errors was not known, but GMM avoids the problem. The structure of errors is captured in the variance-covariance matrix. It was assumed that the moment-functions of observable random variables, and the unknown parameters, have zero expectation when evaluated at the true parameter values.

One drawback of GMM is that it is sensitive to the number of moment-conditions and to sample-size even when the model is correctly specified. The asymptotic standard-errors can understate the finite sample-variances, and the distribution of the over-identifying-restrictions test is not approximated well by asymptotic theory in

[12] A correctly-specified ML-estimator yields the asymptotically most-efficient estimators. In addition, when the statistics in the model are sufficient, then ML is the same as GMM. In the model used here the statistics are not sufficient, hence ML would not be the same as GMM.

moderate sample sizes. Altonji and Segal (1996) examined small-sample properties of the GMM-estimator applied to models of covariance-structures and found that, for moments of any order, they are almost always biased downward in absolute terms, because of correlation between the moments used to fit the model and the weight-matrix. Bias arises because sampling-errors in the second moments are correlated with sampling-errors in the weighting-matrix. Problems can be minimised by using robust estimation-methods to estimate the weighting-matrix or to estimate the moments being modelled using prior information about the appropriate weighting-matrix. In addition, iterative or continuous updating of the weighting-matrix that takes advantage of the link between the weighting matrix and the moments being fitted may be used.

An advantage is that the orthogonality-restrictions implied by the Euler equations can be used to identify and estimate the parameters of the utility function. One then uses the parameter-estimates to calculate risk-premia associated with investing in the various minerals.

Equations (3.1), (3.2), (4.1), and (4.2) cannot be estimated directly since they involve the ratio of a covariance to an expected value. Hence the first order conditions (equations 2.1 – 2.4) were used for estimation. In addition the equations were reformulated to capture the rates of return on the five minerals. The final equations estimated were

$$E_1\left\{\frac{\beta^* C_{t+1}^{\delta-1}(1+\rho_t)}{C_t^{\gamma-1}} - 1\right\} = 0 \qquad (5.1)$$

$$E_2\left\{\frac{\beta^* C_{t+1}^{\delta-1}(1+r_t)}{C_t^{\gamma-1}} - 1\right\} = 0 \qquad (5.2)$$

$$E_{3,j}\left\{\frac{\beta^* \psi_j C_{t+1}^{\delta-1}\lambda_{j,t+1}}{C_t^{\gamma-1}\lambda_{j,t}} - 1\right\} = 0 \qquad j = 1,\ldots,5 \qquad (5.3)$$

where $\beta^* = \beta E\left(C_{t+1}^{\delta}\right)^{(\gamma/\delta)-1}$

$$\psi_j = 1 + \frac{\Delta\beta_j^*}{\beta^*}$$

and $j(1,...,5)$ represents the five minerals chrome, copper, gold, asbestos, and iron. The system, therefore, contains seven equations and eight unknowns. The equations in β^* and ψ are not used directly for estimation; β and ψ are derived from the other parameter estimates.

Equations (5.1) – (5.2) can be summarised as

$$Eh_i(\boldsymbol{x}_i, \boldsymbol{b}) = 0$$

where h_i is the ith Euler equation; $\boldsymbol{b} = (\delta, \gamma, \beta)$ is the vector of parameters; and $\boldsymbol{x}_i = (C_t, C_{t+1}, \lambda_{1,t}, \lambda_{1,t+1}, \lambda_{2,t}, \lambda_{2,t+1}, \lambda_{3,t}, \lambda_{3,t+1}, \lambda_{4,t}, \lambda_{5,t+1}, \lambda_{5,t}, \lambda_{5,t+1}, \rho_t, r_t)$ is the vector of endogenous variables in equations (5.1) – (5.3).

A vector of instrumental variables, z_i, was used to specify the orthogonality-conditions such that

$$E(h_i(\boldsymbol{x}_i, \boldsymbol{b})\mathbf{z}_i) = 0$$

Where $z_{i,} = ([\ln(C_{t-1}) - \ln(C_{t-2})], \lambda_{1,t-1}, \lambda_{2,t-1}, \lambda_{3,t-1}, \lambda_{4,t-1}, \lambda_{5,t-1}, \rho_{t-1}, r_{t-1}, gdp_t, cpi_t)$; gdp_t is per-capita gross domestic product, and cpi_t is the consumer price index.

The sample-counterpart used for estimation was

$$g_i(\boldsymbol{b}) = \sum_{t=1}^{T} h_i(\boldsymbol{x}_{i,t}, \boldsymbol{b})\boldsymbol{z}_{i,t}$$

The GMM-estimator is based on the fact that the forecast-error associated with the Euler equations is additive and is uncorrelated with any information available to agents during the planning period. The unknown parameters were estimated by setting the sample-averages of these moment-functions as close to zero as possible, i.e., by setting $h_i(\boldsymbol{x}_{it}, \boldsymbol{b})\mathbf{z}_t = u_{it}$ where u_{it} denotes the ith error-term at time t.

Thus it was necessary to find parameter-values that would make the sample-analogue of the population-restrictions close to zero. Identification and efficiency depend on the set of instruments used, in this case 10 instruments for each of the 7 Euler equations, hence 10 x 7 = 70 orthogonality-conditions. Eight parameters were estimated from the 63 orthogonality-restrictions. Since the number of orthogonality conditions in the model exceeds the number of parameters estimated, some of the conditions were violated.

Since the number of moment-functions was larger than the number of unknown parameters (i.e. over-identified), it was not possible to set the sample-average of the moment-functions exactly equal to zero. Hence the approach was to set a linear combination of the sample-average equal to zero, with the dimension of the linear combination equal to the number of unknown parameters. If the model is true, there should be 70 – 8 = 62 linearly independent combinations of the orthogonality-conditions that ought to be close to zero but are not actually set to zero. Least squares was used to obtain a covariance-matrix that was then used to adjust the estimator. The adjustment to the covariance-matrix accounts for the moving-average aspect of the disturbance and heteroskedasticity.

The *J*-statistic, the minimised value of the objective-function times the number of observations , was used to test the over-identifying restriction. The *J*-statistic is distributed as χ^2 with degrees of freedom equal to the number of instruments times the number of equations minus the number of parameters ((10 x 7) – 8 = 62).

6. Results

Equations (5.1) - (5.3) were estimated using GMM and estimates were obtained for the parameters elasticity of substitution (γ), risk aversion (δ), utility discounting factor (β), and the differential in the utility-discounting factor due to mineral-resource stock-discounting ($\Delta\beta$). The estimates were then used to calculate risk-premia for the cases using the rate-of-change of real profits, or the rate-of-change of real price adjusted for inflation.

Tables 3, 5 and 6 present results for the parameters γ, δ, β, and $\Delta\beta$ and show that all the estimates except elasticity of substitution, γ, are statistically significant at the 1% level. In addition, the calculated *J*-statistics in the last rows of tables 3-5 do not fall in the rejection region. The values are all smaller than the critical value of 43.77 for χ^2 at a 5% level for 62 degrees of freedom. Hence the model-specification need not be rejected.

The first set of results used the rate-of-change of profits as a proxy for the return on the resource-asset (Table 3). The parameter estimates for elasticity of substitution, γ, are not significant. The null hypothesis H_0: $\delta=\gamma$ equating the parameters for risk-aversion and elasticity of substitution can be rejected.

The risk-aversion parameter, δ has statistically-significant values ranging from -2.05 to -1.43. Relative risk-aversion values $(1 - \delta)$ thus range from 3.05 to 2.43. These values are similar to those found in the literature. Using American data Epstein and Zin (1991) obtain relative risk aversion values that lie between 0.72 and 2.45.[13] Also using United States data Weber (2000) obtains a minimum value of 0.001 and a maximum value of 3.41. Young and Ryan (1996) obtain a value of 3.62 using Canadian data and focusing on the mineral lead. Our results are closer to the maximum values than the minimum values for the developed countries, hence imply a higher level of risk aversion for Zimbabwe. Estimates for the discount-factor, β range from 1.99 to 2.32, implying negative rates of time-preference ($\beta^{-1} - 1$) ranging from -0.41 to -0.57.

Epstein and Zin (1991) obtained discount-factor values of 0.996 to 1.01 giving rates of time-preference from 0.004 to -0.01. Weber (2000) obtained discount-factor values ranging from 1.002 to 1.01 giving rates of time-preference from -0.004 to -0.01. Epstein and Zin explained that the negative results were predictable given the general-equilibrium simulations that have been done with these models. Their results

[13] These have been calculated from Tables 2-5 of their paper.

imply that the present value of future consumption (and therefore of wealth) is high which discourages investment.

Table 3: Regression-results using rate-of-change of profits when $\delta \neq \gamma$

Parameter	Treasury Bills		Government Bonds	
	Dividend Yield	Earnings Yield	Dividend Yield	Earnings Yield
Δβ-chrome	-1.80	-1.58	-1.64	-1.26
	(-4.66)	(-6.59)	(-6.12)	(-3.01)
Δβ-copper	-1.14	-0.98	-1.03	-0.73
	(-4.10)	(-4.90)	(-4.68)	(-2.53)
Δβ-gold	-0.52	-0.44	-0.46	-0.36
	(-4.39)	(-5.68)	(-4.71)	(-3.07)
Δβ-asbestos	-0.72	-0.64	-0.60	-0.45
	(-3.44)	(-4.08)	(-4.01)	(-2.69)
Δβ-iron	-2.28	-1.97	-2.05	-1.68
	(-4.79)	(-7.51)	(-6.37)	(-3.75)
δ	-2.06	-1.54	-1.84	-1.43
1− δ	3.05	2.54	2.84	2.43
	(-2.58)	(-2.74)	(-3.88)	(-3.40)
β	2.32	1.99	2.08	1.69
	-4.75	-7.65	-6.38	-3.75
γ	-0.41	-0.25	-0.41	-0.40
	(-0.58)	(-0.51)	(-0.95)	(-1.07)
1/(1 − γ)	0.71	0.80	0.71	0.71
δ = γ (test)				
Objective	0.65	0.63	0.64	0.60
	16.27	15.79	16.12	15.08

Notes;
t-statistics in brackets;
Δ*β*-chrome = change in discount-factor for chrome;
Δ*β*-copper = change in discount-factor for copper;
Δ*β*-gold = change in discount-factor for gold;
Δ*β*-asbestos = change in discount-factor for asbestos;
Δ*β*-iron = change in discount-factor for iron;
1 − δ = relative risk-aversion ;
β = discount-factor ;
1/(1 − γ) = elasticity of substitution;
Objective = Objective function;
Objective x $N = J$ statistic for testing over-identifying restrictions ($N = 25$);
J is distributed as χ^2 with degrees of freedom equal to 10 instruments x 7 equations - 7 parameters to be estimated = 62;
$\chi^2 = 43.77$ (5%)

The discount factors for the mineral resources were obtained using the equation $\beta\left(1+\frac{\Delta\beta_j}{\beta}\right) = \beta + \Delta\beta_j$ where $j = 1, \ldots, 5$ and refers to the jth mineral[14]. As expected, the difference, $\Delta\beta_j$, was negative in all the cases. The minimum values of $\Delta\beta_j$ for chrome, copper, gold, asbestos, and iron were respectively –1.584, –1.137, –0.516, –0.721, and –2.282, while the respective maximum values were –1.264, –0.726, –0.361, –0.453, and –1.675. The respective average values for the discount factors are 0.22, 0.52, 0.78, 0.70, and 0.01 (Table 4). Iron thus has the lowest and gold the highest values.

Table 4: Average discount-factors for mineral stocks

	chrome	copper	gold	asbestos	iron
profits $\delta \neq \gamma$	0.22	0.52	0.78	0.70	0.01
profits $\delta = \gamma$	0.20	0.48	0.81	0.63	0.02
prices $\delta = \gamma$	0.97	0.90	0.88	0.95	0.71

In order to assess the sensitivity of the estimates to different equation-specifications, we also used the rate-of-change of prices as a proxy for the return to the resource-assets. The parameter estimates failed to converge with different values allowed for elasticity of substitution (γ) and risk-aversion parameters (δ). For that reason the model was re-specified and estimated when $\gamma = \delta$. When elasticity of substitution is the inverse of risk-aversion the model reduces to the expected-utility formulation.

Parameter estimates for elasticity of substitution and risk aversion, δ, were not statistically significant in the equations using the rate-of-change of profits (Table 5), but were statistically significant in equations using the rate-of-change of prices (Table 6). However, estimates of the discount rate, β, obtained from equations using the rate-of-change of profits (Table 5), had lower values when risk-aversion and elasticity of substitution were estimated using the same parameter, δ. Again, the values of the discount rate (β) were greater than unity, however, implying negative rates of time-preference.

[14] A dummy variable with a value of 0 for the period 1969-1991, and 1 for the remainder of the period was tried found insignificant.

As before, the estimates $\beta + \Delta\beta_j$ were combined to obtain the appropriate discount-factors; the average values for the various minerals are presented in the second row of Table 4. The values are close to those obtained using equations which allowed for different parameters for elasticity of substitution and risk-aversion, (first row) and they can thus be compared with those obtained from equations using the rate-of-change of price (third row). Their respective values are 0.97, 0.90, 0.88, 0.95, and 0.71 for chrome, copper, gold, asbestos, and iron. As expected the average discount-rates for mineral-stocks were higher when the rate-of-change of price was used as a proxy for the rate-of-return on the resource-asset. Only the average discount-rate for gold had almost the same value when the rate-of-change of either prices or profits was used. A possible reason for this is price-support guarantee-schemes offered by the reserve bank, which ensure that the price is a constant mark-up over costs.

Table 5: Regression-results using rate-of-change of profits when $\delta = \gamma$

	Treasury Bills		Government Bonds	
Parameter	Dividend Yield	Earnings Yield	Dividend Yield	Earnings Yield
Δβ-chrome	-0.801	-0.781	-0.860	-0.859
	(-9.65)	(-10.52)	(-27.21)	(-21.25)
Δβ-copper	-0.515	-0.488	-0.569	-0.577
	(-6.49)	(-6.19)	(-12.03)	(-15.22)
Δβ-gold	-0.204	-0.187	-0.205	-0.203
	(-8.28)	(-5.18)	(-6.21)	(-7.37)
Δβ-asbestos	-0.398	-0.338	-0.381	-0.395
	(-7.63)	(-5.63)	(-9.42)	(-7.56)
Δβ-iron	-1.012	-1.021	-1.017	-1.023
	(-50.09)	(-99.68)	(-110.89)	(-113.94)
δ	-0.420	-0.188	-0.322	-0.506
1− δ	1.420	1.188	1.322	1.505
	(-1.00)	(-0.44)	(-0.92)	(-1.46)
β	1.030	1.042	1.030	1.035
	(75.49)	(152.87)	(141.03)	(132.87)
γ	-0.420	-0.188	-0.322	-0.506
	-0.299	-3.862	-0.866	-1.109
1/(1 − γ)	0,70	0,84	0,76	0,66
δ = γ	-0.299	-3.862	-0.866	-1.109
(test)	(-0.05)	(-0.65)	(-0.24)	(-0.30)
Obj	0.6413	0.5715	0.6625	0.6656
	16.031	14.289	16.562	16.639

See notes in table 3.

Parameter estimates obtained from equations using the rate-of-change of prices are presented in Table 5, and comparable to those in Tables 4. The parameter estimates for the coefficient of risk-aversion (δ) (Table 6) are, respectively, 0.441, 0.462, 0.323, and 0.471 for chrome, copper, gold, asbestos, and iron, values that are similar to those found in the literature. For example, Young and Ryan (1996) obtained a value of 0.369 using price data for lead.

The parameter estimates were used to calculate risk-premia, shown in Tables 7 to 11. The conventional risk-premium relative to the safe asset is derived from the covariance between marginal consumption and returns to mineral assets, as shown in Equation (4.2). A positive covariance implies that in periods of low consumption the returns on the mineral investments are high. A negative covariance implies the opposite.

Table 7 shows the covariances between marginal utility and returns to mineral assets, which were unambiguously positive for iron, and negative for chrome, copper, and asbestos. For gold the covariances were positive when the rate-of-change of profit was used to measure resource return. However, the covariance was negative when the rate-of-change of price was used instead.

Table 6: Regression-results using rate-of-change of prices when $\delta = \gamma$

Parameter	Treasury Bills: Dividend Yield	Treasury Bills: Earnings Yield	Government Bonds: Dividend Yield	Government Bonds: Earnings Yield
$\Delta\beta$-chrome	-0.037	-0.045	-0.023	-0.033
	(-2.89)	(-4.47)	(-1.28)	(-1.98)
$\Delta\beta$-copper	-0.093	-0.093	-0.115	-0.090
	(-3.54)	(-3.15)	(-4.13)	(-4.40)
$\Delta\beta$-gold	-0.129	-0.127	-0.116	-0.136
	(-5.54)	(-4.74)	(-4.43)	(-5.75)
$\Delta\beta$-asbestos	-0.067	-0.060	-0.040	-0.053
	(-4.78)	(-3.36)	(-3.30)	(-3.19)
$\Delta\beta$-iron	-0.285	-0.278	-0.298	-0.312
	(-7.22)	(-5.80)	(-8.16)	(-8.20)
δ	0.441	0.462	0.323	0.471
$1-\delta$	0.559	0.538	0.677	0.529
	(3.91)	(4.04)	(1.85)	(3.15)
β	1.032	1.036	1.020	1.023
	(268.37)	(225.39)	(136.09)	(127.71)
γ	0.441	0.462	0.323	0.471
	(3.91)	(4.04)	(1.85)	(3.15)
$1/(1-\gamma)$	1,79	1,86	1,48	1,89
$\delta = \gamma$	-6.015	1.665	-5.736	-6.282
(test)	(-0.58)	(0.16)	(-0.50)	(-1.27)
Obj	0.5189	0.5133	0.5333	0.6077
	12.973	12.834	13.332	15.193

See notes in table 3.

Table 7: Covariances between marginal consumption and returns to minerals assets

	chrome	copper	gold	asbestos	iron
profits $\delta \neq \gamma$	-0.028	-0.007	0.002	-0.002	0.004
profits $\delta = \gamma$	-0.030	-0.007	0.002	-0.003	0.004
prices $\delta = \gamma$	-0.0001	-0.003	-0.001	-0.0001	0.004

Hence, using Equation (3.2) and considering the sign of the covariance term, the model suggests that investment in iron or gold did not require a risk-premium, whereas a risk-premium may have been necessary to lure investment into the other minerals, because the covariance was negative. However, these covariance levels do not sufficiently explain the risk-premium, since they exclude the covariance of marginal consumption with the alternative assets (Equation 3.1, and Table 8).

Table 8: Covariances between marginal consumption and returns to alternative assets

	Treasury bills	Government bonds	Dividend yield	Earnings yield
	Real Values			
profits $\delta \neq \gamma$	0.0011	-0.0003	-0.0003	0.0008
profits $\delta = \gamma$	0.0011	-0.0003	-0.0004	0.0008
prices $\delta = \gamma$	0.0007	-0.0002	-0.0003	0.0005
	nominal values			
profits $\delta \neq \gamma$	0.0005	-0.0009	-0.0009	0.0002
profits $\delta = \gamma$	0.0006	-0.0009	-0.0009	0.0002
prices $\delta = \gamma$	0.0004	-0.0006	-0.0006	0.0001

These results can be compared to those obtained by Young and Ryan (1996) using Canadian data. They obtained premium-values for lead ranging from 0.0015 to 0.2004; values for other minerals were lower.

Including the appropriate discount-factors in Equation (4.1) yields the results shown in Table 9. Using the rate-of-change of profits ($\delta \neq \gamma$) as a proxy for the rate-of-return on the resource-asset yields the risk-premium and 0.119 for chrome, 0.029 for copper, −0.005 for gold, 0.011 for asbestos, 0.106 and for iron, relative to the 'safe' assets, treasury bills and government bonds. Similar values were found relative to returns on the risky alternative-asset, stock market shares. These values imply respective risk-premia levels of 11.9, 29, −5, 11, and 10.6 percentage points above returns on the safe assets.

Table 9: Risk premia from covariance terms

Relative to 'safe' alternative-asset using equation

$$A = -\frac{\beta}{\Delta\beta + \beta}\frac{\operatorname{cov}\left\{C_2^{\delta-1}, \left(\frac{\Delta\lambda}{\lambda} + \frac{\Delta\beta}{\beta}\frac{\Delta\lambda}{\lambda} - \rho\right)\right\}}{E\left(C_2^{\delta-1}\right)}$$

	chrome	copper	gold	asbestos	iron
profits $\delta \neq \gamma$	0.119	0.029	-0.005	0.011	0.106
profits $\delta = \gamma$	0.063	0.014	-0.003	0.007	0.021
prices $\delta = \gamma$	0.000	0.004	0.002	0.000	-0.005

Relative to risky alternative-asset using equation

$$B = -\frac{\beta}{\Delta\beta + \beta}\frac{\operatorname{cov}\left\{C_2^{\delta-1}, \left(\frac{\Delta\lambda}{\lambda} + \frac{\Delta\beta}{\beta}\frac{\Delta\lambda}{\lambda} - r\right)\right\}}{E\left(C_2^{\delta-1}\right)}$$

	chrome	copper	gold	asbestos	iron
profits $\delta \neq \gamma$	0.114	0.027	-0.006	0.010	0.075
profits $\delta = \gamma$	0.062	0.014	-0.003	0.006	0.020
prices $\delta = \gamma$	0.000	0.004	0.001	0.000	-0.005

The situation is different when we use the same parameter to estimate risk-premia and elasticity of substitution. In this case the estimates of risk-premia have lower values (Table 9, rows 2 and 3). The lower values obtained using the same parameter to estimate risk-premia and elasticity of substitution is due to a difference in model specification.

However, we can compare when the equations are specified in the same way, i.e., when the same parameter is used to estimate risk-premia and elasticity of substitution, using both the rate-of-change prices and rate-of-change profits as proxies to returns to mineral assets. Using the rate-of-change prices yields lower values (table 9, row 3), again because marginal costs are set to zero when prices rather than profits are used.

In order to determine overall risk-premia values were calculated using the expressions $\frac{\Delta\beta}{\Delta\beta + \beta}(1 + E(r))$ and $\frac{\Delta\beta}{\Delta\beta + \beta}(1 + \rho)$ in Equations (4.1) and (4.2). According to the equations, the investor requires a higher return whenever $\Delta\beta < 0$. Using the rate of change of profits, the calculated values are 3.47 for chrome, 0.90 for copper, 0.28 for gold, 0.42 for asbestos, and 83.78 for iron (Table 10), implying risk-premium levels of 347, 90, 28, 42, and 8378 percentage points. Again, the values are lower when the rate-of-change of prices was used.

Table 10: Risk premia from stock-discounting

Relative to 'safe' alternative asset using equation

$$C = -\frac{\Delta\beta}{\Delta\beta + \beta}(1+\rho)$$

	chrome	copper	gold	asbestos	iron
profits $\delta \neq \gamma$	3.47	0.90	0.28	0.42	83.78
profits $\delta = \gamma$	4.04	1.08	0.24	0.57	66.54
prices $\delta = \gamma$	0.03	0.10	0.14	0.06	0.39

Relative to risky alternative-asset using equation

$$D = -\frac{\Delta\beta}{\Delta\beta + \beta}(1+E(r))$$

	chrome	copper	gold	asbestos	iron
profit $\delta \neq \gamma$	3.38	0.88	0.27	0.41	81.49
profit $\delta = \gamma$	3.92	1.05	0.23	0.55	64.67
price $\delta = \gamma$	0.03	0.10	0.13	0.05	0.38

Combining the risk-premia implied by the covariance terms and the stock-discounting factor yields the values shown in Table 11. Using the rate-of-change of profits, the risk-premia required for investment are 359 percentage points for chrome, 93 for copper, 27 for gold, 43 for asbestos, and 8389 for iron.

The results calculated above are much higher than the actual historical rates of return. However, using the rate-of-change of prices the calculated risk-premia are much lower, 3 percentage points, for chrome, 11 for copper, 14 for gold, 6 for asbestos, and 39 for iron respectively.

Table 11: Total risk premium

Relative to 'safe' alternative asset using equations *A* and *C*

	chrome	copper	gold	asbestos	iron
profit $\delta \neq \gamma$	3.59	0.93	0.27	0.43	83.89
profit $\delta = \gamma$	4.10	1.09	0.23	0.57	66.56
price $\delta = \gamma$	0.03	0.11	0.14	0.06	0.39

Relative to risky alternative asset using equations B and D

	chrome	copper	gold	asbestos	iron
profit $\delta \neq \gamma$	3.49	0.90	0.26	0.42	81.57
profit $\delta = \gamma$	3.99	1.06	0.23	0.56	64.69
price $\delta = \gamma$	0.03	0.10	0.14	0.05	0.38

Note: Equations A, B, C, and D are shown in Tables 9 and 10.

7. Summary and conclusions

This paper used the risk adjusted Hotelling model to examine the level or risk-premia required for mining investment. Results using rate-of-change of prices were obtaining by setting marginal costs to zero, which leaves the cost-structure facing the industry inadequately explained did not reveal the level of profitability of the mineral investments that would depend on the efficiency of the companies involved in extraction. However, the more appropriate results using the rate-of-change of profit, have negative implication, for investment. On the basis of these levels of risk-premia it is highly unlikely that there would be substantial investment in the extractive sector rather than in alternative assets.

Results have to be interpreted with caution, however, since the CCAPM model on which the analysis has based is criticised on grounds that the consumption-ratios which are used are generally too large, relative to average real asset-returns, to be consistent with the theory. Consumption generally had less variation than returns, so that precise estimates for parameters characterising preferences, and hence the covariance of returns and marginal consumption, are sometimes misleading. However, the general indication is that risk-premia required to attract investment into minerals are high. Investments would seem to require the high levels of premia obtained here, which would make investors indifferent between investing in minerals and alternative assets in Zimbabwe. Using the smallest premium, for gold, the returns on mineral investments need to be at least 27 percentage points be higher than those expected from investments in alternative assets for investment to take place. This is an important implication of incorporating a mineral-stock discount-factor into the analysis. Including mineral-stock discounting takes the nature of deposits that tend to be small-scale, appropriately into account, and enables us to more correctly determine the level of returns for mineral investments in Zimbabwe.

Another problem is that the assumption of market efficiency may not be valid. In that case, the assumption that government regulations pertaining to foreign participation in the local stock-exchange and in other investments have insignificant effects may also be invalid, in which case the estimates would not appropriately reflect the risks involved in local investments. In that case, however, risks may actually be higher that those we have been able to capture in the model.

A drawback for the analysis is that no formal statistical tests have been carried out regarding the importance of the risk-premium. In addition, it is possible that the CCAPM on which the model is based is not suitable since most of mining investment is foreign, and the output is geared towards export. It is also possible that discount-factors vary over time. Analysing the effects of such variation on the levels of risk-premia is beyond the scope of this paper. However, the results indicate the importance of risk considerations in analysing the potential for investment in mining in Zimbabwe.

In general the results suggest that high returns would be required by domestic investors for them to invest. This suggests that mining investment will remain low which has negative implications for the indigenisation drive which the government is persuing especially in the light of declining commodity prices.

The results can be generalised to other African countries that have small-scale mineral deposits and an uncertain investment climate. For these countries, high levels of risk-premia are probably also required to lure investment into mining. Increased concern for environmental implications of mining operations further dampens prospects of economic growth through exploitation of mineral resources.

Appendix 1: GMM estimation

The GMM criterion-function minimises the quadratic $m = g(b)'Wg(b)$. The estimator m is consistent for all choices of the weighting matrix, W. Hansen (1982) showed that the most efficient estimator is obtained when W is equal to the inverse of the asymptotic covariance matrix of $NT^{1/2}\, g(b_0)$ Since panel data was used it was expected that the disturbances would be heteroskedastic and also possibly autocorrelated. An appropriate heteroskedasticity and autocorrelation-consistent estimator of the covariance-matrix is the sum of the estimate of the weighted variance-covariance matrix, S_i, for the ith mineral. This is given by

$$S_0 = \sum_{i=1}^{N} S_i$$

where

$$S_i = \Gamma_{i,0} + \sum_{k=1}^{p} \sum_{t=k+1}^{T} (\Gamma_{i,k} + \Gamma'_{i,k})$$

$$\Gamma_{i,k} = E(z_{i,t} u_{i,t} u'_{i,t-k} z'_{i,t-k})$$

z_i = column i in Z' matrix of instrument variables, and

$$\hat{\Gamma}_{i,k} = T^{-1} \sum_{t=k+1}^{T} (z_{i,t} \hat{u}_{i,t} \hat{u}'_{i,t-k} z'_{i,t-k})$$

T is time and u is the disturbance term; p is an autocorrelation lag-length that is zero when disturbances are not autocorrelated. Since the sample was finite there was no guarantee that the variance-covariance matrix would be positive when $p > 0$. The Newey and West (1987) weighting-scheme, in which $\hat{\Gamma}_k$ is multiplied by $1 - \dfrac{k}{p+1}$. guards against the possibility of a negative variance-covariance matrix. In that case

When the disturbances follow a moving average process of known order q such that $u_t = \varepsilon_t + \theta_1 \varepsilon_{t-1} + ... + \theta_q \varepsilon_{t-q}$, then West (1997) suggests using a positive-definite variance-covariance matrix S estimated by $\hat{S}_i = (T-q)^{-1}\sum_{t=1}^{T-q}\hat{d}_{t+q}\hat{d}'_{t+q}$, where

$\hat{d}_{t+q} = (z_{i,t} + z_{i,t+1}\hat{\theta}_1 + ... + z_{i,t+q}\hat{\theta}_q)\hat{\varepsilon}_q$. An optimal weighting-matrix can be estimated using a two-step procedure. In the first step one sets $W = I$ (the identity matrix) and obtains an estimator b_0. In the second step, b_0 is used to compute $W^* = S^{-1}$.

Monte Carlo studies by Kocherlakota (1990) and Ferson and Foerster (1994) suggest that iterating the weighting-matrix yields superior finite-sample properties: At stage $k + 1$ the weighting-matrix is calculated from the k-stage estimates, and $m(b)$ is minimised; the procedure is repeated until convergence is achieved. The asymptotic covariance matrix of the estimator is given by

$$\text{cov}(\hat{b}) = \tfrac{1}{T}\left[\tfrac{\partial g(\hat{b})'}{\partial b} S^{-1} \tfrac{\partial g(\hat{b})}{\partial b}\right] = \tfrac{1}{T}(D'S^{-1}D)^{-1} \ .$$

The model was estimated using SAS procedures.

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Paper III

Mineral extraction costs in Zimbabwe, 1969-1995

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Abstract: This paper examines extraction costs for mining firms in Zimbabwe, reporting on tests of whether the behaviour of firms satisfied optimality-conditions derived from inter-temporal profit-maximisation using parameter-estimates from a dual cost-minimisation problem, during the period 1969-95. Based on the results, the hypothesis that firms were optimising inter-temporal profits can be rejected. A positive relationship was found between cumulative extraction and costs that suggests that ore stock-depletion matters. Due to the small-scale nature of deposits, successful investment in the sector thus depends on raising sufficient funds to cover setting up costs for frequent new operations.

Keywords: investment, mining, minerals, natural resource, cost function, consumption, return, investment, risk premia, user-cost, capital, Zimbabwe.
JEL Classification: O1 O13

1. Introduction

Like many less-developed countries, Zimbabwe's mining sector is export-oriented and exposed to world-market fluctuations. Development of substitute renewable resources is rendering some mineral products redundant, with falling commodity prices. Firms in less-developed countries such as Zimbabwe thus have negligible influence on output prices, and can achieve higher profit-margins only by cutting costs. However, current economic analysis of natural-resource extraction focuses more on resultant environmental degradation than on the efficiency of extraction of the exhaustible resources, while policies for protecting the environment tend to increase production costs. Production costs in turn depend not only on prices of inputs but also on the nature of the ore reserves. Ore-quality is usually assumed to be consistently decreasing over time, and thus it is commonly assumed that there is a negative correlation between reserve-size and per-unit extraction-costs. However, Marvasti (2000) and others have tried to find a positive relationship.

The Zimbabwe government currently intends to increase investment in mineral-extraction in order to increase export-earnings. Yet apart from individual feasibility-studies no empirical work has been done to test the efficiency of firms at the aggregate level. Efficiency in resource-extraction is achieved when the level of output makes the marginal net-benefit of extracting ore in the current period equal to the discounted marginal net-benefit of extracting in the next time-period. This is also known as the opportunity-cost of extraction. Krautkraemer (1998) reviewed some of the issues addressed in the literature on non-renewable resource-scarcity and outlined the basic model.

The basic resource-extraction model assumes a fixed, homogeneous reserve extracted at zero marginal cost by a competitive industry in the absence of externalities. It is assumed that the cost-function at each point in time is independent of the remaining stock of the resource, and that the marginal extraction-cost is simply an increasing function of the extraction-rate. However, as ores are depleted it becomes more costly to extract, unless there are technological changes that improve extraction. Effects of ore-stock depletion can be captured using cumulative extraction, since there is a negative relationship between the amount extracted and the remaining stock.

It is generally hypothesized that a price-taking profit-maximizing firm adjusts mine production so that the difference between price and the marginal cost of production increases at the rate of interest (the Hotelling rule, due to Hotelling, 1931). Indirect tests of the theory have always been used because it is not possible to do a direct test. The economic theory of exhaustible-resource production can use a cost-minimization problem to solve the dual profit-maximisation problem, since they give the same optimal solution. Among others, Chemak and Patrick (1995, 2001) and Halvorsen and Smith (1991) have carried out tests along these lines. However, firms in Zimbabwe do not set the price of the commodities, the existence of resource scarcity could not be tested. Rather, it was tested whether the behaviour of mineral-extraction firms in Zimbabwe conformed to models that imply the existence of resource-scarcity.

The model used was originally proposed and estimated by Farrow (1985). Halvorsen and Smith (1991) estimated an indirect cost-function as the dual of the final-output production-function. They empirically tested their econometric model using aggregate data for the Canadian metal-mining industry. Their results contradicted the empirical implications of the resource-extraction model. They concluded that the problem was either that the model failed to explain the behaviour of firms, or that the firms did not operate optimally. Blackorby and Schworm (1982) showed that aggregation is not always appropriate, and when used, the implied restrictions on technology should be checked.

Chemak and Patrick (1995) found empirical support for the theory that extraction costs are a function both of the extraction rate and of the remaining recoverable reserves. They found that marginal costs decreased with periodic extraction and increased with the depletion of the finite resource. In a later study, Chermak and Patrick (2001) found that, holding other things constant, at any point in time price of the resource in the ground decreased with unprocessed production and increased with final production. They could not reject the theory of exhaustible resources, instead finding that producer behavior was consistent with the theory.

The present study applied the resource-extraction model to test whether, during the period 1969-1995, extracting firms in Zimbabwe were able to maintain levels of production consistent with optimisation-models. An inter-temporal profit-maximisation model was used, which hypothesises that optimality is achieved when the level of output equates the marginal net-benefit of extraction in the current period to the discounted marginal net-benefit of extracting in the next time-period. The cost-minimisation dual to the problem was used to estimate parameters that could be used to test the optimality-hypothesis. The results show the extent to which extraction-costs affected relative returns and investment-decisions.

Optimality-conditions were tested using parameter-estimates of a cost-function that depends on both input-prices and the independent effects of current and cumulative output. Since the results are critically dependent on the level of significance of the parameter-estimates of the cost-function, the stability of coefficients was tested by estimating a cost-function with the optimality-condition explicitly imposed. The optimality-conditions were not satisfied.

The next section describes the mining sector in Zimbabwe, while Section 3 outlines the resource-extraction model. Section 4 discusses the estimation-equations, while Section 5 presents the data and definitions of variables. Section 6 discusses the estimation results, and Section 7 summarises and draws conclusions.

2. The mining sector in Zimbabwe

The importance of mining has been declining in Zimbabwe. In 1999 the mining sector employed 4.5% of the formal sector labour force and produced 4% of the country's gross domestic product (GDP). Two decades earlier, in 1979 mining employed 7.1%, and produced over 10% of the country's GDP. The sector has been adversely affected by declining commodity prices, shortages of spares parts and imported materials for use in mineral extraction, rising energy costs and a hostile domestic political environment.

In spite of the decline, Zimbabwe continues to rely heavily on the export revenues generated from minerals and mineral-related goods. Moreover, geological and engineering studies confirm the availability of ores that could be economically exploited using current technology (see for example the geological survey of Zimbabwe, GoZ 2000).

There has been no unified policy for all firms, with contracts negotiated and revised on a case-by-case basis. For example, in 2000 the government refused to allow one multinational company the same tax breaks given to another for investing in the same mineral. In order to reduce the differences in conditions negotiated by investors, a number of measures were recently adopted. In May 2002 the government revised the fiscal package to set royalties at 3% of total revenues on precious-metal producers and at 2% on base-metal producers. Previously, the royalty had not been fixed and there was a lack of certainty. Some analysts consider the royalties a disincentive that could be replaced by policies that are directed towards the productivity-potential of the firms.

Towards the end of our sample-period in 1995 the corporate tax-rate was 40%, though the effective tax-rate was reduced through several tax-concessions, including allowances for capital-expenditure for buildings, equipment, shaft-sinking, and pre-mining development. In addition there was a depletion-allowance of 5% of the gross value of minerals produced, considered as a working cost (GoZ, 1994). However, some firms tend to hold onto undeveloped claims, that they do not develop which may indicate implicit capital-gains.

The government is focused not only on attracting investment into mining. It also intends to indigenise the economy through deliberate economic empowerment of black Zimbabweans, mainly through economic expansion. It claims that indigenising the economy will eliminate socio-economic development-imbalances, create employment and wealth, eradicate poverty, expand the domestic market, and widen the tax-base. Specifically, the government intends to stimulate the mining sector by making more financial resources available for borrowing at concessionary rates. The opportunity to use profit for reinvestment is improved by the provision of local incentives to offset adverse effects resulting from low prices and other factors that cannot be controlled locally.

The Geology of Zimbabwe favours small-scale mining operations (Hollaway, 1997), hence the minimum efficient scale is reached at low levels of output. But start-up costs for mining operations tend to be high. Hence it is financially burdensome to switch to new proven reserves, so and firms sometimes continue to extract from deposits despite the fact that extraction-costs increase as reserves are depleted

The aim of this study was to estimate the cost-function facing the mining industry and then to use its estimated parameters to test optimality-conditions derived from inter-temporal profit-maximisation. The feasibility of increasing investment and maintaining optimal levels of output was assessed using these conditions. The next section outlines the model that is used to explain the behaviour of firms in resource-extraction industries.

3. The resource extraction model

The resource-extraction model assumes that firms seek to maximise the present value of profits subject to some resource-constraints. It also assumes that firms are price takers in input and output markets, and that there is complete certainty and perfect arbitrage.

The optimisation problem for the firm is then

$$\max \int_0^T e^{-\beta t} \; \{P_t R_t - C(R_t, F_t, \boldsymbol{W}_t, t)\} dt$$

$$s.t. \;\; F_t = S - \int_0^T R_s ds \qquad (\text{or} \quad \dot{F}_t = \; -R_t)$$

with $\quad R_t, F_t, P_t, W_t \geq 0$

and $\quad T \quad$ *unconstrained*

where P_t is the exogenous commodity-price; β is the discount-rate; $R_t = -\dot{F}_t$ is the rate of extraction; F_t is the stock remaining in the mine; $\boldsymbol{W}_t$ is the vector of exogenous input-prices; T is the terminal time-period; S is the known resource-stock; C is the cost- function; and t and s are time indices.

In the model, extraction of ore is limited by the amount of ore remaining in the ground. In addition, output-prices, output, cumulative output, and factor-prices, are restricted to non-negative values. The input variables used here were capital, labour, materials and other services, and energy.

The current value Hamiltonian for the problem is

$$H(R_t, \lambda_t) = P_t R_t - C(R_t, F_t, \boldsymbol{W}_t, t) - \lambda_t R_t = 0$$

where λ_t is the co-state variable, the current-value of the shadow-price of the resource-stock at time t. The shadow-price (scarcity-rent, or net price) measures the approximate decrease in the present-value of profits resulting from a unit-decrease in ore-stocks. The shadow-price is not directly observable.

The first-order necessary conditions include static and dynamic efficiency-conditions. The static efficiency-condition is

$$\lambda_t = P_t - C_{Rt} \tag{1}$$

where C_{Rt} is marginal extraction-cost which captures the increase in extraction costs due to extracting an additional unit of the resource. This condition requires that, at each point in time, the marginal benefit from extracting the resource equal the marginal cost of extraction, including the user-cost of depleting the resource-stock, λ. The dynamic efficiency-condition is

$$\dot{\lambda}_t = \beta\lambda_t - C_{Ft} \tag{2a}$$

where C_{Ft} is the marginal effect of stock-depletion. This condition requires the rate of return for holding the resource-stock to equal the rate of discount. According to the Hotelling rule, the shadow-price of the resource-stock should increase at the discount rate when the marginal effect of stock-depletion is zero (i.e. when $C_{Ft} = 0$). When $C_{Ft} < 0$ then the shadow-price increases faster than the discount rate. The term $\beta\lambda_t$ measures the external opportunity-cost of holding a unit of the resource in the ground.

It is the value of the forgone interest that could have been earned by extracting the resource during the previous period and investing the return in an alternative asset.

In current literature reserve-size and other reserve-characteristics are considered to be major cost-determinants for mining firms. Cumulative output may be used to measure the effects of stock-depletion that are not directly observable. There is an inverse relationship between cumulative extraction and the stock of ore in the mine. Hence, the partial derivatives of the cost-function with respect to cumulative extraction and stock-depletion have opposite signs. When firms operate optimally, costs increase, implying a fall in quality as deposits are depleted.

4. Estimation equations

In order to test whether mining firms in Zimbabwe were optimising, the optimality-conditions derived from the profit-maximising model were tested. As suggested by Farrow (1985) the empirical test of optimality for the resource-extraction model is the degree of consistency between the data and the necessary (optimality) conditions in equations (1) and (2a). Fisher (1981) showed that the continuous-time specification in equation (2a) is the limiting case of the discrete-time model[1]

$$\Delta\lambda_t = \beta\lambda_{t\text{-}1} - C_{Ft} \tag{2b}$$

Optimality requires the estimated coefficient for C_{Ft} to be approximately unity and that of λ_{t-1} to be equal to the discount-rate (Farrow, 1985). The estimates of λ and C_{Ft} used to test the optimality-conditions are obtained by differentiating an appropriate cost-function.

The marginal cost of current extraction is

$$C_{Rt} = \frac{\partial \ln C_t}{\partial \ln R_t}\frac{C_t}{R_t} = \phi\, AC_t \tag{3}$$

[1] This equation may also be written as $\lambda_t = (1+\beta)\lambda_{t-1} - C_{Ft}$.

where AC_t is the average cost and ϕ is the derivative of the log of the cost function with respect to the log of output.

The marginal effect of stock-depletion is

$$C_{Ft} = \frac{\partial \ln C_t}{\partial \ln F_t}\frac{C_t}{F_t} = \phi_1 \frac{C_t}{F_t}, \tag{4}$$

where ϕ_1 is the derivative of the log of the cost-function with respect to the log of the ore-stock remaining in the ground.

Economies of scale are implied when marginal cost is less than average cost, which can also be measured by the reciprocal of the elasticity of cost with respect to output. Sweeny (1993) showed that the discrete cost-function is consistent with the existence of the cost function used in the profit-maximisation problem when

$$\frac{\partial C_t}{\partial R_t} + \frac{\partial C_t}{\partial F_t} > 0 \quad \text{and} \quad \frac{\partial^2 C_t}{\partial R_t^2} + \frac{\partial^2 C_t}{\partial F_t \partial R_t} > 0$$

The marginal effect C_{Rt} can be eliminated from equation (1) and the result can be substituted into equation (2a) to yield

$$AC_t = \gamma_0 + \gamma_1(P_t - (1 + \beta_{t-1})P_{t-1}) + \gamma_2(1 + \beta_{t-1})\, AC_{t-1} + \gamma_3 \frac{C_t}{F_t} + \xi_t \tag{5}$$

where $\gamma_1 = \frac{1}{\phi}$, $\gamma_3 = \frac{\phi_1}{\phi}$.

The construction of equation (5) implies that its parameter-estimates should be consistent with those of an appropriate cost-function. In particular, the estimated coefficient for the variable $(P_t - (1 + \beta_{t-1})P_{t-1})$ is the reciprocal of the elasticity of cost with respect to output, ϕ, and is expected to be negative. The estimated coefficient of $\frac{C_t}{F_t}$ is also expected to be negative (when ϕ is positive) since the stock-effect is measured using cumulative extraction as a proxy. Reducing the stock of ore, i.e., an increase in cumulative extraction would increase the cost of extraction. The estimated coefficient for the variable $(1 + \beta_{t-1}) AC_{t-1}$ is expected to be close to unity. It is possible to estimate equation (5) using the equation

$$AC_t = \gamma_0 + \gamma_1(Pyldt) + \gamma_2(TcoyLdt) + \gamma_3(Tcocypt) + R1 + \xi_t \qquad (5a)$$

where R_1 are transformed errors from the first stage of estimation. However, there is a potential problem of biased results, since the data are not transformed and trends over time. In addition, parameter-estimates do not necessarily capture the optimality-conditions unless cost-minimisation holds.

In order to impose cost-minimisation, a cost-function was estimated then the derivatives of the cost-function with respect to output and to cumulative output were used in conjunction with output prices to estimate the optimality-conditions[2].

Under the assumption of competitive market-behaviour, the theory of duality makes it possible to model production-characteristics of firms using a weakly separable cost-function. The cost-function assumes that input-prices are exogenous and that producers choose quantities that minimise costs. The production function can be uniquely represented by the cost-function

$$C = C(R_t, F_t, \boldsymbol{W}_t, t) = \Sigma_j w_j x_j (R, w, t), \qquad j = L, K, E, M, F, R$$

where C denotes total cost of the mining sector; $\boldsymbol{X} = [x_j]$ is a vector of inputs (labour, L; capital, K; energy, E; and materials, M); and $\boldsymbol{W}$ is the vector of input-prices. If $C(R_t, F_t, \boldsymbol{W}_t, t)$ gives the minimum total cost of production, then applying Shepard's (1970) lemma gives the cost-minimising set of factor-demands

[2] Crowson (1992) argues that the techniques and cost-structures of mineral-extraction are usually quite similar regardless of the metals or minerals being mined, hence it has been assumed here that it is possible to form a panel using the short series of individual mineral data.

$$X_j^* = \frac{\partial\ (C_t,R_t,F_t,\boldsymbol{W}_t,t)}{\partial\, w_j}$$

where X^* is the j-th factor-demand.

The cost-minimising factor-cost shares are the logarithmic derivatives of the cost-function with respect to input-prices

$$S_j = \frac{\partial \ln(C_t,R_t,F_t,\boldsymbol{W}_t,t)}{\partial \ln w_j}$$

where S_j is the cost-share for the j-th input in total cost.

In order to estimate the cost-function, a flexible functional-form was adopted that places few restrictions on the underlying production-technology, and can also approximate a wide variety of functional forms[3]. The function is a second-order Taylor series approximation to either a generalised quadratic or to an arbitrary logarithmic cost-function (Christensen et al. 1973, and Diewert 1974).[4]

The translog cost-function and the associated cost-share equation for the j-th input are

$$c_{it} = a_0 + \Sigma_n a_j w_{jit} + a_r r_{it} + a_f f_{it} + a_T t + ½\{\Sigma_j \Sigma_k a_{jk} w_{jit} w_{kit} + a_{rr} r^2_{it} + a_{ff} f^2_{it} + a_{TT} t^2\} + \Sigma_j a_{jr} w_{jit} r_{it} + \Sigma_j a_{jf} w_{jit} f_{it} + \Sigma_j a_{jT} w_{jit} t + a_{rf} r_{it} f_{it} + a_{rT} r_{it} t + a_{fT} f_{it} t + \varepsilon_{it} \quad (6)$$

$$S_{jit} = \frac{\partial c_{it}}{\partial w_{jit}} = a_j + \Sigma_k a_{jk} w_{kit} + a_{jr} r_{it} + a_{jf} f_{it} + a_{jT} t + e_{jit} \quad (7)$$

$$\varepsilon_{it} = u_i + v_{it} \quad (8)$$

---

[3] Cobb-Douglas, CES, and Leontief forms can be represented by introducing appropriate parameter restrictions.
[4] To be consistent with linear homogeneity, the sum of shares of logarithmic derivatives must sum to one, i.e., $1 = \Sigma_k S_{jit} = \Sigma_k a_j + \Sigma_k \Sigma_k a_{jk} w_{kit} + \Sigma_k a_{jr} r_{it} + \Sigma_k a_{jf} f_{it} + \Sigma_k a_{jT} t$, hence the restrictions are required for the function to hold globally.

where c, w, f, and r are logarithmic; ε_{it} is the error-term composed of mineral-specific effects, u_i, and a white-noise component v_{it}; and e_{jit} is the error-term associated with the j-th share-equation. It was assumed that the unobservable mineral specific effects, u_i, were fixed. Linear homogeneity in factor prices and symmetry impose the restrictions

$$a_{jk} = a_{kj};\quad \Sigma_j a_j = 1; \text{ and } \Sigma_j a_{kj} = \Sigma_j a_{jk} = \Sigma_j a_{jr} = \Sigma_j a_{jf} = \Sigma_j a_{jT} = 0$$

The time-variable in the cost-function represents shifts in the production-technology, while the a's are the parameters of the model to be estimated. The cost-function is non-increasing (or non-decreasing) in t if technical change is progressive (or regressive). The coefficient a_r on the logarithm of extraction was expected to be positive since, as noted earlier, mining costs go up as more ore is mined, and the ore-stock is depleted.

Such cost-specifications have been used by Young (1991), and Halvorsen and Smith (1991), Frechette (1999), and Chenmak and Patrick (1995, 2001). Young used the GMM technique, whereas Halvorsen and Smith used three-stage least-squares to reduce the statistical problems arising from the first two indirect methods of estimation. Chemak and Patrick used feasible generalised least-squares. Frechette used nonlinear seemingly unrelated (Zellner) regression.

The cost-functions and share-equations were jointly estimated in order to improve the efficiency of the parameter-estimates. Since the cost-system is singular the energy share-equation was dropped and total cost, and remaining prices were divided the by the energy price. Iterative SUR was used to estimate the transformed equations in order to make parameter-estimates invariant to the choice of the dropped equation. The equations used for estimation were

$$\begin{aligned} \text{Costs} = {} & aONE*1 + aD1*D1 + aD2*D2 + aD3*D3 + aD4*D4 + aD5*D5 \\ & + aW_K1*W_K + aW_L*W_L + aW_M*W_M + aY*Y + aCY*CY \\ & + aTIM*TIME + aW_K2*W_K^2 + aW_L2*W_L^2 + aW_M2*W_M^2 + aY2*Y^2 \end{aligned}$$

$$+ aCY2*CY^2 + aTIM2*TIME^2 + aW_{KL}*W_{KL} + aW_{KM}*W_{KM} + aW_{LM}*W_{LM}$$
$$+ aW_{KY}*W_{KY} + aW_KCY*W_KCY + aW_KT*W_KT + aW_LY*W_LY + aW_LCY*W_LCY$$
$$+ aW_LT*W_LT + aW_MY*W_MY + aW_MCY*W_MCY + aW_MT*W_MT + aYCY*YCY$$
$$+ aYTIM*YTIME + aCYT*CYT \quad (6a)$$

$$\text{Capital share} = aW_K + aW_K2*W_K + aW_{KL}*W_L + aW_{KM}*W_M + aW_{KY}*Y$$
$$+ aW_KCY*CY + aW_{KT}*TIME \quad (7a)$$

$$\text{Labour share} = aW_L + aW_{KL}2*W_K + aW_L2*W_L + aW_{LM}2*W_M + aW_{LY}*Y$$
$$+ aW_{LC}Y*CY + aW_{LT}*TIME \quad (7b)$$

$$\text{Material share} = aW_M + aW_{KM}3*W_K + aW_{LM}3*W_L + aW_M2*W_M + aW_{MY}*Y$$
$$+ aW_MCY*CY + \mathrm{aW_{MT}*TIME} \quad (7c)$$

Elasticity with respect to output (ϕ) is

$$aY + aY2 * Y + aW_KY * WK + aW_LY * W_L + aW_MY * W_M + aYCY * CY$$
$$+ aYTIM * TIME \quad (8)$$

and elasticity with respect to cumulative output (ϕ_1) is

$$\mathrm{aCY + aCY2 * CY + aW_KCY * W_K + aW_LCY * W_L + aW_MCY * W_M}$$
$$\mathrm{+ aYCY * Y + aCYT * TIME} \quad (8)$$

where the dummy-variables are $D1$ for chrome, $D2$ for copper, $D3$ for gold, $D4$ for asbestos, and $D5$ for the ESAP period; K is capital, L is labour, M is material, E is energy, Y is extraction, CY is cumulative extraction, and T is time and they are also represented by their respective subscripts. With the exception of time and dummy variables all the other variables were measured in logs.

The parameter-estimates from the cost-equation (6a) were then used to estimate the optimality-conditions given by equations (1) and (2a).

For estimation-purposes appropriate time-subscripts for ϕ and ϕ_1 were included, and the following modified version of equation (5) was adopted

$$AC_t = \eta_0 + \eta_1 \frac{1}{\phi_t}(P_t - (1+\beta_{t-1})P_{t-1}) + \eta_2 (1+\beta_{t-1}) \frac{\phi_{t-1}}{\phi_t} AC_{t-1}$$

$$+ \eta_3 \frac{\phi_{1,t}}{\phi_t} \frac{C_t}{F_t} + \xi_t \qquad (5b)$$

Since the equation includes lagged average-costs instruments for the variable were formed. Predictions from a regression of average cost on current and lagged values of all the strictly exogenous variables were used including a transformed errors from the first stage of estimation.

The estimation-equation becomes

$$AC_t = \eta_0 + \eta_1 (Pyldt) + \eta_2 (TcoyLdt) + \eta_3 (Tcocypt) + R1 + \xi_t \qquad (5c)$$

where R_1 are the transformed errors. The estimated coefficients η_1, η_2, and η_3 are expected to be close to unity if the optimality-condition is satisfied. Hence the null hypothesis that they are equal to unity was tested.

The problem with this method of estimation is that the results depend on the level of significance of the estimates in the first stage of estimation. It was thus checked whether the coefficients changed if the optimality-condition was explicitly imposed by estimating the system jointly. This method does not yield the required estimates of equation (5c) however, but only serves to check that the parameter of the cost-function did not change significantly due to the imposition of the optimality-condition. The optimality-condition condition included is

$$aY + aY2*Y + aW_K1Y*W_K + aW_LY*W_L + aW_MY*W_M + aYCY*CY$$
$$+ aYTIM*TIME)*(-AC_t) + (aY + aY2*YL + aW_KY*W_{KL} + aPLY*PLL$$
$$+ aW_MY*W_ML + aYCY*CYL\ aYTIM*TIMEL)*aTcoyL*(\mathbf{1}+DISCL)*TcoyLdt$$
$$+ aP*PY - PYL*(\mathbf{1}+DISCL) + (aCY + aCY2*CY + aW_KCY*W_K + aW_LCY*W_L$$
$$+ aW_MCY*W_M + aYCY*Y + aCYT*TIME)*aTcocy*Tcocypt \qquad (5d)$$

where *DISCL* is the lagged discount rate.

By jointly estimating the cost-system and the optimality-condition the appropriate parameter-estimates of the optimality-conditions and the cost-function were set to the same values under the null hypothesis. Under the null hypothesis the time-path of the shadow-price of the natural resource is expected to conform to the dynamic conditions. In this case the parameter-estimates of 5(b) would be a subset of the parameters of the cost-system. The alternative hypothesis is that the dynamic optimality-conditions would not be satisfied. Estimation of the cost-system in isolation provides consistent estimates of parameters of the cost-function under both the null and alternative hypotheses. Joint estimation of the dynamic optimality-conditions and the cost-system yields consistent and asymptotically-efficient estimates of the parameters of the cost-function under the null hypothesis but inconsistent estimates under the alternative. Under such circumstances, a Hausman (1978) specification-test can be used to test the null hypothesis that the optimality-condition is satisfied. This is done by comparing estimates of the parameters obtained by joint estimation with estimates obtained by estimating the cost-system in isolation.

The test-statistic is the quadratic $\chi_d^2 = (\boldsymbol{A}^* - \boldsymbol{A})'(\boldsymbol{V}_A - \boldsymbol{V}_{A^*})^{-1}(\boldsymbol{A}^* - \boldsymbol{A})$, where $\boldsymbol{A}$ is the vector of the parameter-estimates under the null hypothesis, $\boldsymbol{A}^*$ is the vector of parameter-estimates under the alternative hypothesis, and $\boldsymbol{V}$ is the variance-covariance matrix of the parameter-estimates. The test-statistic is asymptotically distributed as χ^2 with degrees of freedom equal to the number of parameters being tested. Its weakness is that there are many parameters to estimate because the number of instruments required for estimation is large.

5. Data and definition of variables

Annual aggregate data was used for five mineral products; chrome, copper, gold, asbestos, and iron for which data were available for the period 1969-1995; after that some of the sectors were aggregated. The total number of observations is 135. The major source of data is the Central Statistical Office, Zimbabwe. There is not much bias in the aggregation since the commodities can reasonably be viewed as homogeneous, except for differences in ore grades that fetch different prices on the market. The data are also of a reasonable quality for estimation and inference purposes.

Observations for the rate of extraction, cumulative extraction, input and output prices and cost shares for the various inputs were required for estimation. Input categories used were capital, labour, materials, and other services, and energy. Implied investment deflator was used measure the price of capital, P_K. National-accounts data was used to calculate the deflator as the ratio of investment values in current and constant prices. It was assumed that the user-cost of capital facing the firms was the same for all sectors during any particular year; this is justifiable because the extraction process is usually done with similar types of equipment. In order to calculate the user-cost of capital, the equation $W_K = (r+\delta-\dot{P}_k) P_K$ was used, where r is the real interest-rate, and δ is the depreciation-rate of capital. The rate of interest during the sample period was 4.4%, and an assumed 5% rate of depreciation of capital was used. This is the value that is used in most studies on resource economics.

Cumulative investment was used to represent capital-stock. It was assumed that the capital-stock for the year 1969 was the real investment during that year. The cost of capital for the firm was taken as the stock of capital multiplied by the user-cost of capital. Calculations gave an average value of 4.92 for the cost of capital during the sample period, as shown in Table 1. Wage-per-worker was used to define the price of labour, W_L. The average value over the sample period was 8.28. Since the bulk of intermediate mining inputs are imported the price of imports in Zimbabwe dollars was used as a proxy for the price of material inputs, W_M. The average value during the sample period was 12.23, assumed the same for all minerals since they all import most of their major intermediate inputs. The price of energy, W_E, was calculated as a weighted average of all types of energy inputs. The weighted average for the sample period was 4.81. Total cost is the sum of the costs of capital, labour, materials, and energy.

Table1: Average price indices

	output	capital	labour	materials	energy
chrome	0.52	4.92	8.21	12.23	4.56
copper	0.56	4.92	6.84	12.23	4.93
Gold	0.31	4.92	11.78	12.23	4.49
asbestos	0.54	4.92	8.14	12.23	4.62
Iron	0.38	4.92	6.41	12.23	5.46

Source: computations from Zimbabwe Central Statistical Office Reports.

Cost shares were obtained by dividing the appropriate input-cost by total cost; Table 2 shows the shares for each of the minerals. Copper had the highest cost-share at 37%. The largest cost-share for all minerals was for materials. Energy had the smallest share in all cases because energy is provided by government-owned companies that keep prices low; coal is provided by a company whose major shareholder is the government, and electricity is provided by a parastatal.

Table 2: Proportion (shares) of total cost

	capital	labour	materials	energy	total
chrome	0.20	0.36	0.36	0.08	1.00
copper	0.37	0.16	0.38	0.10	1.00
gold	0.15	0.34	0.41	0.11	1.00
asbestos	0.22	0.29	0.40	0.10	1.00
iron	0.25	0.19	0.42	0.14	1.00
average	0.24	0.27	0.39	0.10	1.00

Source: computations from Zimbabwe Central Statistical Office Reports.

An index for the rate of extraction was calculated using gross output value divided by the price index.[5] The averages were 2.68 for chrome, 32.38 for copper, 44.04 for gold, 19.01 for asbestos, and 81.6 for iron. Dummies for each of the commodities were also used when estimating the fixed coefficients model. A regime shift dummy-

[5] Gross output figures supplied in the census of production are net of the amount of capital expenditure – see note on page 3 of the 1995/96 report.

variable was used for the socialist period between 1980 and 1990, and another for the ESAP period.

6. Estimation results and discussion

First the optimality equation (5a) was directly estimated with results as presented in Table 3. The estimate of the reciprocal of the derivative of the logarithmic value of cost with respect to the logarithmic value of output (γ_1) had a statistically significant value of –0.028. Hence, the elasticity of cost with respect to output has a value of –35.7, also statistically significant. Since estimates are derived from the assumption that firms operate optimally, the result implies that firms had an incentive to increase production, as they could have lowered costs. This suggests that, holding other things constant, increasing output could makes firms operate more efficiently.

Table 3: Estimation results for equation (5a)

Parameters	Estimates	t-statistics
γ_0	0.023	41.74
γ_1	-0.028	-18.95
γ_2	-0.060	-3.44
γ_3	2.528	36.57

Null Hypothesis	F-Value	Pr > F
$\gamma_2 = 1$	319.26	<.0001

The estimate of the parameter of the ratio of total cost to cumulative output (γ_3) had a statistically significant value of 2.528 with an unexpected sign. The estimate is the quotient of the elasticity of cost with respect to output and the elasticity of cost with respect to cumulative output. Thus $\gamma_3 = 2.528 = \dfrac{\phi_1}{-0,28}$. Hence, the elasticity of cost with respect to cumulative output is –0.07. This suggests that increasing cumulative extraction decreased extraction cost, and that the problem of exhaustion of mineral deposits was not of paramount importance. This is also consistent with the argument that Zimbabwean mineral deposits are not fully exploited, which could be one of the reasons why firms hold claims that they do not develop. The coefficient on the lagged value of average costs (γ_2) has a statistically significant value of –0.60, which suggests that average costs are autocorrelated over time. The null hypothesis that the value of the parameter γ_2 is unity can be rejected. A value of unity would imply that optimality as identified by equation (5a) holds, and the results suggest that this was not the case.

Generally, all the coefficients suggest that the optimality-conditions are not satisfied. However, the parameter-estimates capture some aspects of the cost-function from which they were derived. In addition, the *t*-statistics appear too large, suggesting that they might not be reliable. Hence it is not possible to tell with certainty whether it is the optimality-conditions or cost-minimisation or both which were being violated. Equation (5c) was estimated in order to reduce this ambiguity problem and make the optimality-condition depend on cost-minimisation.

The parameter-estimates for the translog cost-function and the complete system are presented in table 4. The dummies D1, D2, D3, and D4 were included in the cost-equation in order to capture any mineral specific effects. They take care of the differences the cost-shares shown in Table 2; they were included only in the cost-equations, but not in share-equations.

Table 4: Estimation results for translog cost-function using equation (6a) and share equations, then using equations (5d) and (6a) and share equations

	Equation (6a)		Equations (5d) and (6a)	
Parameter	estimate	*t*-statistic	estimate	*t*-statistic
*A*One	-2.026	-5.67	-1.830	-8.34
aD1	-0.181	-1.12	-0.214	-1.39
aD2	-0.377	-5.68	-0.365	-5.71
aD3	-0.254	-2.73	-0.270	-3
aD4	-0.357	-3.94	-0.359	-4.1
aW_K	0.138	2.9	0.135	2.85
aW_L	0.386	12.93	0.391	13.23
aW_M	0.395	11.05	0.392	11.13
AY	0.876	2.21	1.090	4.67
ACY	0.370	0.82	0097	0.42
ATIM	-0.254	-5.62	-0.232	-7.04
AW_K2	0.015	4.56	0.014	4.44
AW_L2	0.112	9.66	0.111	9.77
AW_M2	0.097	4.24	0.094	4.21
*AY*2	0.077	0.69	0.143	1.94
*ACY*2	0.193	1.38	0.280	3.44
*ATIM*2	-0.001	-0.51	-0.0003	-0.3
aW_{KL}	-0.001	-0.31	-0.001	-0.39
aW_{KM}	-0.015	-3.69	-0.014	-3.51
aW_{LM}	-0.057	-3.97	-0.056	-3.98
aW_{KY}	-0.132	-5.81	-0.135	-6
aW_KCY	0.162	5.76	0.165	5.94
aW_KT	-0.018	-8.07	-0.019	-8.31
aW_LY	0.030	2.03	0.034	2.33
aW_LCY	-0.084	-4.75	-0.088	-5.06
aW_LT	0.011	7.1	0.011	7.39
aW_{MY}	0.090	5.07	0.087	5.01
aW_MCY	-0.078	-3.67	-0.076	-3.62
aW_MT	0.005	2.56	0.005	2.61
AYCY	-0.364	-1.5	-0.509	-3.3
AYTIM	0.022	1.39	0.029	2.42
ACYT	-0.002	-0.08	-0.012	-0.84
aTCOYL			-0.240	-0.33
aTCOCY			7.323	1.52
aPY			1.000	0
aPYL			0.964	32.68
Restrict	*APYL* = 1		115.080	10.49

Based on a joint test of the significance of the variables, we cannot reject the null hypothesis (Table 5). In fact, the Wald χ^2-tests for the joint significance of the dummy variables that capture firm-specific effects in the cost-function are both individually and jointly statistically significant. The Wald χ^2 statistic is 34.37 and has a probability value less than 0.0001.

Table 5: Tests of joint significance of parameter-estimates, Wald statistics

	Wald χ^2 - statistic	Pr > χ^2
Output:	59.07	<.0001
aY, $aY2$, aW_KY, aW_LY, aW_MY, $aYCY$, $aYTIM$		
Cumulative output:	139.27	<.0001
aCY, $aCY2$, aW_KCY, AW_LCY, aW_MCY, $aYCY$, aYT		
Mineral specific effect dummies:	34.37	<.0001
D1, D2, D3, D4		

Based on the Wald χ^2-statistics for the joint significance of parameters used to determine the elasticities of cost with respect to both output and cumulative output we can reject the hypotheses that they are jointly equal to zero. The statistics are 59.07 for output, and 139.27 for cumulative output. Since their probability values are less than 0.0001 we cannot reject the null hypothesis that the parameters are jointly significant. Also, most of the parameters required to estimate the elasticities with respect to output and cumulative output are individually significant.

The estimates of the parameters were used to calculate the average elasticities of cost with respect to output and cumulative output (Table 6). With the exception of chrome, the cost-elasticities with respect to output suggest that increasing output would have reduced costs of production. These results confirm the finding from the method used earlier. Hence we can unambiguously say that there was a negative relationship between the level of output and costs that suggest that economies of scale were not being achieved. Firms could have benefited from producing more output. However, this might not have been possible because of the small-scale deposits. Chrome presents a different case and the same cannot be said. However, empirical evidence from other sources suggests that chromium deposits are also small, so again it might be difficult to attain economies of scale although the elasticity suggests that they should have cut back on chrome output.

The average elasticities of cost with respect to cumulative output suggest a positive relationship between cumulative extraction and cost. This contradicts the finding reported earlier because it suggests that depletion of ore stocks does matter. The results here are also more reliable because they take into account other factors that influence cost, considered in the simple average-cost estimation-method.

Table 6: Average elasticities of cost with respect to output and cumulative output

	output	cumulative output
chrome	0.350	0.406
copper	-0.228	0.013
gold	-0.044	-0.065
asbestos	-0.058	0.081
iron	-0,361	-0,151

The parameters of the optimality-conditions embedded in the simple average cost estimator given in equation (5c), were also estimated after appropriately adjusting the variables (Table 7). Only the coefficient of the price variables (η_1) was statistically significant though it had an unexpected sign. The null hypothesis that its value was equal to one, as required by the optimality-condition, can also be rejected based on the tests provided. Since the other parameter-estimates were statistically insignificant, we can reject the hypothesis that the results satisfy the optimality-conditions. A contributory factor to the statistical insignificance of the estimates is probably the insignificance of some of the parameter-estimates in the cost-system. However, these were found to be jointly statistically significant; it is thus most likely that firms failed to minimise costs of production.

Table 7: Parameter estimates for equation (5c)

Parameters	Estimates	*t*-statistic
η_0	0.040	29
η_1	0.002	1.88
η_2	3.65E-05	0
η_3	-0.058	-1.25
R1	0.113	3.82

Null hypothesis	*F*-Value	Pr > *F*
Pyldt =1	1293371	<.0001
TcoyLdt =1	3426.10	<.0001
Tcocypt =1	521.21	<.0001

In order to check the sensitivity of the results to the imposition of the optimality constraint, the cost-system and the optimality-condition equation (5d) were jointly estimated. The variables in the cost-system were measured as previously. The remaining variables in the optimality-equation (5d) were measured in levels.

The Hausman test was used to determine whether the parameter-estimates in the two models were statistically different. A χ^2-value of 40.69 with a *p*-value of 0.86 was found. Hence we cannot reject the null hypothesis that the relevant parameters in the two models have the same mean values. We can conclude that the parameter-estimates give elasticities that are not significantly different. Since the optimality-requirement we have specifically imposed, it was satisfied by construction. The test of its significance is a test that the parameters used to derive the appropriate statistics are themselves statistically significant. The results confirm that firms were not attaining economies of scale. In addition, depletion of deposits does matter for individual firms.

7. Summary and conclusions

This paper used a dual cost-function to test optimality conditions derived from a profit-maximisation model. Since some of the mining firms are vertically integrated they incur costs in both the extraction and production process. Data available did not distinguish the two processes and attributed most effects to the extraction process. This limitation is likely to understate the level of costs for the firms since the extraction process is expected to be increasingly costly over time because of declining ore grades. Hence it is likely that extracting firms incur higher costs that those captured by the model.

The analysis in this paper is based on the Hotelling relationship that cannot be directly tested. The issue is further complicated by the fact that mining firms in Zimbabwe have negligible effect on world mineral production. In addition new technology and production of substitutes depresses mineral prices and the effects have not been factored out. Only the condition that firms are expected to reduce their costs of production for the optimality condition to hold could be tested. Historically mineral products from Zimbabwe have not failed to find a market so the substitution effect on production is expected to be minimal. However, results suggesting failure to minimise costs would in fact be more negative since firms would be failing to utilise the new technologies available.

The costs of mineral extraction in Zimbabwe were adversely affected by the low scale of production. In addition, cumulative output that captures the effect of ore-stock depletion had a positive effect suggesting that ore stock depletion matters. The two factors suggest that capital investment may have been inappropriately geared towards large-scale production than the nature of the ore reserves merits. Thus the mining firms failed to attain economies of scale.

New mining investment should take into account the small-scale nature of the deposits where that investment will be used. In addition, successful mining investment is mostly likely dependent on the possibility of raising sufficient funds to cover setting-up costs for frequent new operations, due to the small-scale deposits. Without taking these factors into account, it is not likely that new mining investment will yield returns that will make it possible to reinvest and that in turn hampers the indigenisation-process that the government desires.

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Paper IV

Irreversibility and mining investment in Zimbabwe, 1969-1995

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Abstract: This paper examines the effects of irreversibility on mining investments in Zimbabwe. The path of reversible investment determined by the equality of the marginal-revenue-product of capital to its user-cost is used to predict irreversible investment based on individual-firm uncertainty. It is assumed that the level of capital-stock deviates from its desired level and that the distribution of the deviations can be derived. The distribution is then used to estimate the implied effects of the uncertainty which underly the observed regular investment-pattern. Results show that individual-firm uncertainty must have deviation values greater than 0.166 contrasted with values less than 0.04 for the observed investment-ratio. The result implies that the irreversible capital stock was less than 68% of the reversible level when there was positive investment.

Keywords: investment, mining, minerals, natural resource, uncertainty, irreversibility, return, investment, risk, user-cost, capital, Zimbabwe.
JEL Classification: O1 O13

1. **Introduction**

Like many other less-developed countries, Zimbabwe relies to a great extent on export of mineral commodities to generate foreign exchange. Mining in fact generates more than 30% of export earnings, and the government wants to use it to increase the growth-rate of the economy.

The scale of mining investment required in Zimbabwe is not large; Hollaway (1997) explained that the geology of Zimbabwe favours many small-scale investments. But Strongman (1994) argued that Zimbabwe has an attractive mineral potential warranting increased exploration expenditures by the private sector. He also noted that investors require competitive terms and conditions, including assurances that the investment environment will be stable. Relative to most other industries, mining is characterised by high risk. Bertola (1998) pointed out that risk-neutral firms are sometimes reluctant to invest when the future is uncertain and projects are irreversible. Ericson and Gibbon (1992) noted that political factors in Zimbabwe had had negative effects on investment decisions during the period 1980-1990, and the situation has deteriorated further in recent years.

The negative political climate has tended to reinforce the geological factors and increase the uncertainty of the investment climate. In addition, investors face uncertainty over future product-prices, future interest-rates and operating costs that determine cash-flows. All this uncertainty has a major impact on investments that could be very costly to reverse once undertaken.

The literature suggests that irreversible investment takes place when expected discounted returns exceed user-cost by the opportunity-cost or option-value of the investment. Under uncertain conditions, firms undertake levels of irreversible investment that are lower than they would if it were possible to disinvest. Pindyck (1988, 1991) argued that most investments are lumpy and largely irreversible, making most investment-expenditures sunk costs. Firms also tend to delay irreversible investments in order to wait for new information about prices, costs, or market conditions. The existence of sunk costs in an uncertain future reduces the optimum level of investment expenditures. Hence, a high internal rate of return will be required to stimulate the expansion of such an industry.

Recent theoretical developments relating to investment under uncertainty thus highlight the importance of irreversibility and expected returns for the timing of investment expenditures. Most methods used to investigate the empirical implications of irreversibility of investment focus on the relationship between investment flows and proxy measures of uncertainty. At the aggregate level, the proxies normally used are the risk-premium computed from the interest-rate, or autoregressive conditional heteroskedasticity-estimates of conditional variances of inflation, real wages, and real profits. At the disaggregate level, the proxies include volatility of the exchange rate, real wages, material prices, and output prices from autoregressive moving-average-model residuals. The general conclusion is that increased uncertainty, at both aggregate and disaggregate levels, leads to lower investment rates. Caballero (1991) pointed out imperfect competition is a necessary condition for any irreversibility-driven negative relationship between investment and uncertainty.

According to Arrow (1985) and Nickell (1974) firms take into account future cost conditions and demand conditions when making investment decisions, since installed capital is valuable only to the extent that it is actually used in production. Demers (1991) pointed out that the interaction of irreversibility of investment in physical capital, anticipation of receiving information, and the currently unknown state of future demand, lead to cautious investment behaviour. This reduces investment levels and creates a time-varying risk-premium or marginal adjustment-cost, with gradual adjustment of the capital-stock to the desired level. The modelling of investment has recently been surveyed by Carruth et al. (2000).

Irreversibility of investment is not observable at the macro-level because data reveals only information about firms that have invested not about those with more capital than they might desire. What is observable are usually low variation and autocorrelated investment-series that do not show the importance of uncertainty and irreversibility. The series have low variation and show autocorrelation because observed values occur only when there is actual investment, implying that irreversibility is an irrelevant constraint. Hence uncertainty and irreversibility are often regarded as unimportant at the macro-level, and their negative effects are ignored.

If firm or industry fluctuations in uncertainty do not coincide then they are expected to cancel out at the aggregate level. However, it is possible that they do not cancel out since macro-economic factors such as uncertainty about future interest-rates, exchange-rates, and inflation rates, or shocks in monetary, fiscal, or regulatory policy, may be important in determining micro-level decisions. In addition, aggregate uncertainty may be generated or propagated by individual decision makers. Bernanke (1983) pointed out that if an individual-firm is uncertain about whether an aggregate demand-shock is transitory or permanent, then the decision to invest might be delayed in order to learn more.

In order to explain the low variation and autocorrelation of aggregate investment-series, most studies have focused on adjustment-costs under various assumptions, including the idea that technology and market structure make it costly to adjust capital-stock. Adjustment-costs are convex when capital-producing firms face decreasing returns, or when firms face increasing costs and invest frequently. However, unit-costs may be constant in the rate of investment, or decreasing when lump-sum adjustment-costs are present. Since industrial plant is difficult to convert to other users due to the specific nature of production processes, and the sale of used machinery faces thin markets and heavy discounts, installed capital is only useful when used in production. Thus when the cost for negative investment can be assumed infinite irreversibility can be regarded as another form of adjustment-cost.

Bertola and Caballero (1994) developed and tested a model of sequential irreversible investment that can link individual-firm uncertainty with the low variation and autocorrelation of macroeconomic investment-series. They determined the hypothetical path of reversible investment implied by the equality of the marginal-revenue-product of capital to its user-cost, and used it to predict the path of irreversible investment They showed that investment-processes that has low variation and autocorrelation at the aggregate level could be characterised by more volatile and intermittent investment-decisions at the micro-economic level.

In a less-developed country such as Zimbabwe, mining investments are more intermittent than those in more highly developed countries. It is thus likely that firms in less-developed countries will be more particular about the irreversibility of their investments.

This paper examines the effects of uncertainty and irreversibility on mining investments in Zimbabwe using the method proposed by Bertola and Caballero (1994) where regular investment-patterns are decomposed into their desired and irreversible components using the distribution of the deviations of capital from its desired level. Bertola and Caballero used a diffusion-process to model their investment-series in order to establish the deviations of capital from its desired level. Since investment in Zimbabwe is more intermittent, the model has here been modified to include a jump-process to account for periods when there are unusually high investment growth-rates. Results show that these uncertainty effects led to a reduction in investment expenditures.

Analysing the effects of uncertainty is important because it sheds light on investment decisions and the process of capital accumulation that must be taken into account when formulating policies to develop the mining sector. The importance of uncertainty suggests that it is important to reduce the volatility of the investment climate. Factors that affect the investment climate include prices, interest-rates, exchange rates, taxes, tariff structures, and regulatory policies.

The next section outlines the model, while Section 3 describes the estimation method. Data and definitions of variables are covered in Section 4. Section 5 then presents the estimation results, and Section 6 summarises and draws conclusions.

2. The model

The paper uses a model developed by Bertola and Caballero (1994), where a firm chooses an investment policy that maximises its present market-value depending on the price of capital goods, other business conditions, and the possibility of recovering investment expenditure when necessary to do so. The price of capital and other business conditions are assumed to be uncertain. The other business conditions are defined by demand, productivity, and the cost of optimally-used flexible factors of production other than capital. The productivity of the firm and the level of demand for its output have a positive effect on an index that measures its business conditions, whereas the cost of factors other than capital has a negative effect.

An investment policy that maximises the present value of the firm is

$$V(K(t), B(t), P(t)) \equiv \max_{\{G(\tau)\}} E_t \left\{ \int_t^\infty e^{-r(\tau-t)} (K(\tau)^\alpha B(\tau) d\tau - P(\tau) dG(\tau)) \right\} \tag{1}$$

s.t.

$$dK(\tau) = dG(\tau) - \delta K(\tau)\, d\tau \tag{2}$$

$$dG(\tau) \geq 0 \tag{3}$$

$$dB(\tau) = B(\tau)[\mu_1\, d\tau + \sigma_1'\, dW(\tau)] \tag{4}$$

$$dP(\tau) = P(\tau)[\mu_2\, d\tau + \sigma_2'\, dW(\tau)] \tag{5}$$

$$Y[K(\tau), B(\tau)] = K(\tau)^\alpha B(\tau) \tag{6}$$

$$0 < \alpha < 1$$

where V is the value of the investment programme; K is capital-stock; B is the index of business conditions; P is the price of capital; and G is cumulative investment. dW is a random increment of a two-dimensional Wiener-process defined by $\varepsilon_t \sqrt{dt}$ where ε_t is a serially-uncorrelated and normally-distributed random variable with zero mean and a standard deviation of one[1]. The process is two-dimensional because there are two sources of uncertainty: the price of capital; and business conditions. Y is revenue; α is elasticity; δ is a depreciation-parameter; r is the rate of discount; t and τ are time; μ_1 is the mean of the change in business conditions dB; μ_2 is the mean of the change in the price of capital dP; σ_1 and σ_2 are the respective standard deviations of dB and dP;[2] E is the expectations operator. The conditional-expectation E is taken at time t over the joint distribution of the processes of $B(\tau)$, $P(\tau)$, and $K(\tau)$.

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[1] For a small time-interval, dt, the change in the standard deviation is larger than the movement in the mean because $\sqrt{dt}$ is larger than dt.

[2] σ_1 and σ_2 have dimensions (2 x 1).

The integrand in equation (1) is the present value of revenue minus the cost of investment. Equation (2) shows that changes in capital occur through investment and depreciation. The distribution of the process of $K(\tau)$ is determined endogenously. The inequality (3) introduces irreversibility by constraining investment to non-negative values.

The business conditions index and price-processes are assumed to follow geometric Brownian-motions specified in equations (4) and (5), respectively. Their future values are assumed to be log-normally distributed with variances that grow linearly with time. The business conditions index, B_t, has an expected value of $E(B_t) = B_0 \exp(\mu_1 t)$ (where B_0 is its initial value). The index follows the stated process if demand, productivity, and the cost of flexible factors of production grow at some constant mean rate, μ_1. The price of capital, P_t, has an expected value of $E(P_t) = P_0 \exp(\mu_2 t)$ (where P_0 is its initial value). The price of capital follows the stated process if it grows at the constant mean rate, μ_2.

These processes are exogenous to the firm's problem. Mean-reversion processes have been considered the natural choice for commodities because values are expected to settle to long-run levels. Metcaff and Hasset (1995) empirically found that cumulative investment was unaffected by the use of a geometic Brownian-motion rather than mean-reversion. Given the assumptions of the model, the logarithm of the cumulative investment-process follows an arithmetic Brownian-motion.

The firm is assumed to have a constant-elasticity production-function and to face a constant elasticity-of-demand function that enables it to derive the revenue specified in equation (6). The assumptions conveniently simplify the optimisation problem in order to yield analytical solutions. For the firms' infinite-horizon problem to be finite, for given-capital and the expected-rate of deflation in the price of capital, the rate of return must be large relative to the growth-rate of operating profits.

If investment is unconstrained and reversible, then optimality-conditions imply that the marginal-revenue-product of capital equals its user-cost (Jorgenson, 1963), i.e.,

$$\frac{\partial Y}{\partial K} = \alpha \frac{Y}{K} = (r + \delta - \mu_2)P(t) \tag{7}$$

The neo-classical user-cost of capital is defined as the sum of the rate of discount, r, the rate of depreciation, δ, and the expected-rate of deflation in the purchase price of capital $-\mu_2$, multiplied by its price, $P(t)$. Capital, $K(t)$, is not a state-variable. Equations (6) and (7) yield the unconstrained and reversible capital-stock

$$K^u[B(t),P(t)] = \left(\frac{r+\delta-\mu_2}{\alpha}\frac{P(t)}{B(t)}\right)^{1/(\alpha-1)} \tag{8}$$

This equation holds if the opportunity-cost of capital equals operating cash-flows from production.

When investment is irreversible, the firm invests until its capital-stock reaches its desired level[3]. No investment takes place when the current level of capital-stock is higher than its desired level, but rather the capital-stock depreciates. Irreversibility makes firms undertake lower levels of investment than would be necessary to equate capital-stock to its desired level. The investment-functions are non-linear when the irreversibility-constraint is binding. Thus the marginal-revenue-product of capital must not exceed a constant proportion of its purchase price, $P(t)$, which condition can be expressed as[4]

$$\frac{\partial}{\partial K}Y(K(t),B(t),P(t))\begin{cases}\leq cP(t) & \forall t\\ = cP(t) & \forall t \quad \textit{such that}\; dG(t) > 0\end{cases} \tag{9}$$

where $\quad c \equiv r + \delta - \mu_2 + ½\, \Sigma^2 A.$ (10)

c is a constant of proportionality, the ratio of the flow of marginal profits to the purchase-price of capital. $\Sigma^2 \equiv (\sigma_1 - \sigma_2)' \, (\sigma_1 - \sigma_2)$ is the variance per unit of time in

[3] If $K^d(t)$ is the desired capital-stock, the firms invest until $K(t) = K^d(t)$. If $K(t) > K^d(t)$ then the irreversibility-constraint is binding and there is no investment

[4] This is a standard result obtained in most of the literature on irreversible investment. Precise results differ with model-specification.

the growth-rate of the process $\{B(t)/P(t)\}$. A is a strictly positive constant that solves the characteristic equation associated with the change in the value of the firm as a result of changes in both profit-flows and capital-gains.[5] As shown by Bertola and Caballero (1994), its value is defined by

$$\frac{A}{A-1}=\frac{r+\delta-\mu_2+\Sigma^2 A}{r+\alpha\delta-\mu_1} \tag{11}$$

where $\frac{A}{A-1}$ is the option-value multiple. Thus, the marginal-revenue-product of capital at which irreversible investment takes place is larger than the neoclassical user-cost of capital by the value $\Sigma^2 A$ when $\Sigma^2 > 0$.

Inverting the marginal condition (9) gives the firm's desired capital-stock,

$$K^d(B(t),P(t))=\left(\frac{c}{\alpha}\frac{P(t)}{B(t)}\right)^{1/(\alpha-1)} \tag{12}$$

In this case, capital-stock, K, is a state-variable in equation (1).

Equations (8) and (12) show that reversible and irreversible capital-stocks differ only by a constant of proportionality, c, so their dynamics coincide. This feature makes it possible to use the mean growth-rate of the implied reversible-ratio to estimate the predicted path of irreversible investment. The quotient of the reversible capital, K^u, to the irreversible capital, K^d, is $c/(r+\delta-\mu_2)$, which is decreasing in μ_1, and increasing in μ_2, δ, and Σ^2.

If the values of α, r, μ_1, μ_2 , σ_1, and σ_2 are the same for all firms, then the natural logarithmic value of frictionless capital, $\ln(K_i^u(t)) = k_i^u$, for each one of them follows a Brownian-motion such that $dk_i^u(t) = \theta\, dt + \sigma\, dW_i(t)$ where $W_i(t)$ is a univariate process constructed as a combination of the processes of $P_i(t)$ and $B_i(t)$; θ and σ are,

[5] The changes in profits are a result of changes in the drift and volatility of the price of capital and of the business-condition index.

respectively, the mean of the growth-rate of capital and the standard deviation of that rate. By Itô's lemma[6] the values of θ and σ can be calculated as

$$\theta = \frac{\mu_1 - \mu_2 - \frac{1}{2}(\sigma'_1\sigma'_1 - \sigma'_2\sigma_2)}{1-\alpha}$$

and

$$\sigma = \frac{\Sigma}{1-\alpha}$$

3. Estimation method

Even though the level of reversible capital is not observable, its hypothetical level can be inferred from capital-stock, production-data using observed values of revenue, the price of capital, and interest-rates.

The process followed by aggregate reversible investment is $dk_A^u(t) = \theta\, dt + \sigma_A\, dW_{Ai}(t)$ where $\sigma\, dW_i(t) = \sigma_A\, dW_{Ai}(t) + \sigma_I\, dW_{Ii}(t)$; σ_A is the aggregate component of the standard deviation of the expected growth of capital; and σ_I is the firm-specific component. Equation (8) can be used to calculate the level of reversible capital-stock, since $dW_{Ii}(t)$ will aggregate to zero. In the absence of individual-firm uncertainty, investment problems for individual-firms differ only due to their initial conditions. Equation (6) can be used to eliminate from equation (8) the index of business conditions, $B(t)$, which is not observable thus obtaining[7]

$$\ln K_i^u(t) = \frac{1}{1-\alpha}(\ln Y_i(t) - \ln b_i(t)) - \frac{\alpha}{1-\alpha}\ln K(t) + \frac{1}{1-\alpha}\alpha \qquad (13)$$

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[6] Itô's lemma: $dF = \frac{\partial F}{\partial V}dV + \frac{\partial F}{\partial t}dt + \frac{1}{2}\frac{\partial^2 F}{\partial V^2}\left[f(V,t)dt\right]$ where $f(V, t)$ depends on the adopted stochastic process of V.

[7] One can eliminate $B(t)$ in equation (8) using equation (6) then linearise by taking logs and using the equation $d\ln K(t) = \frac{dK(t)}{K(t)} = \frac{dG(t) - \delta K(t)dt}{K(t)}$ to yield equation (13). Equation (14) is then obtained by taking differences.

$$\Gamma^*_i(t) = \frac{1}{1-\alpha}(\Delta \ln Y_i(t) - \Delta \ln b_i(t)) - \frac{\alpha}{1-\alpha}\Gamma_i(t) + \frac{1}{1-\alpha}\delta \quad (14)$$

where $b = (r + \delta - \mu_2)P(t)$; $K(t)$ is the capital-stock; $K^u(t)$ is the reversible capital-stock; $dG(t)$ is investment; $dK^u(t)$ is reversible investment; $\Gamma(t) \equiv \frac{dG(t)}{K(t)}$ is the investment-ratio; and $\Gamma^*(t) = \frac{dK^u(t)}{K(t)} + \delta$ is the reversible investment-ratio plus depreciation. The relationships are assumed to hold at both the individual and aggregate levels, with the subscripts dropped from the latter. Equation (14) shows that the hypothetical reversible investment-ratio, Γ^*, is proportional to the observed level, Γ, adjusted for depreciation, δ, when the growth-rate of the revenue of capital, $\Delta\ln(Y)$, and that of the cost of capital, $\Delta\ln(b)$, are equal.

If the irreversibility-constraint is binding, then investment-functions are non-linear, and direct logarithmic aggregation is not possible because individual-firms' responses to uncertainty are relevant. Not all the firms will be able to maintain their capital-stock at the desired level, but every firm is expected to invest when its capital-stock is below that desired level. Thus, aggregate investment depends on the distribution of the firms between those with more capital than they desire and those with less. Since the actual numbers of firms in each category are not available, Bertola and Caballero (1994) suggested using stochastic aggregation to derive a cross-sectional distribution-function of the deviation of capital from its desired level. For each firm, the deviation of the logarithm of the capital-stock from its desired irreversible level is given by $s_i \equiv k_i(t) - k^d_i(t)$, where s_i is log-deviation[8]. Each deviation will be a Brownian-motion with standard deviation σ_I and drift $\mu \equiv (-\theta + \delta)$. Bertola and Caballero show that the limiting steady-state of the cross-sectional density is exponential.

At the aggregate level, the mean of the density-function would be the average deviation of capital-stock from its desired irreversible level. Thus, the deviation of the

[8] $ds = \begin{cases} -\delta\, dt - dk^d(t) & when \quad dG(t) = 0 \\ 0 & otherwise. \end{cases}$ if s can be broken down into sufficiently small intervals compatible with individual investments that in aggregate fail to equate K and K^d.

logarithm of the capital-stock from its desired irreversible level, $\overline{s}(t) \equiv k(t) - k^d(t)$, where $\overline{s}(t)$ is log-deviation, is calculated using the mean value obtained from the density. In differential form we have $dk(t) = dk^d(t) + d\overline{s}(t)$ which can be used to compute the investment-ratio

$$\Gamma(t) = \hat{\Gamma}(t) + d\overline{s}(t) \tag{15}$$

where $\hat{\Gamma}(t) = \frac{dK^d(t)}{K(t)} + \delta$ is the irreversible investment-ratio.

The equation shows that the investment/capital ratio differs from its desired irreversible-ratio by the change in the average deviation, $d\overline{s}(t)$.

It is assumed that when there is no uncertainty and there are no problems with synchronising and timing investment i.e. $d\overline{s}(t) = 0$, then capital-stock would be maintained at its desired level. If the standard deviation of the desired investment-process is large then changes in that deviation will smooth out the responsiveness of the investment-ratio, Γ, to its desired level, Γ^*. One can use the equation to estimate the predicted irreversible investment-path when the deviations are not zero.

The path of the cross-sectional density is obtained from the solution of the forward Kolmogorov-equation

$$\partial_t f(s,t) = \tfrac{1}{2}\sigma^2\, \partial_{ss} f(s,t) - \mu\, \partial_s f(s,t)$$

However, this equation does not take into account the intermittent nature of the investment-process in Zimbabwe. In order to account for it, one can modify the equation to include a jump-process. Das and Sundaram (1999), Das (2002), and Martzoukos (2003) showed that jump-processes enhance diffusion-models by capturing empirical features of data that the diffusion-models do not. Most studies of these enhanced models are simulation based.

The new equation is

$$\partial_t f(s,t) = \tfrac{1}{2}\sigma^2\, \partial_{ss} f(s,t) - \mu\, \partial_s f(s,t) + \varphi\, E[f(s+J) - f(s)] \tag{16}$$

where J is a jump-term; φ is the frequency of the jumps; and E is an expectations-operator. The stochastic differential for ds is

$$ds = \theta dt + \sigma dW + J d\xi(\varphi)$$

where ξ is the process that governs the jumps. This procedure is analogous to the use of shift-variables in ordinary regression-equations.

Appendix 1 shows how to derive the probability-density that is a solution to equation (16)

$$f(s,t) = \sum_{n=0}^{\infty} A_n f(s) e^{-\lambda_n t} \quad - \quad \frac{2\varphi MJ}{\mu}$$

$$\lambda_0 = 0$$

$$\lambda_n = \frac{\sigma^2}{2}\left(\beta^2 + \frac{\varepsilon^2}{4}\right)$$

$$b = \frac{n\pi}{S}$$

$$f_0 = e^{-\varepsilon s}$$

$$f_n = e^{-\frac{1}{2}\varepsilon s}\left[Cos(\beta s) - \frac{\varepsilon}{2\beta} Sin(\beta s)\right] \qquad n = 1, 2, ..$$

subject to the initial conditions

$$\sum_{n=0}^{\infty} A_n f_n(s) - \frac{2\varphi MJ}{\mu} = \bar{g}(s) = f(s,0)$$

The values for ε are given by[9]

$$\varepsilon = \frac{2\Gamma^*}{\sigma_I^2} \tag{17}$$

A's are functional constants that have to be estimated so that the initial conditions are satisfied. As $n \to \infty$ the functional constants converge to zero, the λ's diverge to positive infinity, and the series for $f(s,t)$ can be truncated to give an approximation for $f(s,t)$. The solution-equations can be numerically approximated for different time periods and the results used to calculate the deviations in equation (15).

Since $\lim_{s \to \infty} f(s, t) = 0$, one can choose a value at which to truncate the limit-value S. Good values were found by observing those generated by the limiting distribution of the cross-sectional density for the highest and lowest values of ε for each mineral. The functional constants, A, were calculated over a set of 500 grid-points on the range 0 to S.

The mean of the distribution, $\bar{s}(t)$ at each point in time was obtained as the sum of the products of each point on the grid, $s(t)$ and its probability, f(s,t), then used to calculate the deviation, $d\ \bar{s}(t)$. The solution outlined above was a set of recursive relations that, at discrete times, track the convergent path of the cross-sectional density to its stable form. It links successive observations of the investment-ratios and will predict an irreversible investment-series that matches the observed one. The cross-sectional density at the end of each period was treated as the initial condition for the next period. Since individual-firm uncertainty, σ_I, was unknown, different values of the deviation were tried until one was found to match the standard deviations of predicted irreversible investment ratios and the observed investment-ratios. SAS procedures were used to do the numerical approximations.[10]

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[9] The equation giving the value of ε in the paper by Bertola and Caballero (1994) shown as $\varepsilon = -\frac{2\Gamma^*}{\sigma_I^2}$, has a typographical error. This specification is inconsistent with positive density-functions specified in their equation (19) on page 232. The specification of $A(\beta)$ has another typographical error but the $A(\beta; h)$ function in their equation (B13) is the correct one.

[10] Numerical integration could be used to track the same densities using the specification in the paper by Bertola

4. Data and definition of variables

This paper uses annual aggregate data for five mineral products – chrome, copper, gold, asbestos and iron – for which data were available for the 27 years 1969-1995; the total number of observations is thus 135. The series stop in 1995 because some of the sectors were aggregated after that. The major source of data was the Zimbabwean Central Statistical Office. There should not be much bias in the aggregation since the commodities can reasonably be viewed as homogeneous, except for the differences in ore grades that fetch different prices on the market. The data seem to be also of a reasonable quality for purposes of estimation and inference.

The variables for the model are revenue; the elasticity-coefficient, α; the price of capital; aggregate investment; capital-stock; and capital-costs and jump-parameters.

The revenue variable was measured by the gross output of the respective mining firms. Since equation (12) was used to calculate an approximation of the business-index it could not be used to calculate the elasticity coefficient, α. However, using production data and following Bertola and Caballero (1994) its value was inferred indirectly from the ratio of the share of investment in value-added and the mark-up coefficient (elasticity of capital = $\frac{share\ \ of\ \ investment}{1\ +\ markup-coefficient}$). The average coefficients were 0.10 for chrome and iron, 0.11 for copper, gold, and asbestos.

The ratio of the implied investment and GDP-deflators was used to calculate the price of capital, P_i. National-accounts data were used to calculate the deflators as the ratios of their respective values in current and constant prices. It was assumed that the user-cost of capital facing the firms was the same for all the sectors during any particular year. This is justifiable because the extraction-process is usually done with similar types of equipment and firms classified in the mining industry are generally engaged

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and Caballero (1994). However, in some cases there are breakdowns in the algorithm over some regions of integration, and one has to define different functional constants over ranges of the sub-integrals of s. Since discrete densities can be used to approximate the continuous versions, this paper uses them. Thus there is no need to define the functional constants over intervals of s, but only for specific values of s.

only in extraction-activity. The user-cost of capital is $P_k = P_i\,(r+\delta-\dot{P}_i)$ where r is the real interest-rate, δ is the depreciation-rate of capital, and $\dot{P}$ is the growth of the price of capital. As in other similar studies, a 5% depreciation-rate of capital was assumed. The cost of capital for the firm was taken as the stock of capital multiplied by the user-cost of capital. The average rate of interest during the sample-period was 4.4%.

Capital-stock was defined as cumulative investment plus a benchmark-level of capital-stock at the beginning of the sample-period. The benchmark capital was calculated using the value of capital that stabilised the growth-rate of the investment-rate series, using multiples between 1 and 46 times the investment in the first year, 1969. The drawback of using this method is that using a multiple with a low value creates an impression that there was a high and increasing growth-rate of capital-stock when there might not have been. Using a high value for the multiple has the opposite effect. The effects can be graphed and the value at which they stabilise can be observed. The multiple of 25 times the value of investment in 1969 gave the most stable investment-ratios[11].

Most firms had been in the extraction-business for some years prior to 1969, so it is likely that their levels of capital were higher than the benchmark-capital calculated for that year. Documented development of Zimbabwe's mineral potential started about 1923, by settlers from Europe. Mining of chrome, iron, and other ferrous metals, copper, and gold increased during the years 1939 and 1945. However, production data for the minerals separately presented started only in 1969 for Zimbabwe. This means that mining operations had been in existence for over 46 years, during which period there must have been huge investments, because start-up capital for mining operations is generally high.

As can be seen in figures 1-5 below there were three years in which there were no investments in chrome; otherwise there, was positive investment for all years for all

[11] An option is to obtain benchmark-capital using cost-ratios suggested in UNIDO investment-studies. These worked well only for chrome, which suggested that the implied cost of capital did not match the Zimbabwean value in 1969.

minerals, which would imply that the irreversibility constraint is irrelevant. The patterns are characterised by some high values that cannot be modelled using a diffusion-process, but should include a jump-process, which can be used to augment the diffusion-process used to account for low variation in the investment-series.

Figure 1: Chrome investment-ratios, 1969-95, actual

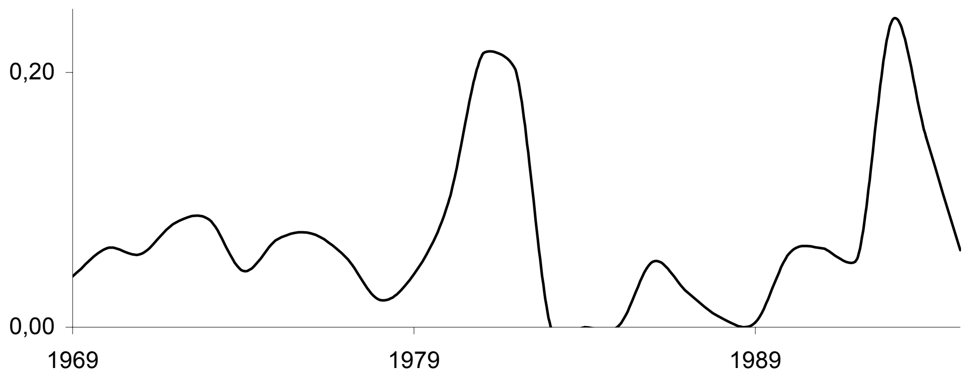


Figure 2: Copper investment-ratios, 1969-95, actual

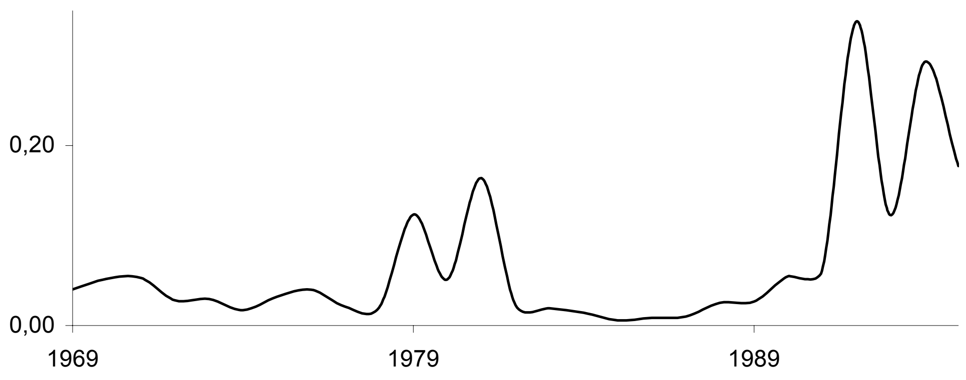


Figure 3: Gold investment-ratio, 1969-95, actual

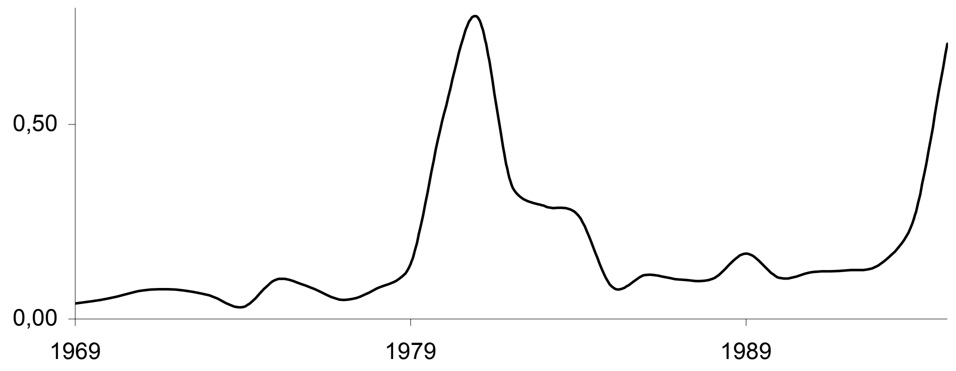


Figure 4: Asbestos investment-ratios, 1969-95, actual

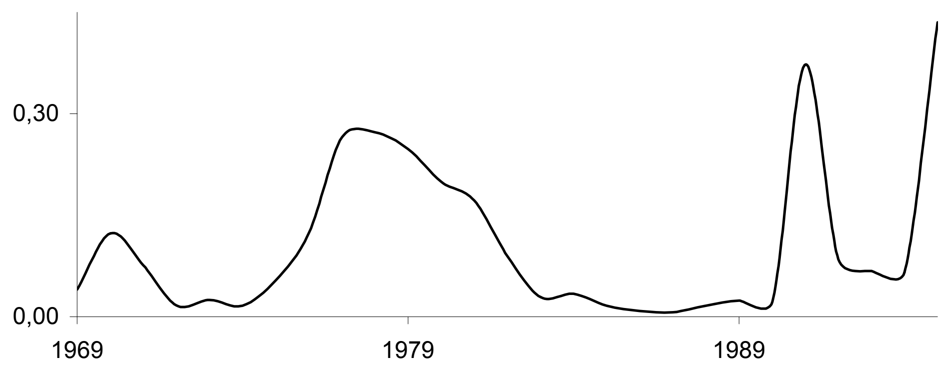
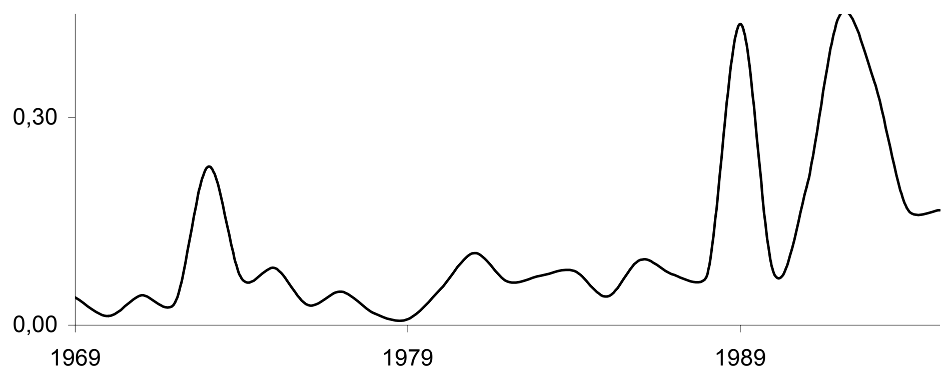


Figure 5: Iron investment ratio,1969-95, actual



Chrome had an average investment-ratio of 0.06 for the periods 1969-1980 and 1995, and 0.03 for 1983-92 (Table 1), which are lower than those observed for 1981-82 (0.21) and for 1993-94 (0.20). This means that there were major investments in 1981-82 and 1993-94. For the other minerals as well, there were periods of major investment outlays, in 1979, 1981, and 1992-95 for copper; in 1980-84 and 1994-95 for gold; in 1977-81, 1991, and 1995 for asbestos; and in 1973, 1989, and 1991-95 for iron.

Table 1: Mean, standard deviation, and trend for investment-ratios

	period	average	standard deviation
chrome:	69-80	0.06	0.02
	81-82	0.21	0.01
	83-92	0.03	0.03
	93-94	0.20	0.06
	95	0.06	
copper:	69-78	0.03	0.01
	79	0.12	
	80	0.05	
	81	0.16	
	82-91	0.02	0.02
	92-95	0.23	0.10
gold:	69-79	0.07	0.03
	80-84	0.44	0.21
	85-93	0.12	0.03
	94-95	0.48	0.32
asbestos:	69-76	0.06	0.04
	77-81	0.23	0.04
	82-90	0.03	0.02
	91	0.37	
	92-94	0.07	0.01
	95	0.44	
iron:	69-72	0.03	0.01
	73	0.23	
	74-88	0.06	0.03
	89	0.44	
	90	0.08	
	91-95	0.27	0.13

The high levels of investment during those years raise the average values of the growth of capital for the sample-period. However, apart from these few episodes, the investment-process varies around generally low mean-values. As can be seen from

Table 1, the average levels of investment during the remaining periods were 0.03 during 1969-1978, 0.05 in 1980, and 0.02 during 1982-1991 for copper. Similar patterns can be observed for gold, asbestos and iron. Except for iron, the average length of the periods in which investment was low was almost 10 years. The standard deviations for all the years are 0.06 for chrome, 0.08 for copper, 0.19 for gold, and 0.12 for asbestos, and iron (Table 2). There is a positive serial correlation in the investment-ratios with values of 0.43 for chrome, 0.40 for copper, 0.62 for gold, 0.36 for asbestos, and 0.39 for iron. These values do not suggest that the irreversibility-constraint was binding although the constraint would be expected to be relevant to investors.

Table 2: Summary statistics

				auto-correlation coefficient	
	standard deviation	skewness	kurtosis	estimate	Standard deviation
chrome	0.06	1.41	4.33	0.43	0.03
copper	0.08	2.04	6.33	0.40	0.04
gold	0.19	1.97	5.91	0.62	0.001
asbestos	0.12	1.38	3.94	0.36	0.07
iron	0.12	1.74	4.97	0.39	0.05

5. Estimation and Results

Equation (14) was used to calculate the hypothetical reversible investment-ratios for the five mineral products. For purposes of comparison, the observed deviations from the mean of the investment-ratios of the high values were used for the years with unusually high levels of growth as required by equation (16). In the years when the hypothetical investment-ratio was below the observed level some of the mining firms would have sold assets. If the value of assets sold by disinvesting firms were larger the value of investment (by investing firm) then the hypothetical investment-ratios would be negative at the aggregate level.

The negative values have implications for the procedure used to derive the cross-sectional densities. The investment-ratios (growth-rates) must be positive in order to yield densities with positive values. The solution to this problem was to redefine the ratios such that their values would be above 0. Mathematically, the adjustment should not change the gradient used to calculate the necessary deviations.

The hypothetical reversible investment-ratios have higher standard deviations than did the observed investment-ratios, almost double, as shown in Table 3.

Table 3: Summary statistics for observed, hypothetical reversible, and predicted irreversible investment-ratios

				autocorrelation	
		mean	standard deviation	coefficient	Standard deviation
observed:	chrome	0.04	0.03	0.43	0.03
	copper	0.03	0.02	0.39	0.05
	gold	0.08	0.04	0.37	0.06
	asbestos	0.04	0.03	0.26	0.20
	iron	0.05	0.03	0.29	0.15
hypothetical reversible:	chrome	0.05	0.06	0.15	0.46
	copper	0.05	0.07	-0.09	0.67
	gold	0.05	0.08	-0.01	0.96
	asbestos	0.05	0.08	-0.13	0.52
	iron	0.05	0.08	-0.21	0.31
predicted irreversible:	chrome	0.03	0.03	0.19	0.37
	copper	0.03	0.02	0.06	0.78
	gold	0.05	0.04	0.05	0.82
	asbestos	0.05	0.03	0.01	0.94
	iron	0.04	0.03	-0.07	0.74

The correlation-coefficient values for the hypothetical reversible investment ratios are lower than the observed values for all the minerals. The variation in the coefficients is sometimes accounted for using convex adjustment-cost functions. However, the procedure used in this paper was to make these differences partly a result of the mathematical properties of a process derived from the uncertainty of the investment environment. Figures 6-10 show that the hypothetical reversible investment-ratios were much more variable than the actual investment-series. The hypothetical reversible investment-ratios are shown with dashed lines, and the observed ratios with solid lines. The more extreme patterns of the hypothetical series are a reflection of the mathematical properties of the equations used to derive it.

Figure 6: Chrome investment ratios, 1969-95

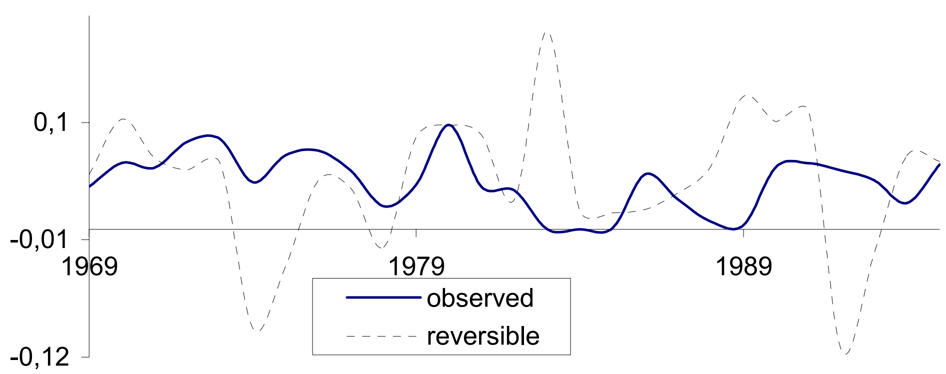


Figure 7: Copper investment ratios, 1969-95

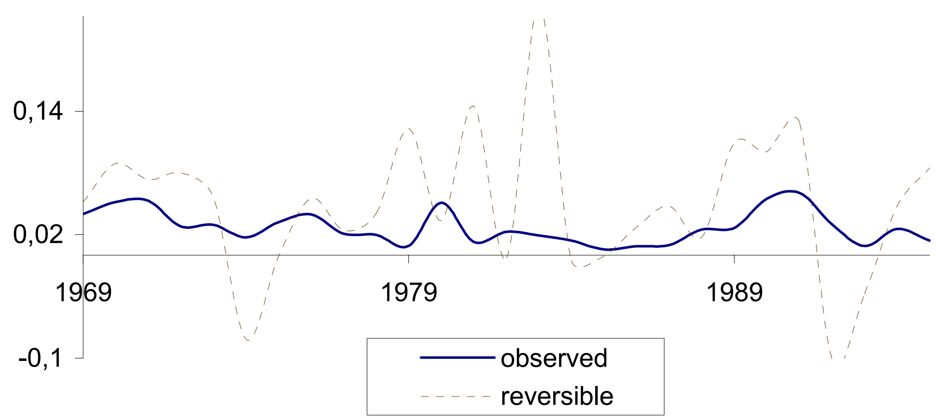


Figure 8: Gold investment ratios, 1969-95

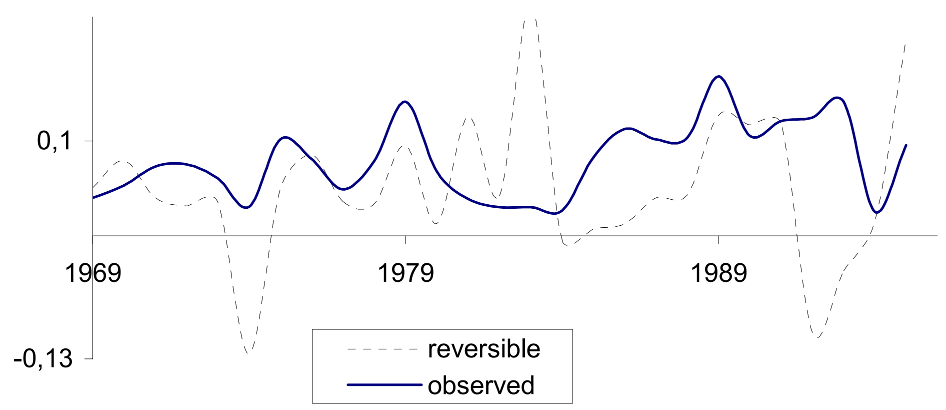


Figure 9: Asbestos investment ratios, 1969-95

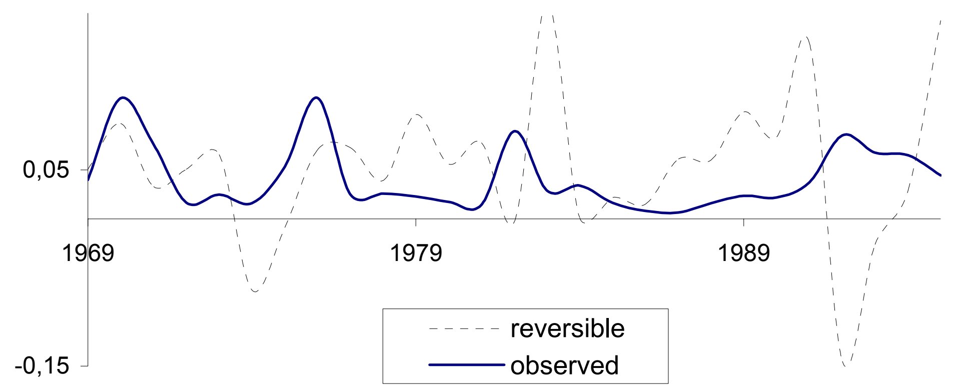
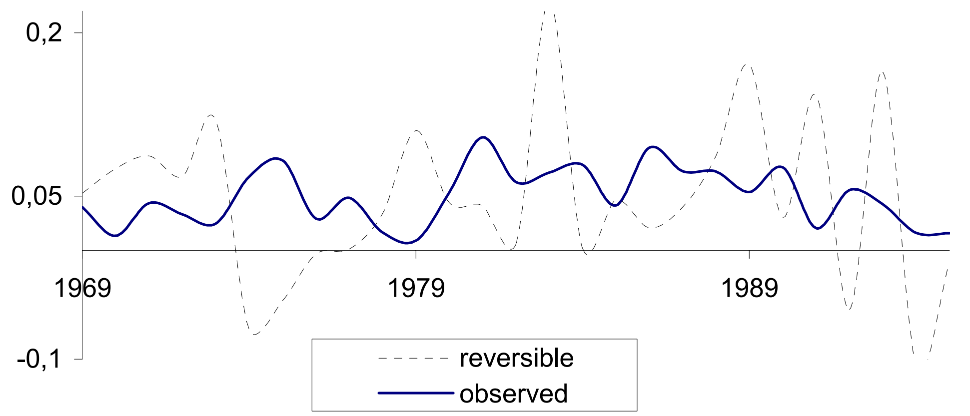


Figure 10: Iron investment ratios, 1969-95



In order to estimate the deviation of the predicted investment ratios, $\hat{\Gamma}$, from the observed ones, using the hypothetical reversible investment-ratios, Γ^*, one needs to calculate the values of the parameter, ε, using the ratio of Γ^* to deviation caused by individual firm uncertainty, σ_I (equation 17). The values of σ_I ranging between between 0 and 1 were used scale down the growth-rate of the hypothetical reversible investment-ratios, Γ^*. Values of σ_I close to 1 imply high levels of uncertainty. σ_I values of 0.21 for chrome, 0.166 for copper, 0.241 for gold, 0.283 for asbestos chrome, and 0.207 for iron make the ratios of the standard deviations of observed investment-ratios and the predicted reversible investment- ratios of equal to unity. The standard deviations are 0.06 for chrome, 0.02 for copper, 0.04 for gold, 0.10 for asbestos, and 0.04 for iron.

Table 4: Standard deviations of observed and predicted investment ratios and the standard deviation of s_i, (σ_I)

	Observed and predicted investment ratios	Individual firm uncertainty, (σ_I)
chrome	0.04	0.210
copper	0.03	0.166
gold	0.08	0.411
asbestos	0.04	0.283
iron	0.05	0.207

The deviations $d\overline{s}(t)$ of the irreversible investment-ratios from the observed ratios were estimated using the solution to equation (16). Adding the deviations to the hypothetical investment-ratios gives the predicted irreversible investment-ratios shown in figures 11-15. Predicted irreversible-ratios are shown with dashed lines, and the observed ratios again with solid lines. Thus a decomposition of the investment-series into desired investment levels and deviations from the observed levels can be used to reconcile the two to some extent, without taking into account other forms of adjustment-costs.

Figure 11: Chrome investment ratios, 1969-95

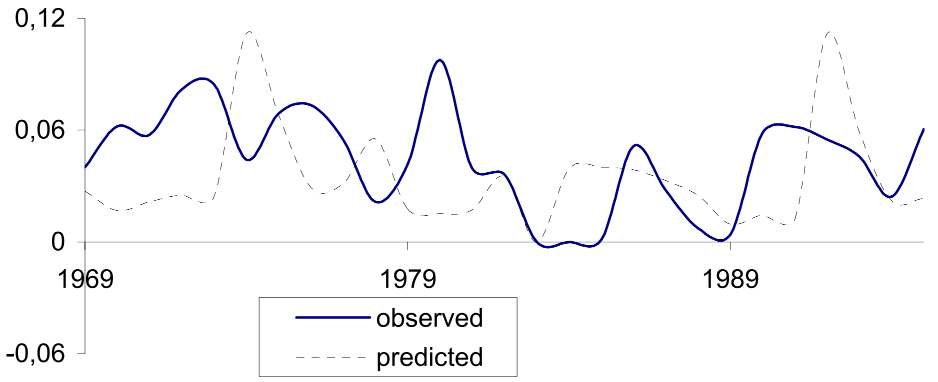


Figure 12: Copper investment ratios, 1969-95

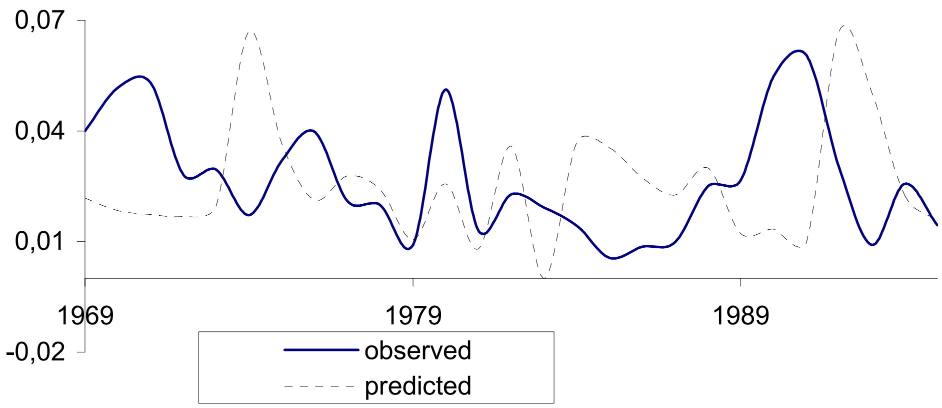


Figure 13: Gold investment ratios, 1969-95

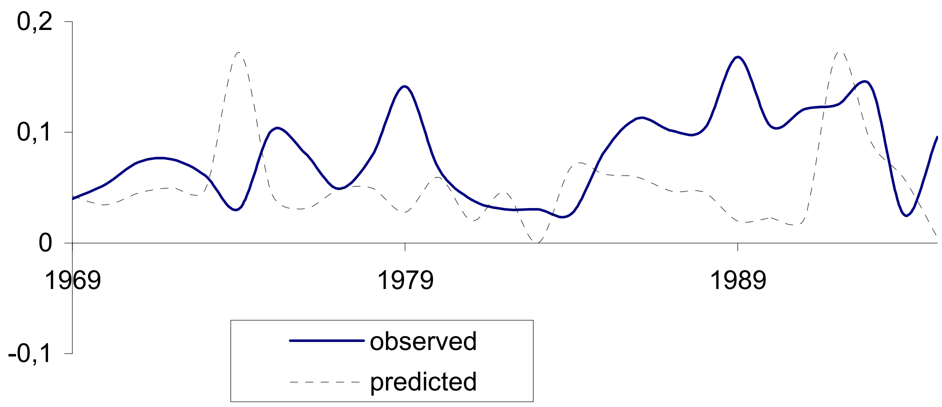


Figure 14: Asbestos investment ratios, 1969-95

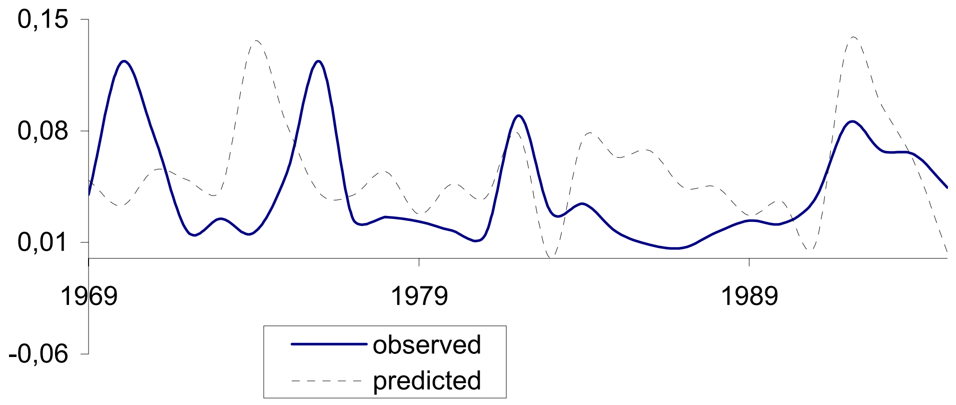
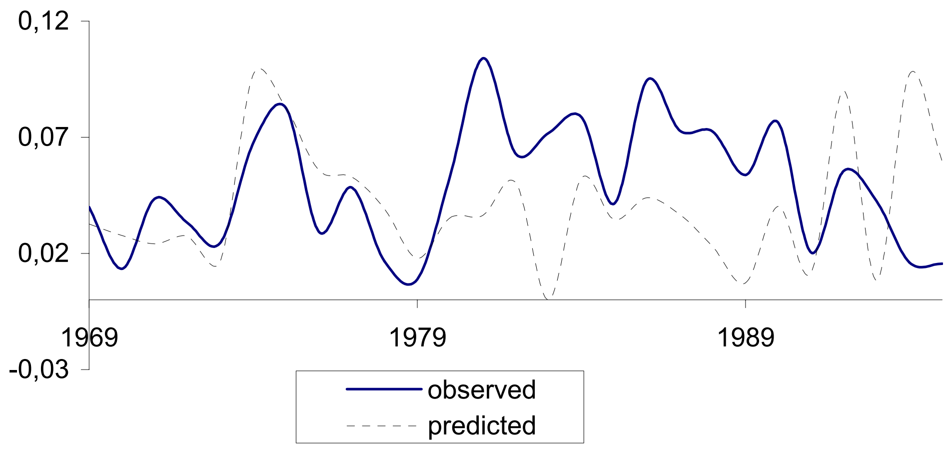


Figure 15: Iron investment ratios, 1969-95



The relationship between the actual capital and irreversible capital can be analysed further using equations (8) and (12), and using the values of c, A, μ_1, Σ^2, σ_1 and σ_2. The values of c, and A are calculated using equations (10) and (11), and using values of μ_1 and Σ^2 obtained as outlined below.

The logarithmic rates of annual growth of the revenue of capital growth are -0.03 for chrome, -0.04 for copper, 0.05 for gold, 0.02 for asbestos and 0.10 for iron (Table 5). Using these values, together with the corresponding growth rates of the capital, θ, (equivalent to the average of the observed investment ratios), and using the relationship between the logarithmic growth rates of capital, marginal revenue of capital, and the business conditions index defined by equation (6), gives the implied rate of growth of the business conditions index, μ_1 shown in Table 5. The table also shows the values of Σ^2 calculated using the $\sigma_I = \frac{\Sigma}{1-\alpha}$.

Table 5: Parameters used to compare actual, hypothetical reversible, and predicted irreversible capital stocks

	g_{Rk}	μ_1	μ_2	σ_I	Σ^2	A	½ AΣ^2	c	K^d/K^u
chrome	-0.03	-0.04	0.01	0.21	0.04	1.12	0,02	0.062	0.61
copper	-0.04	-0.04	0.01	0.17	0.02	1.67	0,02	0.057	0.68
gold	0.05	0.05	0.01	0.41	0.13	0.59	0,04	0.079	0.47
asbestos	0.02	0.02	0.01	0.28	0.06	0.90	0,03	0.067	0.56
iron	0.10	0.09	0.01	0.21	0.03	1.31	0,02	0.060	0.64

Notes: g_{Rk} = average growth of revenue of capital. Other parameters are as defined previously.

The results show that the marginal-revenue-product of capital at which irreversible investment took place was larger than the neoclassical user-cost of capital (value 0.04 for all minerals) by values of ½ AΣ^2 greater than 0.02 (2%). In addition the ratio of the desired irreversible capital stocks (equation 12) to the hypothetical reversible capital stocks (equation 8) was less than 0.68. This result implies that the desired irreversible capital stocks would be less than 68% of the hypothetical reversible level when there was positive investment. For gold the desired capital stocks were 47% of the hypothetical reversible levels.

The above results are obtained at the aggregate level where the covariance of the ratio of the price of capital to the business conditions index has a specific value. However, it is not possible to obtain the values of σ_1 and σ_2 separately from the equation $\theta = \frac{\mu_1 - \mu_2 - \frac{1}{2}(\sigma'_1\sigma'_1 - \sigma'_2\sigma_2)}{1-\alpha}$ as would be necessary to obtain the range of values that the growth of capital might take. However, it can be shown that when the volatility of the price of capital relative to the business conditions index is greater than unity then the irreversible capital stocks would be much lower than the values obtained above.

6. Summary and conclusion

The level of mining investment in Zimbabwe was lower than the level that would equate the user costs to marginal revenue of capital, The level would be expected to be even lower if the variation factored out by the jump process is included. The level of investment was lower because of individual-firm uncertainty that did not cancel out at the aggregate level. Macroeconomic policies that are not sensitive to the negative effects of uncertainty of investment would thus depress the level of investment, a feature that is not apparent in observed low variation and autocorrelated macroecomic investment series. There is a tendency to ignore the negative effects since investment would appear to be insensitive to policy-changes. Since investment in Zimbabwe is more intermittent than the case explained by the model, much lower levels of investment were undertaken due to uncertainty and irreversibility.

The model used does not take into account other types of adjustment costs that are important and do affect investment spending. In addition the model variation used for estimation did not track back to the Bellman equation for the irreversible investment problem the effects of including jump processes. The form of solution would become more complicated than the one used in this paper and perhaps more intractable. However, the results do indicate that there was lower investment due to uncertainty and irreversibility of investment before taking into account all the other important factors into account.

The analysis here shows that the values of the deviation of individual-firm uncertainty that make predicted irreversible investment behaviour match observed investment were above 0.16 whereas the actual investment-series had far lower volatility, less than 0.04. The values show the importance of reducing uncertainty in the investment climate. The results imply that the irreversible capital stock was less than 68% of the reversible level when there was positive investment. However, the study still leaves unexplained the functional form that can be used to derive σ_I. It also does not incorporate other aspects of adjustment-costs, that might be important. However, the results show that if the level of uncertainty is not taken into account, it is highly unlikely that substantial investment will be undertaken in mining in Zimbabwe.

Appendix 1: Deriving the path of the cross-sectional distribution and its mean.

Let $f(s, t)$ denote the probability-density of the process $s(t)$ with stochastic differential

$$ds = \theta dt + \sigma dW + Jd\xi(\varphi)$$

The path of the cross-sectional distribution can be found from the solution of the forward Kolmogorov equation

$$\partial_t f(s,t) = ½\sigma^2 \partial_{ss} f(s,t) - \mu \partial_s f(s,t) + \varphi E[f(s+J) - f(s)] \quad \text{(A1)}$$

with boundary and initial conditions

$$f(0, t) = \frac{\sigma^2}{2} f_s(0, t) + \varphi E[f(s+J) - f(s)] \quad \text{(A2)}$$

$$f(S, t) = \frac{\sigma^2}{2} f_s(S, t) + \varphi E[f(S+J) - f(S)] \quad \text{(A3)}$$

and initial conditions

$$f(S, 0) = \bar{g}(s) \qquad \sum_0^S \bar{g}(s) = 1 \quad \text{(A4)}$$

$$\text{Lim}_{s\to\infty} f(s, t) = 0$$

Separating the variables s and t we write

$$f(s, t) = g(s)\, h(t) + E[f(s+J) - f(s)] \quad \text{(A5)}$$

Differentiating (A5) partially and substituting the results in equation (A1) we can obtain the equations in the t and s directions:

$$h'(t) + \lambda h(t) = 0 \quad \text{(A6)}$$

$$g''(\text{s}) - \frac{2\mu}{\sigma^2} g'(s) + \frac{s\lambda}{\sigma^2} g(s) + \frac{2E\left[f(s+J) - f(s)\right]}{\sigma^2} = 0 \quad \text{(A7)}$$

where λ is a constant. Equation (A6) has the solution $h(t) = h_0 e^{-\lambda t}$.

We can define a simple function $f(s; J) = Ms + M_0$ where M and M_0 are constants such that the expectation in the jump-process is not zero. Since the data here has identifiable jumps it was not necessary to construct a probability-distribution for them, as the simple jump suffice for the known points where the function had large deviations. Hence

$$f(s + J) - f(s) = E[f(s + J) - f(S)] = MJ$$

Equation (A7) reduces to

$$g''(s) - \frac{2\mu}{\sigma^2} g'(s) + \frac{s\lambda}{\sigma^2} g(s) + \frac{2MJ}{\sigma^2} = 0 \qquad \text{(A8)}$$

The equation (A8) has the complete solution $g(s) = v_1(s) + g^*$, where $v_1(s)$ is the solution to the characteristic equation for A(8) and g^* is the particular solution,

$$g^* = -\frac{2\varphi MJ}{\mu}.$$

The characteristic equation for (A8) has real solutions for $\lambda \le \frac{\varepsilon\mu}{4}$ where $\varepsilon = \frac{2\mu}{\sigma^2}$ and can be written as

$$g(s) = A_1 e^{\alpha_1 s} + A_2 e^{\alpha 2 s} \qquad \text{(A9)}$$

which gives

$$g(s; \lambda = 0) = \varepsilon e^{-\varepsilon s}$$

However, when $\lambda > \frac{\varepsilon\mu}{4}$ we have complex roots and the solution has the form

$$g(s;\lambda) = e^{-\frac{1}{2}\varepsilon s}\left[Cos(\beta s) - \frac{\varepsilon}{2\beta} Sin(\beta s)\right] \qquad \text{(A10)}$$

where

$$\beta(\lambda) \quad = \quad \sqrt{-\varepsilon\left(\frac{\lambda}{\mu}+\frac{\varepsilon}{4}\right)} \quad = \quad \frac{n\pi}{S} \qquad \text{and,} \qquad n = 1, 2, \ldots$$

The probability-density that we obtain as a solution to equation (16) is given by

$$f(s,t) = \sum_{n=0}^{\infty} A_n f(s) e^{-\lambda_n t} \; - \; \frac{2\varphi MJ}{\mu}$$

$$\lambda_0 = 0$$

$$\lambda_n = \frac{\sigma^2}{2}\left(\beta^2 + \frac{\varepsilon^2}{4}\right)$$

$$b = \frac{n\pi}{S}$$

$$f_0 = e^{-\varepsilon s}$$

$$f_n = e^{-\frac{1}{2}\varepsilon s}\left[Cos(\beta s) - \frac{\varepsilon}{2\beta} Sin(\beta s)\right] \qquad n \; = \; 1, 2, ..$$

subject to the initial conditions

$$\sum_{n=0}^{\infty} A_n f_n(s) - \frac{2\varphi MJ}{\mu} = \overline{g}(s) = f(s,0)$$

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